Productivity and Misallocation in General Equilibrium

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Aggregation Theorems for Efficient Economies

- For efficient economy, Solow (1957):

\[ \frac{d \log Y}{d \log TFP} = \Lambda_L \frac{d \log L}{d \log K} + \Lambda_K \frac{d \log K}{d \log TFP}. \]

- For efficient economy, Hulten (1978):

\[ \frac{d \log TFP}{d \log A_k} = \sum \lambda_k \frac{sales_k}{GDP}, \quad \text{where} \quad \lambda_k = \frac{sales_k}{GDP}. \]

- Ex-ante (structural counterfactuals) and ex-post (growth accounting) content.
What We Do

- Extend these results to inefficient economies and other shocks.
- General reduced-form, non-parametric formula.
- Mapping from micro to macro using a general structural model.
  - micro wedges.
  - structural micro elasticities of substitution.
  - returns to scale.
  - factor market reallocation.
  - network linkages.

- Wide range of applications in different contexts: sources of TFP growth, impact of misallocation, macro impact of micro shocks, effects of monetary policy with nominal rigidities, etc.

- Some selected numbers:
  - 50% of TFP growth 1997-2014 from improved allocative efficiency.
  - 20% rise in TFP from eliminating markups.
Related Literature

- **Efficient Network Production Economies:**

- **Inefficient Network Production Economies:**

- **Misallocation**
Related Literature

- **Falling Labor Share, Increasing Markups, Productivity Slowdown:**

- **Nominal Rigidity with intermediate inputs:**
Agenda

General Non-parametric Result

General Parametric Result

Applications
  Growth Accounting
  Quantitative Model

Extensions (see paper)

Conclusion
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General Framework

- Final demand as maximizer of homothetic aggregator:
  \[ Y = \mathcal{D}(c_1, \ldots, c_N), \]
  with \( c_k \) final consumption of good \( k \).

- Budget constraint:
  \[
  \sum_k (1 + \tau^c_k) p_k c_k = \sum_f w_f F_f + \sum_k \pi_k + \tau,
  \]
  with \( p_k \) prices, \( \pi_k \) profits, \( \tau^c_k \) consumption wedges, \( w_f \) wages, \( F_f \) factors, \( \tau \) lump-sum rebate.
General Framework

- Good $k$ produced with constant-returns cost function:

$$\frac{y_k}{A_k} C_k \left( (1 + \tau_{k1}) \rho_1 , \ldots , (1 + \tau_{kN}) \rho_N , (1 + \tau_{k1}^f) w_1 , \ldots , (1 + \tau_{kF}^f) w_F \right),$$

with $y_k$ total output, $A_k$ Hicks-neutral productivity shock, $\tau_{kl}$ input-specific wedge, $\tau_{ki}^f$ factor-specific wedge.

- Markup $\mu_k$ over marginal cost.

- Equilibrium: all markets clear.
Generality

- Captures factor augmenting productivity shocks with relabeling.
- Captures demand shocks as mix of productivity shocks.
- Captures decreasing returns with fixed quasi-factors.
- Can capture “technical” adjustment costs and capacity utilization.
- See later for increasing returns.
- Can be applied to final demand within period, or intertemporally.
Notation and Accounting Convention

- Represent all wedges as markups with relabeling.

- Assume that in *data*, expenditures by $i$ on $j$ and revenues of $i$ recorded *gross* of wedges and markups.

- If not, for ex. with implicit wedges (e.g. credit constraints), re-write expenditures gross of these wedges.
Revenue-Based vs. Cost-Based

Definition

Ω and ˜Ω are $N \times N$ input-output matrices with $ij$th element:

$$\Omega_{ij} = \frac{p_j x_{ij}}{p_i y_i}, \quad ˜\Omega_{ij} = \frac{p_j x_{ij}}{\sum_k p_k x_{ik} + \sum_f w_f F_{if}}.$$ 

Ψ and ˜Ψ are $N \times N$ Leontief inverse matrices:

$$\Psi = (I - \Omega)^{-1}, \quad ˜\Psi = (I - ˜\Omega)^{-1}.$$ 

$b$ is $N \times 1$ consumption-shares vector with $i$th element:

$$b_i = \frac{p_i c_i}{\sum_j p_j c_j}.$$ 

λ and ˜λ are $N \times 1$ Domar weights:

$$\lambda = b' \Psi, \quad ˜\lambda = b' ˜\Psi.$$
Revenue-Based vs. Cost-Based

Cost-based definitions capture correct notion of exposure:

- $\tilde{\Omega}_{ij}$ is direct exposure of $i$ to $j$.
- $\tilde{\Psi}_{ij}$ is direct and indirect exposure of $i$ to $j$.
- $\tilde{\lambda}_k$ is direct and indirect exposure of household to $k$. 
Macro Impact of Micro Shocks

- \( Y(A, X) \): output \( Y \) given productivities \( A \) and shares \( X_{ij} = x_{ij}/y_j \).

- Change in equilibrium in response to shocks:
  \[
  d \log Y = \frac{\partial \log Y}{\partial \log A} d \log A + \frac{\partial \log Y}{\partial X} dX
  \]
  \( \Delta \) Technology \( \Delta \) Allocative Efficiency

- For efficient economies, macro-envelope implies Hulten:
  \[
  d \log Y = \lambda' d \log A + 0
  \]
  \( \Delta \) Technology \( \Delta \) Allocative Efficiency

- Inefficient economies: no macro-envelope, only micro-envelope.
Macro Impact of Micro Productivity Shocks

Theorem

\[
\frac{d \log Y}{d \log A_k} = \tilde{\lambda}_k - \sum_f \tilde{\Lambda}_f \frac{d \log \Lambda_f}{d \log A_k}.
\]

\(\Delta \text{Technology} \quad \Delta \text{Allocative Efficiency}\)

- Yields Hulten’s theorem for efficient economies:
  \[\tilde{\lambda}_k = \lambda_k \quad \text{and} \quad - \sum_f \tilde{\Lambda}_f \frac{d \log \Lambda_f}{d \log A_k} = 0.\]

- See later for structural formula for \(- \sum_f \tilde{\Lambda}_f \frac{d \log \Lambda_f}{d \log A_k}\).
Macro Impact of Micro Markup Shocks

Theorem

\[
\frac{d \log Y}{d \log \mu_k} = -\tilde{\lambda}_k - \sum_f \tilde{\Lambda}_f \frac{d \log \Lambda_f}{d \log \mu_k}.
\]

\(\Delta\text{Allocative Efficiency}\)

- Also applies to shocks to other wedges.
- Can be applied to endogenous wedges via chain rule.
- See later for structural formula for \(-\sum_f \tilde{\Lambda}_f \frac{d \log \Lambda_f}{d \log \mu_k}\).
Example of multiple marginalization taken from Baqae (2016):

\[ \tilde{\lambda}_k = 1 \neq \lambda_k = \prod_{i=1}^{k-1} \mu_i^{-1} \quad \text{and} \quad \Lambda_L = \prod_{i=1}^{N} \mu_i^{-1} \neq 1. \]

Productivity shocks:

\[ \frac{d \log Y}{d \log A_k} = \tilde{\lambda}_k - \frac{d \log \Lambda_L}{d \log A_k} = 1 \]

Markups/wedges shocks:

\[ \frac{d \log Y}{d \log \mu_k} = -\tilde{\lambda}_k - \frac{d \log \Lambda_L}{d \log A_k} = 0 \]
Ex. Simple Horizontal Economy

\[ \tilde{\lambda}_k = \lambda_k \text{ and } \Lambda_L = \sum_j \lambda_j \mu_j^{-1} \neq 1. \]

Productivity shocks:

\[
\frac{d \log Y}{d \log A_k} = \tilde{\lambda}_k - \frac{d \log \Lambda_L}{d \log A_k} = \lambda_k - (\theta_0 - 1) \left( \frac{\mu_k^{-1}}{\sum_j \lambda_j \mu_j^{-1}} - 1 \right) \lambda_k.
\]

Markup/wedge shocks:

\[
\frac{d \log Y}{d \log \mu_k} = -\tilde{\lambda}_k - \frac{d \log \Lambda_L}{d \log A_k} = \theta_0 \left( \frac{\mu_k^{-1}}{\sum_j \lambda_j \mu_j^{-1}} - 1 \right) \lambda_k.
\]
Ex. Cobb-Douglas Economies

- Productivity shocks:

\[ \frac{d \log Y}{d \log A_k} = \tilde{\lambda}_k - \sum_f \tilde{\Lambda}_f \frac{d \log \Lambda_f}{d \log A_k} = \tilde{\lambda}_k. \]

- Markup/wedge shocks:

\[ \frac{d \log Y}{d \log \mu_k} = -\tilde{\lambda}_k - \sum_f \tilde{\Lambda}_f \frac{d \log \Lambda_f}{d \log A_k} = -\tilde{\lambda}_k + \lambda_k \sum_f \tilde{\Lambda}_f \psi_{kf}/\Lambda_f. \]

- Cobb-Douglas functional forms very popular in literature.

- For an efficient economy, first-order approximation equivalent to Cobb-Douglas (not true at higher order).

- For inefficient economies, first-order approximation not equivalent to Cobb-Douglas, and so assumption even more problematic!
Productivity shocks:
\[
\frac{d \log Y}{d \log A_k} = \tilde{\lambda}_k - \tilde{\Lambda}_L \frac{d \log \Lambda_L}{d \log A_k} - \tilde{\Lambda}_K \frac{d \log \Lambda_K}{d \log A_k}.
\]

Markup/wedge shocks:
\[
\frac{d \log Y}{d \log \mu_k} = -\tilde{\lambda}_k - \tilde{\Lambda}_L \frac{d \log \Lambda_L}{d \log \mu_k} - \tilde{\Lambda}_K \frac{d \log \Lambda_K}{d \log \mu_k}.
\]
Sources of Growth and Solow Residual

- Easy extension to changing factor supplies:
  \[
  d \log Y - \tilde{\Lambda}' d \log L = \tilde{\lambda}' d \log A - \tilde{\lambda}' d \log \mu + \check{\lambda}' d \log \Lambda. \\
  \Delta \text{Technology} \quad \Delta \text{Allocative Efficiency}
  \]

- Solow residual:
  \[
  d \log Y - \hat{\Lambda}' d \log L = \tilde{\lambda}' d \log A - \tilde{\lambda}' d \log \mu + \check{\lambda}' d \log \Lambda + (\tilde{\Lambda} - \hat{\Lambda})' d \log L, \\
  \Delta \text{Technology} \quad \Delta \text{Allocative Efficiency} \quad \text{Miscounting Factor Growth}
  \]

  where \( \hat{\Lambda} \) adjusts \( \Lambda \) to count profit share in capital share.

- Can perform decomposition without imposing any parametric assumptions on production functions. Example: handles factor augmenting productivity and demand shocks with no modification.
Alternative Decompositions

- Do not use input-output information.
- Revealing example of acyclic economies:

![Diagram of acyclic economies]

- These decompositions detect changes in allocative efficiency, even though allocation is efficient. Ours does not.
Measuring Allocative Efficiency

- Measure of change in allocative efficiency along equilibrium path.
- Different from change of distance to frontier a la Restuccia and Rogerson (2008) or Hsieh and Klenow (2009).

Relation between the two concepts:

\[
\log \left( \frac{Y(A, 1)}{Y(A, \mu)} \right) = - \int_0^1 \frac{d \log Y(A, \hat{\mu}(t))}{d \log \hat{\mu}} \frac{d \log \hat{\mu}(t)}{dt} dt
\]

\[
= \frac{1}{2} \sum_i \frac{d \log Y(A, \mu)}{d \log \mu_i} \left( \frac{1 - \mu_i}{\mu_i} \right) + O(\|\mu - 1\|^3),
\]

where \( \hat{\mu}_k = \tau \mu_k + (1 - \tau) \).

Ex. for a horizontal economy:

\[
\log \left( \frac{Y(A, 1)}{Y(A, \mu)} \right) = \frac{1}{2} \theta_0 \frac{\text{Var}_{\lambda}(\mu^{-1})}{E_{\lambda}(\mu^{-1})}.
\]
General Non-parametric Result

General Parametric Result

Applications
  Growth Accounting
  Quantitative Model

Extensions (see paper)

Conclusion
Parametric Model

- Final demand:
  \[
  \frac{Y}{Y} = \left( \sum_k b_k \left( \frac{c_k}{\bar{c}_k} \right) \frac{\theta_0^{-1}}{\theta_0} \right)^{\frac{\theta_0}{\theta_0-1}}.
  \]

- Production of good \( k \):
  \[
  \frac{y_k}{y_k} = A_k \left( a_k \left( \frac{l_k}{l_k} \right) \frac{\theta_k^{-1}}{\theta_k} + (1 - a_k) \left( \frac{X_k}{X_k} \right) \right) \frac{\theta_k^{-1}}{\theta_k}.
  \]

- \( X_k \) composite intermediate input given by
  \[
  \frac{X_k}{X_k} = \left( \sum_l \omega_{kl} x_{lk} \right) \frac{\varepsilon_k^{-1}}{\varepsilon_k},
  \]
  where \( x_{kl} \) intermediate inputs from industry \( l \) used by industry \( k \).
Parametric Model

- Relabel network so that each node corresponds to one CES nest.
- Structure can actually represent any nested CES economy with arbitrary pattern of nests and wedges.

**Definition**

\[
\text{Cov}_{\tilde{\Omega}(j)} \left( \tilde{\Psi}^{(k)}, \Psi^{(L)} \right) = \sum_i \tilde{\Omega}_{ji} \tilde{\Psi}_{ik} \Psi_{iL} - \left( \sum_i \tilde{\Omega}_{ji} \tilde{\Psi}_{ik} \right) \left( \sum_i \tilde{\Omega}_{ji} \Psi_{iL} \right).
\]
Macro Impact of Micro Productivity Shocks: One Factor

Proposition

Suppose there is only one factor (with index L). Then

\[
\frac{d \log Y}{d \log A_k} = \tilde{\lambda}_k - \frac{d \log \Lambda_L}{d \log A_k},
\]

where

\[
\frac{d \log \Lambda_L}{d \log A_k} = \sum_j (\theta_j - 1) \mu_j^{-1} \lambda_j \text{Cov}_{\tilde{\Omega}(j)} \left( \tilde{\Psi}(k), \frac{\Psi(L)}{\Lambda_L} \right).
\]

- Centrality measure mixing network and elasticities.
- Upstream and downstream distortions matter.
Explaining Covariance Operator

\[
\frac{d \log Y}{d \log A_k} = \tilde{\lambda}_k - \sum_j (\theta_j - 1) \mu_j^{-1} \lambda_j \text{Cov}_{\tilde{\Omega}(j)} \left( \tilde{\Psi}(k), \frac{\Psi(L)}{\Lambda_L} \right).
\]

- High \( \tilde{\Psi}_{ik} \): i’s highly exposed to k.
- High \( \Psi_{iL}/\Lambda_L \): most of i’s revenues are ultimately paid to workers.
Ex. Back to Simple Horizontal Economy

Change in technology and change in allocative efficiency:

\[
\frac{d \log Y}{d \log A_k} = \lambda_k - (\theta_0 - 1) \left( \frac{\mu_k^{-1}}{\sum_j \lambda_j \mu_j^{-1}} - 1 \right) \lambda_k.
\]

Key: markup vs. average and elasticity minus one.
Macro Impact of Micro Markup Shocks: One Factor

Proposition

Suppose there is only one factor indexed by $L$. Then

\[
\frac{d \log Y}{d \log \mu_k} = -\tilde{\lambda}_k \left( -\frac{d \log \Lambda_L}{d \log A_k} \right),
\]

where

\[
\frac{d \log \Lambda_L}{d \log \mu_k} = \sum_j (1 - \theta_j) \mu_j^{-1} \lambda_j \text{Cov}_{\tilde{\Omega}_j} \left( \tilde{\Psi}_k, \frac{\psi_L}{\Lambda_L} \right) - \lambda_k \frac{\psi_{KL}}{\Lambda_L}.
\]

- Positive markup shock like negative productivity shock...
- ...but also releases labor.
Change in allocative efficiency:

\[
\begin{align*}
\frac{d \log Y}{d \log \mu_k} &= -\tilde{\lambda}_k - (1 - \theta_0)\lambda_k \left( \frac{\mu_k^{-1}}{\Lambda_L} - 1 \right) + \frac{\lambda_k \mu_k^{-1}}{\Lambda_L}, \\
&= \theta_0 \left( \frac{\mu_k^{-1}}{\sum_j \lambda_j \mu_j^{-1}} - 1 \right) \lambda_k.
\end{align*}
\]

Key: markup vs. average and elasticity.
Macro Impact of Micro Productivity Shocks: Multiple Factors

Proposition

The following linear system describes the elasticities of factor shares:

\[
\frac{d \log \Lambda}{d \log A_k} = \Gamma \frac{d \log \Lambda}{d \log A_k} + \delta_{(k)},
\]

with

\[
\Gamma_{f,g} = -\sum_j (\theta_j - 1) \lambda_j \mu_j^{-1} \text{Cov}_{\tilde{\Omega}(j)} \left( \tilde{\Psi}_{(f)}, \frac{\Psi_{(g)}}{\Lambda_g} \right),
\]

and

\[
\delta_{fk} = \sum_j (\theta_j - 1) \lambda_j \mu_j^{-1} \text{Cov}_{\tilde{\Omega}(j)} \left( \tilde{\Psi}_{(k)}, \frac{\Psi_{(f)}}{\Lambda_f} \right).
\]

Given the elasticities of factor shares, we have

\[
\frac{d \log Y}{d \log A_k} = \tilde{\lambda}_k - \sum_f \tilde{\lambda}_f \frac{d \log \Lambda_f}{d \log A_k}.
\]

\(\Delta\text{Technology}\)\(\Delta\text{Allocative Efficiency}\)
Multiple Factors


- Can derive similar formula for markups/wedge shocks.
Ex. Multiple Factors

Diagram:
- Node L points to 1 and 2.
- Node 1 points to 2.
- Node 2 points to HH.
- Node HH points to 3.
- Node K points to 4.
- Node 4 points to HH.
Ex. Multiple Factors

- Change in technology and change in allocative efficiency:

\[
\frac{d \log Y}{d \log A_k} = \lambda_k + \lambda_k (\theta_0 - 1) \left( 1 - \frac{\mu_k^{-1}}{\frac{\lambda_1}{\lambda_1 + \lambda_2} \mu_1^{-1} + \frac{\lambda_2}{\lambda_1 + \lambda_2} \mu_2^{-1}} \right) \quad (k = 1, 2),
\]

\[
\frac{d \log Y}{d \log A_3} = \lambda_3,
\]

\[
\frac{d \log Y}{d \log A_4} = \tilde{\lambda}_4 = \mu_3 \lambda_4.
\]

- No change in allocative efficiency between (1+2) and (3+4).
Agenda

General Non-parametric Result

General Parametric Result

Applications
  Growth Accounting
  Quantitative Model

Extensions (see paper)

Conclusion
Sources of Growth and Solow Residual

- Suppose markups are only distortions.

- Use annual IO tables from BEA from 1997-2015.

- Use markups from Gutiérrez and Philippon (2016), De Loecker and Eeckhout (2017), and Lerner Indices for firms in Compustat.

- All measures show large increases in markups, from composition effects across firms, not from effects within firms: high markup firms getting bigger, not large firms getting higher markups. Also consistent with Autor et al. (2017).

- Perform decomposition.
Using the Gutiérrez and Philippon (2016) markup data.

Similar with De Loecker and Eeckhout (2017) and Lerner Indices.
Using the De Loecker and Eeckhout (2017) markup data.
Sources of Growth and Solow Residual

- Solow Residual
- Allocative Efficiency
- Factor Undercounting
- Technology

Using Lerner indices.
General Non-parametric Result

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Quantitative Results

Calibrate parametric model.

- Benchmark elasticities of substitution:
  - consumption 0.4;
  - value and intermediates 0.3;
  - across intermediates be 0.01;
  - between labor and capital 1;
  - within industries 8.

- Use IO table from BEA from 2015.

- Robustness checks: role of elasticities and input-output structure.
# Gains from Eliminating Markups

<table>
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<tr>
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<th>Gutierrez-Philippon</th>
<th>Lerner Index</th>
<th>De Loecker-Eeckhout</th>
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<td>2014</td>
<td>20%</td>
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<tr>
<td>1997</td>
<td>3%</td>
<td>5%</td>
<td>21%</td>
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</table>

- Measures show big increase between 1997 and 2014.
- Contrast with 0.1% estimate of Harberger (1954) triangles!

“*It takes a heap of Harberger triangles to fill an Okun gap.*”  
— Tobin
Gains from Shrinking Markups: Robustness

<table>
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<tr>
<th></th>
<th>Benchmark</th>
<th>CD+CES</th>
<th>CD+CD</th>
<th>VA Benchmark</th>
<th>VA CD + CES</th>
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<td>GP</td>
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<td>21%</td>
<td>4%</td>
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<td>8%</td>
<td>1%</td>
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<tr>
<td>LI</td>
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<td>18%</td>
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<td>7%</td>
<td>7%</td>
<td>1%</td>
</tr>
<tr>
<td>DE</td>
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<td>38%</td>
<td>7%</td>
<td>18%</td>
<td>18%</td>
<td>3%</td>
</tr>
</tbody>
</table>

- Elasticities matter.
- Input-output structure matters.
- Value-added production functions misleading!
Macro-Volatility from Micro Shocks

\[ \text{Var}(\log Y) \approx \| D_{\log A} \log Y \|^2 \text{Var}(d \log A) + \| D_{\log \mu} \log Y \|^2 \text{Var}(d \log \mu). \]

- Thought experiment: i.i.d. shocks to Compustat firms, not others.
- Tabulate diversification factors \( \text{std}(\log Y)/\text{std}(d \log A) \) and \( \text{std}(\log Y)/\text{std}(d \log \mu) \).

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Competitive</th>
<th>Cobb-Douglas</th>
<th>Passive</th>
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<tbody>
<tr>
<td>Firm Productivity Shocks (GP)</td>
<td>0.0491</td>
<td>0.0376</td>
<td>0.0396</td>
<td>0.0396</td>
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<tr>
<td>Firm Markup Shocks (GP)</td>
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<td>0.0000</td>
<td>0.0391</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
Macro Impact of Micro Shocks

Output elasticity productivity and markup shocks relative to size.
For firm shocks and for sectoral shocks.
Distortions matter!
General Non-parametric Result

General Parametric Result

Applications
   Growth Accounting
   Quantitative Model

Extensions (see paper)

Conclusion
Extensions (see paper)

- Endogenous markups/wedges.
- Elastic Factors.
- Fixed costs and entry.
- Nonlinearities.
General Non-parametric Result

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Conclusion

- Reduced-form aggregation theorem for economies with frictions.
- Structural aggregation theorems.
- Wide range of applications in different contexts.
- Work in progress: structural models of frictions (IO, financing constraints, search and matching, nominal rigidities, etc.), fixed costs, entry and exit, dynamics, non-homotheticities, endogenous innovation, other models of network formation, etc.
- Part of a broader research agenda.
Example of multiple marginalization taken from Baqee (2016):

\[ b'\tilde{\Psi} = \tilde{\lambda}_k = 1 \text{ and } b'\Psi = \lambda_k = \prod_{i=1}^{k-1} \mu_i^{-1} < 1. \]
Example of multiple marginalization taken from Baqae (2016):

\[ \frac{d \log Y}{d \log A_k} = \lambda_k = 1 > \lambda_k = \prod_{i=1}^{k-1} \mu_i^{-1}. \]

In accounting sense, Hulten’s theorem fails.

In economic sense, Hulten’s theorem survives!
Shares and factor shares:

\[ \tilde{\lambda}_k = \lambda_k, \quad \tilde{\Lambda}_L = 1 > \Lambda_L = \sum_j \lambda_j \mu_j^{-1}. \]

Change in technology and change in allocative efficiency:

\[ \frac{d \log Y}{d \log A_k} = \tilde{\lambda}_k + \frac{d H(\tilde{\Lambda}, \Lambda)}{d \log A_k} = \lambda_k - \frac{d \log \Lambda_L}{d \log A_k}. \]


Autor, D., D. Dorn, L. Katz, C. Patterson, and J. Van Reenen (2017). The fall of the labor share and the rise of superstar firms.


Bouakez, H., E. Cardia, and F. J. Ruge-Murcia (2009). The


Nakamura, E. and J. Steinsson (2010). Monetary non-neutrality in a


