

Systemic Risk-Taking: Amplification Effects, Externalities, and Regulatory Responses

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Abstract

This paper analyzes the risk-taking behavior of agents in an economy that is prone to systemic risk, captured by financial amplification effects that involve a feedback loop of falling asset prices, tightening financial constraints and fire sales. It shows that decentralized agents who have access to a complete set of Arrow securities take on socially excessive exposure to such risk because of pecuniary externalities that are triggered during financial amplification. The paper develops an externality pricing kernel that reflects the state-contingent magnitude of such externalities and provides foundations for macro-prudential regulation to correct the distortion. Furthermore, it derives conditions under which agents employ ex-ante risk markets to fully undo anticipated government bailouts. Finally, it finds that financially constrained agents face socially insufficient incentives to raise costly equity during episodes of systemic risk.

JEL Codes: E44, G21, G28, D62

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1 Introduction

The financial crisis of 2008/09 has powerfully highlighted the vulnerability of modern economies to financial amplification, whereby falling asset prices, deteriorating balance sheets, tightening financial constraints and fire sales mutually reinforce each other (see figure 1). Such dynamics capture an essential element of what policymakers and market participants describe as *systemic risk*.¹ More broadly, financial amplification effects (or financial accelerator effects) have not only played an essential role during the near-meltdown of the financial sector in Fall 2008, but also in the ensuing credit crunch and the protracted slump of the housing sector (see Brunnermeier, 2008; Shleifer and Vishny, 2011). After the financial crisis, policymakers and regulators have argued for the introduction of new financial regulations with a *macro-prudential* focus, designed to limit aggregate risk-taking of the financial sector and by implication to reduce the incidence of crises that involve financial amplification.

This paper shows that financial amplification effects create pecuniary externalities that induce financial market participants (“bankers”) to allocate aggregate risk inefficiently. They take on socially excessive exposure to such risk or, equivalently, insure insufficiently against it. This finding provides a natural rationale for macro-prudential regulation to limit the risk-taking of the financial sector.

We describe a group of competitive bankers who choose how much insurance to buy against an aggregate shock based on their privately optimal trade-off between risk and return. If they are hit by an adverse realization of the shock, they become financially constrained and are forced to sell assets, triggering financial amplification. Pecuniary externalities arise because each individual banker does not internalize that his asset sales reduce the prices at which other bankers can sell assets. A planner or regulator could make everybody better off by inducing bankers to buy more insurance against such shocks. This would reduce fire sales and price declines and thereby mitigate the financial amplification effects.²

The paper develops an *externality pricing kernel* that captures the social benefit of liquidity in the banking sector that is not internalized by individual bankers. The social benefit of liquidity, i.e. of holding an additional unit of liquid net worth, consists

¹The Bank for International Settlements, for example, defines systemic risk as a situation when exogenous shocks to financial institutions that have common risk exposure are endogenously amplified because of wide-spread financial distress (see e.g. Borio, 2003).

²Pecuniary externalities are welfare-relevant in our setting even though bankers have access to a complete set of Arrow securities ex-ante, i.e. before financial amplification occurs. The reason is that they experience binding financial constraints when an adverse shock is realized and when financial amplification effects are triggered. This market incompleteness violates the conditions of the first welfare theorem.

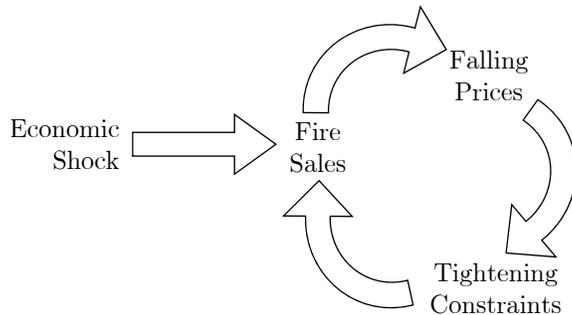


Figure 1: Financial amplification under binding financial constraints

of reduced fire sales, increased asset prices and relaxed financial constraints across the sector. We use the externality pricing kernel to develop a framework of macro-prudential regulation to correct the distortion. We analyze how the magnitude of externalities is affected by the degree of risk aversion and of aggregate risk in the economy.

We also derive conditions under which bankers employ ex-ante risk markets to fully undo anticipated transfers (“bailouts”) that are intended to relax binding constraints. Finally, we show that financially constrained bankers face socially insufficient incentives to raise costly equity during episodes of financial amplification because they do not internalize the social benefits in terms of reducing aggregate fire-sales.

The specific setting in which we describe our results is an economy with three time periods $t = 0, 1, 2$ and two categories of agents, bankers and households. The economy experiences an aggregate shock that is realized at the beginning of period 1. Bankers represent the combined productive and financial sector of the economy.³ They are risk-neutral and raise finance in a complete market of Arrow securities in period 0 for an investment project that yields a risky payoff in period 1 and a safe payoff in period 2. Bankers can use the asset value but not the contemporary return on the project as collateral for their financial promises.⁴ This implies that they have sufficient collateral to back up the Arrow securities that come due in period 1, but since the terminal asset value of all projects is zero, no borrowing between periods 1 and 2 can be sustained.

The borrowing constraints on bankers imply that their liquid net worth in period 1 (i.e. the net worth that they can access immediately) is less than the present discounted value of their earnings. Although bankers cannot borrow in period 1, they can

³For simplicity, we do not separately model bankers who allocate capital and firms who employ capital in production. Instead we focus our analysis on the risk sharing problem of the combined financial/productive sector with the household sector.

⁴Kiyotaki and Moore (1997), building on Hart and Moore (1994), motivate this by observing that the owners of a project could threaten to withdraw their labor and thereby destroy the contemporary return.

raise finance by selling a fraction of their project at the prevailing market price to the household sector. However, this is inefficient from a first-best perspective because the household sector has an inferior production technology. In period 2, bankers consume the payoff on their remaining asset holdings and perish.

Households come in two generations. First-generation households live from period 0 to 1 and provide finance to bankers in the market for Arrow securities. They are risk-averse so their demand for securities contingent on a particular state of nature is downward-sloping. This makes it costly for bankers to share risk with them. Second-generation households live from period 1 to 2 and have access to a technology that employs the assets of bankers but that is less productive and subject to decreasing returns-to-scale. Therefore the asset demand of second-generation households is downward-sloping, and it is costly for bankers to sell assets to them.

If the initial investment requirement of bankers is sufficiently small, they promise a fixed payment to first-generation households, which they finance from their period 1 payoff. Bankers absorb all aggregate risk and do not engage in fire sales in period 1. In this case the decentralized equilibrium in the economy is constrained socially efficient.

For a larger initial investment requirement, bankers cannot afford their repayment obligations in low states of period 1 without resorting to costly asset sales. Bankers therefore need to find the optimal trade-off between costly risk sharing with first-generation households and asset sales at a price below their marginal product to second-generation households.

Our main result is that bankers in the decentralized equilibrium of the described economy insure too little in ex ante risk markets and engage in excessive fire sales when an adverse shock materializes. The reason for this distortion is that atomistic bankers take prices in the economy as given and do not internalize the pecuniary externalities that their fire sales give rise to. Under complete markets, pecuniary externalities do not have efficiency implications because the relative marginal valuations of all goods among all agents in the economy are equated, and a redistribution cannot achieve a Pareto improvement. In the described setting, by contrast, binding financial constraints and the inability of second-generation households to participate in period 0 insurance markets imply that bankers value productive assets more highly than households. A constrained social planner internalizes that reducing fire sales keeps asset prices more elevated, which mitigates the financial constraints on bankers. Therefore the planner engages in more ex ante insurance and fewer fire sales than decentralized agents.

Our constrained inefficiency result relies crucially on two market imperfections. First, bankers cannot borrow against the payoffs of their asset holdings in the final period. They are the natural holders of capital assets because they have the most pro-

ductive technology. If they could borrow against the full payoffs they receive in period 2, no fire sales would occur, and the first-best equilibrium would be restored. Secondly, the second-generation households who buy up fire sales are not alive in period 0 and cannot participate in ex-ante risk markets in which bankers insure against binding constraints. If they could participate in that market, they would optimally insure the liquidity risk of bankers and remove the need for fire sales in low states of nature of period 1, leading to a constrained efficient equilibrium. However, in the absence of their participation in this market, the only way they can provide liquidity to bankers in constrained states is the less efficient route of fire-sales. A constrained planner can emulate insurance transfers by curtailing bankers' fire sales in low states of nature, which raises asset prices and provides an implicit wealth transfer to constrained bankers.

We employ our model to shed light on a number of policy issues that have been debated in the aftermath of the recent financial crisis:

First, we use the identified pecuniary externalities to develop a conceptual framework of macro-prudential regulation that induces individual bankers to internalize their contribution to systemic risk. We characterize an *externality pricing kernel* that captures the state-contingent magnitude of pecuniary externalities and that can be used to price the externalities imposed by financial claims or real investment opportunities, in analogy to the standard pricing kernels employed in the literature. In states of nature when financial constraints are loose, the externality kernel is zero since no amplification effects occur; in constrained states of nature the externality kernel captures the social cost of financial amplification effects. A policymaker who induces bankers to internalize these pecuniary externalities via Pigouvian taxes, capital requirements, or other regulatory measures can restore constrained Pareto efficiency in the described economy.

Second, we derive a bailout neutrality result: we characterize conditions under which bankers will employ ex-ante risk markets to fully undo any expected lump-sum government transfer that aims to relieve binding constraints and mitigate financial amplification effects. Undoing such transfers is optimal for bankers since the equilibrium with excessive systemic risk constitutes their private optimum.

Third, we find that individual bankers undervalue the social benefits of raising new equity capital during episodes of financial amplification because they do not internalize the positive effects of reducing their fire sales on the rest of the banking system. This provides a policy rationale for mandatory capital injections.

Our paper also illustrates an important conceptual difference between systematic risk and systemic risk: Bankers in our model are always subject to systematic risk (i.e. to aggregate, undiversifiable market risk). By contrast, systemic risk only arises when the banking sector as a whole experiences binding financial constraints and financial

amplification effects are triggered.

Literature Our work builds on the literature on financial amplification and fire sales as described by Fisher (1933), Bernanke and Gertler (1990), Shleifer and Vishny (1997), Kiyotaki and Moore (1997) and Brunnermeier and Pedersen (2009). Specifically, our model is a simplified version of Kiyotaki and Moore (1997). In this literature, it is common to assume that financially constrained bankers/entrepreneurs only have access to uncontingent forms of finance. If they had access to complete and risk-neutral insurance markets, bankers/entrepreneurs would fully insure against the risk of becoming constrained and no financial amplification effects would occur in case of adverse shocks (Krishnamurthy, 2003). This paper shows that risk aversion among the providers of finance is sufficient to break this result, as bankers trade off the costs of binding financial constraints and of purchasing insurance and choose an interior optimum.

The paper also builds on the literature on the generic inefficiency of the decentralized equilibrium under incomplete markets (Stiglitz, 1982; Geanakoplos and Polemarchakis, 1986), which includes more recent seminal contributions by Gromb and Vayanos (2002), Caballero and Krishnamurthy (2003) and Lorenzoni (2008). Gromb and Vayanos (2002) analyze financially constrained arbitrageurs and show that they generally fail to engage in the socially efficient amount of arbitrage between two risky assets because they do not internalize the pecuniary externalities involved in fire sales when financial constraints are binding. Aside from the two risky assets, arbitrageurs in their model only have access to uncontingent bonds.

In Caballero and Krishnamurthy (2003) and Lorenzoni (2008), entrepreneurs raise finance in a risk-neutral security market and face the risk of binding financial constraints in a subsequent period. Caballero and Krishnamurthy (2003) investigate the financing and investment decisions in a small open emerging economy in which binding future constraints result in exchange rate depreciations. Lorenzoni (2008) focuses on the aggregate level of investment in a simplified Kiyotaki-Moore economy similar to ours, in which binding constraints lead to fire sales and asset price declines.⁵ In both works, entrepreneurs engage in excessive investment because of pecuniary externalities that arise from future binding constraints.

What distinguishes our paper is that we introduce risk-aversion into such a framework, which allows us to study the trade-off between risk and return that bankers face when they have access to a complete set of Arrow securities. The focus on risk versus return makes our framework well suited for studying the risk-taking decisions of financially constrained agents such as banks and derive implications for price-based

⁵A similar framework with an extension to multiple equilibria is developed in Gai et al. (2008).

macro-prudential regulation of the resulting pecuniary externalities. Our paper also presents a number of additional new results, including on the incentives for constrained bankers to raise new equity capital, and a “bailout neutrality” result that derives conditions under which bankers undo expected transfers that are aimed at relaxing binding financial constraints.⁶

Gersbach and Rochet (2012) describes a setting in which pecuniary externalities induce banks to excessively reallocate capital after sectoral shocks. Acharya et al. (2011) analyze the incentives of banks to hold liquidity to buy up the fire sales of competitors and find that these incentives are in general inefficient. This finding relies on ex-post heterogeneity among banks. By contrast, our work focuses exclusively on fire sales between a homogenous banking sector and the rest of the economy.

A number of recent papers document the empirical importance of financial amplification effects. For example, Adrian and Brunnermeier (2011) show that VaR – a measure for the riskiness of a financial institution’s assets – rises strongly when another institution is in distress. They also document that financial institutions that increase their exposure to systemic risk raise their expected return, consistent with our theoretical model. Adrian and Shin (2010) find that leverage among investment banks is strongly pro-cyclical, implying that they take on more risk in good times and sell off risky assets in bad times. Benmelech and Bergman (2011) provide evidence for fire-sale externalities in the airline sector.

The rest of the paper is structured as follows. The following section describes our model setup. Section 3 analyzes the decentralized equilibrium of the economy and the dynamics of financial amplification when financing constraints are binding. Section 4 analyzes the social efficiency of the decentralized equilibrium and presents our framework for macro-prudential regulation. In section 5 we study extensions of our baseline model to develop our results on bailout neutrality and on the incentives for raising equity. Section 6 concludes. The appendix contains a detailed discussion of some of the technical assumptions and proofs of our model.

2 Model

Our model economy consists of three time periods $t = 0, 1, 2$ and is inhabited by two categories of atomistic agents of mass 1, bankers and households. Bankers represent the

⁶There is a significant literature stretching back at least to Bagehot (1873) arguing that the expectation of bailouts distorts the risk-taking decisions of banks. Recent noteworthy contributions include e.g. Farhi and Tirole (2012) and Acharya et al. (2011). Our result stands out in that we characterize conditions under which anticipated bailouts are neutral, i.e. they are fully undone by bankers.

consolidated productive sector of the economy and could alternatively be interpreted as entrepreneurs – the important characteristic is that they make financing decisions and are subject to business risk and financial constraints. Households come in two generations; they value productive capital less than bankers, but they receive endowments and therefore have the ability to provide finance to bankers.

There are two types of goods, a homogeneous consumption good and a productive capital asset. In period 1, a random state of nature $\omega \in \Omega$ is realized, where Ω is a set of all possible outcomes. The productivity of bankers' capital assets in period 1 is given by a random variable A_1^ω , which is continuously distributed over the interval $[A^{\min}, A^{\max}] \subseteq \mathbb{R}_0^+$ with density function $g(A)$, and which satisfies the normalization $E[A_1^\omega] = 1$.

Bankers Bankers are risk-neutral and value consumption in periods 1 and 2, c_1^ω and c_2^ω , according to the function

$$V = E[c_1^\omega + c_2^\omega] \tag{1}$$

In period 0, they have access to a lumpy investment technology that allows them to invest αt_1 consumption goods and obtain t_1 units of productive capital assets. We can think of this as planting a seed that costs α on t_1 units of land. They have no endowment, so they need to finance their period 0 investment by selling financial claims in a complete one-period market of Arrow securities that are contingent on the state of nature $\omega \in \Omega$. We denote the amount to be repaid in state ω of period 1 as b_1^ω and the stochastic discount factor (or pricing kernel) at which the claims are priced in period 0 as m_1^ω . The resulting period 0 budget constraint is

$$\alpha t_1 = E[m_1^\omega b_1^\omega] \tag{2}$$

In period 1, each unit of the capital asset produces a stochastic net dividend A_1^ω , which depends on the state of nature ω .

Bankers are subject to a commitment problem that limits what they can pledge to repay. Specifically, we follow Kiyotaki and Moore (1997) in assuming that when they enter financial contracts, they can only pledge the market value but not the dividend income of their asset holdings next period.⁷ Since the economy ends after period 2, the price of capital assets ex dividend is zero in that period and bankers have no collateral to pledge in period 1, i.e. no borrowing between periods 1 and 2 can be sustained. We therefore set w.l.o.g. $b_2^\omega = 0$. Following the same argument, bankers do have collateral to offer between periods 0 and 1, which they use to back up their promises b_1^ω .

⁷Kiyotaki and Moore (1997) motivate this by the notion that bankers could threaten to withdraw their labor in the period in which lenders try to seize the assets, which would destroy all contemporaneous output.

In period 1, bankers cannot borrow, but they have access to a market in which they can trade productive assets at price q_1^ω . As we will see below, sales of bankers in this market share certain characteristics of fire-sales; therefore we denote the quantity of assets that bankers sell as fire-sales f_1^ω .

We make the simplifying assumption that the optimal amount of security issuance is such that $b_1^\omega < s^{\max} \forall \omega$, where s^{\max} is the maximum amount of funds that bankers can raise if they fire-sell their entire asset holdings t_1 in period 1 (which will be formally defined below). This assumption is not critical but it guarantees (i) that bankers have enough collateral in period 1 to issue the optimal amount of securities b_1 in period 0 and (ii) that bankers never fire-sell their entire asset holdings, avoiding a corner solution. The assumption allows us to omit the two constraints embodied by (i) and (ii) from the optimization problem below.

Accounting for the promised repayment b_1^ω on the Arrow securities that they issued, the period 1 budget constraint of bankers is

$$c_1^\omega + b_1^\omega = A_1^\omega t_1 + q_1^\omega f_1^\omega \quad (3)$$

Given their linear preferences, bankers would like to substitute consumption between periods 1 and 2 at a rate of unity. We impose a non-negativity constraint on period 1 consumption $c_1^\omega \geq 0$ to prevent them from using this device to circumvent the borrowing constraint that they face.

In period 2, bankers employ their remaining asset holdings $(t_1 - f_1^\omega)$ in production, and they consume the resulting output $c_2^\omega = \bar{A}_2(t_1 - f_1^\omega)$, where $\bar{A}_2 > E[A_1^\omega]$ since period 2 reflects the entire future of the economy. The resulting optimization problem for bankers is

$$\max_{\{b_1^\omega, c_1^\omega, f_1^\omega\}} E [c_1^\omega + \bar{A}_2(t_1 - f_1^\omega)] \quad \text{s.t. (2), (3) and } c_1^\omega \geq 0 \quad (4)$$

First-Generation Households We assume that there are two generations of households that live for two periods each. The first generation lives across periods 0 and 1. They are risk averse and derive utility from consumption according to the function

$$U = u(c_{0,h}) + E[u(c_{1,h}^\omega)]$$

where $u(\cdot)$ is a standard neo-classical utility function. We use the sub-index ‘ h ’ for first-generation households. They receive an endowment e every period that satisfies $e > \alpha t_1$. In period 0 they buy a bundle $\{b_{1,h}^\omega\}$ of Arrow securities that offer a contingent repayment $b_{1,h}^\omega$ in period 1. Given the stochastic discount factor $\{m_1^\omega\}$ at which Arrow securities are priced in the market, the total outlay of first generation households in period 0 is $E[m_1^\omega b_{1,h}^\omega]$.

We denote their optimization problem as

$$\max_{\{b_{1,h}^\omega\}} u(e - E[m_1^\omega b_{1,h}^\omega]) + E[u(e + b_{1,h}^\omega)] \quad (5)$$

The Euler equation that captures their demand for Arrow securities contingent on state ω is

$$FOC(b_1^\omega) : m_1^\omega = \frac{u'(c_{1,h}^\omega)}{u'(c_{0,h})} \quad (6)$$

This defines a demand function for Arrow securities $m(b)$ that is downward-sloping, implying that $dm/db < 0$. Furthermore, we assume that the functional form of $u(\cdot)$ and the parameters of the model are such that $d(m(b) \cdot b)/db > 0$, i.e. that the funds raised by selling Arrow securities increase in the amount of securities sold. The technical condition for this is listed as assumption A.1 in the appendix.

Remark : First generation households could alternatively be interpreted as entrepreneurs who are unconstrained and who have a competing use for funds in a production technology with declining marginal product that mirrors the declining marginal rate of substitution of households in our example.

Second-Generation Households Second-generation households live from period 1 to period 2. They value consumption according to the linear utility function

$$W = E[c_{1,l}^\omega + c_{2,l}^\omega]$$

where the sub-index ‘ l ’ denotes variables of second-generation households. They receive an endowment e every period and buy $f_{1,l}^\omega$ units of productive capital assets at the given market price q_1^ω in period 1. As in Lorenzoni (2008), they employ their assets in period 2 production using a decreasing returns-to-scale production function $F(\cdot)$ that satisfies $F'(0) = \bar{A}_2$ and $F'' < 0$, i.e. their marginal productivity is equal to the productivity of bankers at zero, but declines in the amount of assets purchased – households are less productive than bankers for any positive amount of assets employed.

The resulting optimization problem for second-generation households is

$$\max_{\{f_{1,l}^\omega\}} E[(e - q_1^\omega f_{1,l}^\omega) + (e + F(f_{1,l}^\omega))] \quad (7)$$

The first-order condition yields an inverse demand curve for productive assets

$$q_1^\omega = F'(f_{1,l}^\omega)$$

The inverse demand curve defines a function $q(f)$ that is downward-sloping, $dq/df < 0$, since the production technology exhibits decreasing returns to scale. Denote by $s(f)$

the dollar amount that second-generation households are willing to spend to purchase f units of capital assets,

$$s(f) = q(f) \cdot f \quad (8)$$

We assume that the price elasticity of assets with respect to fire-sales satisfies $\eta_{qf} < 1$, i.e. the price function $q(f)$ does not decline too fast in the amount of assets fire-sold. This guarantees that the function $s(f)$ is strictly increasing in the quantity of assets purchased and that the equilibrium is unique. The technical condition for this is discussed as assumption A.2 in the appendix.

The strictly monotonic relationship $s(f)$ allows us to define an inverse function $f(s)$, which expresses the quantity of assets that bankers need to fire-sell in order to obtain $s \geq 0$ units of liquidity from second generation households. If bankers sell their entire productive asset holdings t_1 , the asset price declines to $q^{\min} = q(t_1)$ and bankers can raise a maximum amount of funds

$$s^{\max} := s(t_1) = t_1 q^{\min}$$

Observe that s^{\max} is also the collateral of bankers, since it is what creditors could obtain if they seize all t_1 assets from bankers in period 1 and re-sell them to second-generation households.

For non-negative values $s \in [0, s^{\max}]$, $f(s)$ increases in a strictly convex fashion from 0 to t_1 and $q(f(s))$ decreases from \bar{A}_2 to q^{\min} . For later use, we define $f(s) = 0$ for $s < 0$, which implies that the asset price is at its first-best level $q(f(s)) = \bar{A}_2$ for such values of s . The function $f(s)$ is then defined over the entire interval $(-\infty, s^{\max}]$.

Remark : In the described setup, the demand of second-generation households for productive assets is downward-sloping because their production technology exhibits decreasing returns to scale. As we show in appendix B.1, similar results hold if second-generation households have concave utility $w(\cdot)$. In that case, asset demand would be defined by an optimality condition $q = \frac{w'(e+\bar{A}_2 f)}{w'(e- q f)} \cdot F'(f)$ and would be downward-sloping because households dislike an unsmooth consumption profile. The analysis is more complicated, but our basic results continue to hold.

3 Decentralized Equilibrium

An equilibrium in the economy consists of allocations $(c_1^\omega, c_2^\omega, c_{0,h}, c_{1,h}^\omega, c_{1,l}^\omega, c_{2,l}^\omega, b_1^\omega, b_{1,h}^\omega, f_1^\omega, f_{1,l}^\omega)$ and prices (m_1^ω, q_1^ω) which satisfy the maximization problems (4), (5), (7) of all three agents as well as the market-clearing conditions for Arrow securities $b_1^\omega = b_{1,h}^\omega$ and the asset market $f_1^\omega = f_{1,l}^\omega \forall \omega$.

3.1 Backward Induction: Period 1 Equilibrium

We solve the problem of bankers by backward induction: we first analyze their optimal period 1 and 2 allocations, given that the state of the world ω is realized at the beginning of period 1; then we proceed to solve for the optimal financing decision in period 0.

After the productivity shock ω has been realized, denote by $V(a^\omega)$ the utility that a banker obtains from his net liquid asset holdings $a^\omega = A_1^\omega t_1 - b_1^\omega$ in the beginning of period 1. We denote the Lagrangian of the associated optimization problem as follows. (Since there are no further shocks after period 1, we drop the superscript ω for ease of notation.)

$$V(a) = \max_{\{c_1, f_1\}} c_1 + \bar{A}_2(t_1 - f_1) - \mu [c_1 - a - q_1 f_1] + \lambda c_1 \quad (9)$$

The first order conditions are

$$\begin{aligned} \text{FOC}(c_1) : \quad & \mu = 1 + \lambda \\ \text{FOC}(f_1) : \quad & \bar{A}_2 = \mu q_1 \end{aligned}$$

Depending on the amount of initial liquid assets a at the beginning of period 1, we distinguish two equilibria:

Unconstrained equilibrium for $a \geq 0$: For non-negative liquid asset holdings at the beginning of period 1, the optimum allocation of bankers is unconstrained: they consume their liquid wealth in period 1 $c_1 = a$ and do not engage in fire sales $f_1 = 0$. In period 2, they consume their production $c_2 = \bar{A}_2 t_1$. The shadow prices satisfy $\mu = 1$ and $\lambda = 0$. The allocation $f_1 = 0$ together with a price $q_1 = \bar{A}_2$ also constitutes an optimum for second-generation households.

Constrained equilibrium for $-s^{\max} \leq a < 0$: For negative liquid asset holdings, i.e. when the output of bankers in period 1 is insufficient to cover their payment obligation b_1 , bankers would like to roll over debt into period 2 but are prevented from doing so by the binding borrowing constraint. We denote the liquidity shortfall $s = -a$. Bankers choose period 1 consumption $c_1 = 0$ and engage in asset sales of $f_1 = f(s)$ to cover the liquidity shortfall s .

In period 2, they consume the output from their remaining asset holdings $c_2 = \bar{A}_2(t_1 - f_1)$. Second-generation households are willing to buy a level $f(s)$ of assets if the price declines to $q_1 = q(f(s))$ according to their optimality condition. Since bankers sell assets at prices that are below their marginal product, we call their sales ‘‘fire sales.’’ The shadow price of liquidity of bankers is

$$V'(a) = \mu = \bar{A}_2/q_1 > 1 \quad (10)$$

This reflects that an additional unit of liquidity could buy up $1/q_1$ assets and earn a return of \bar{A}_2 .

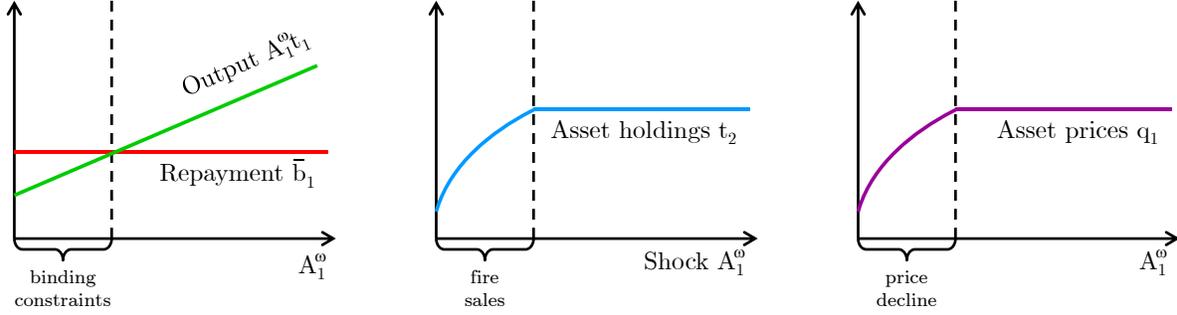


Figure 2: Fire sales and price declines as a function of A_1^ω for $b_1^\omega = \bar{b}_1$

Financial Amplification Effects

Figure 2 depicts a comparative static analysis of the economy's equilibrium in period 1 for a fixed repayment obligation \bar{b}_1 . The lower the productivity A_1^ω , the lower the liquidity of bankers (left panel). If bankers produce less than the debt level $A_1^\omega t_1 < \bar{b}_1$, they experience binding constraints. As a result, they have to engage in fire sales of some of their productive asset holdings (center panel), which reduce the equilibrium price q_1 (right panel).

The effects of shocks under this constrained regime are magnified by financial amplification: suppose that bankers are constrained, selling $f_1 > 0$ to meet their period 1 repayment obligation, and suddenly experience a small shock $ds > 0$ to their liquidity position. The partial equilibrium effect is that they are forced to fire-sell an additional $\frac{ds}{q_1}$ of their productive assets. This sale depresses the price q_1 by $\frac{ds}{q_1} \cdot \frac{\partial q_1}{\partial f_1} < 0$. By implication bankers receive $\frac{ds}{q_1} \cdot \frac{\partial q_1}{\partial f_1} \cdot f_1 < 0$ less on their prior fire sales f_1 and need to increase sales by $\frac{ds}{q_1} \cdot \left(\frac{\partial q_1}{\partial f_1} \cdot \frac{f_1}{q_1} \right) = \frac{ds}{q_1} \cdot \eta_{qf}$, leading to further price declines $\frac{ds}{q_1} \cdot \eta_{qf} \cdot \frac{\partial q_1}{\partial f_1}$, a further reduction in revenues from asset sales, further fire sales $\frac{ds}{q_1} \cdot \eta_{qf} \cdot \left(\frac{\partial q_1}{\partial f_1} \cdot \frac{f_1}{q_1} \right) = \frac{ds}{q_1} \cdot (\eta_{qf})^2$ and so on. Summing up the resulting geometric series, we find that the shock leads to total asset sales of

$$\frac{df_1}{ds} = \frac{1}{q_1} \cdot \frac{1}{1 - \eta_{qf}} \quad (11)$$

Note that this expression can also be obtained by implicitly differentiating equation (8). The second factor in the expression is (by assumption A.2) greater than 1 and captures the effects of financial amplification.

3.2 Period 0 Financing Decisions

First-generation households consume $c_{0,h} = e - \alpha t_1$ in period 0 and $c_{1,h}^\omega = e + b_1^\omega$ in state ω of period 1. Following optimality condition (6), their pricing kernel m_1^ω is a function

of the payment b_1^ω they receive in state ω of period 1,

$$m_1^\omega = m(b_1^\omega) = \frac{u'(e + b_1^\omega)}{u'(e - \alpha t_1)} \quad (12)$$

The period 0 optimization problem of bankers can be reformulated by employing the definition of V in equation (9),

$$\max_{\{b_1^\omega\}} E \{V(A_1^\omega t_1 - b_1^\omega)\} \quad \text{s.t.} \quad \alpha t_1 = E[m_1^\omega b_1^\omega]$$

Assigning a shadow price of ν to the period 0 budget constraint, the first-order condition of the Lagrangian to this problem for security issuance in a given state ω is

$$V'(a^\omega) = \nu m_1^\omega \quad (13)$$

or, substituting for $V'(a^\omega) = \mu^\omega = \bar{A}_2/q_1^\omega$ and for m_1^ω ,

$$\frac{\bar{A}_2}{q_1^\omega} = \nu \cdot \frac{u'(e + b_1^\omega)}{u'(e - \alpha t_1)} \quad (14)$$

Optimality requires that the marginal valuations of liquidity of bankers and of first generation households are proportional across all ω , with the factor of proportionality ν reflecting the shadow cost of raising funds in period 0.

Since bankers are risk-neutral and first-generation households are risk-averse, let us first consider the case that bankers promise a risk-free payment \bar{b}_1 to first generation households that is constant across all states of nature. In order to cover the initial investment of bankers, such a payment would have to satisfy

$$\alpha t_1 = m(\bar{b}_1) \bar{b}_1 = \frac{u'(e + \bar{b}_1)}{u'(e - \alpha t_1)} \cdot \bar{b}_1 \quad (15)$$

Bankers can afford this period 1 payment without incurring fire sales as long as $\bar{b}_1 \leq A^{\min} t_1$ in the lowest state of nature where $A_1^\omega = A^{\min}$. We define the maximum period 0 investment $\hat{\alpha} t_1$ such that they will not occur binding constraints as

$$\hat{\alpha} t_1 = m(A^{\min} t_1) A^{\min} t_1 = \frac{u'(e + A^{\min} t_1)}{u'(e - \hat{\alpha} t_1)} \cdot A^{\min} t_1 \quad (16)$$

Note that this threshold is higher the greater the minimum period 1 return A_1^{\min} and the higher the elasticity of substitution of first generation households. (A higher elasticity of substitution implies that households require less compensation to accept an unsmooth consumption profile.)

We characterize the period 0 equilibrium of the economy as follows:

Proposition 1 (Decentralized Equilibrium) 1. If $\alpha \in [0, \hat{\alpha}]$, the equilibrium exhibits loose constraints in all states of nature. Bankers absorb all risk and make an uncontingent payment \bar{b}_1 to first-generation households as determined by equation (15). They do not engage in fire sales $f_1^\omega = 0$ and consume the remainder of their period 1 income $c_1^\omega = A_1^\omega t_1 - \bar{b}_1$.

2. If $\alpha > \hat{\alpha}$, the equilibrium exhibits occasionally binding constraints and is characterized by a productivity threshold \hat{A}_1 such that:

- for $A_1^\omega \geq \hat{A}_1$, bankers are unconstrained, provide a constant payment $\bar{b}_1 = \hat{A}_1 t_1$ to households, do not engage in fire sales $f_1^\omega = 0$ and consume the remainder of their period 1 income $c_1^\omega = A_1^\omega t_1 - \bar{b}_1$,
- for $A_1^\omega < \hat{A}_1$, bankers are constrained, provide a reduced repayment $b_1^\omega < \bar{b}_1$ to households, engage in positive fire sales $f_1^\omega > 0$ and consume zero $c_1^\omega = 0$. The lower A_1^ω in this region, the more bankers reduce their payment b_1^ω and the larger their fire sales f_1^ω .

Proof. The proof including a detailed derivation of b_1^ω and \hat{A}_1 is given in the appendix.

■

Our results are illustrated graphically in figure 3: The left panel depicts case 1 in which bankers can afford a constant payment \bar{b}_1 to households in all states of nature and absorb all risk in their consumption. This arrangement can be interpreted as risk-free bond finance.

The right panel depicts the situation in which the financing requirement αt_1 of bankers is so high that they cannot afford a constant payment without fire sales. By implication, they become constrained in low states of nature $A_1^\omega < \hat{A}_1$ and no longer find it optimal to absorb all risk. The resulting payment profile is similar to a defaultable bond: bankers make a fixed payment in high states of nature – as long as their output is sufficient to cover this fixed payment. They pay their entire output plus receipts from fire sales in low states of nature when they are in financial distress.

Reducing the payment b_1^ω or engaging in fire sales f_1^ω in constrained states of nature are two alternative costly ways of obtaining liquidity: when bankers engage in fire-sales, asset prices decline so that their proceeds are less than the marginal product that they could have earned on the assets. Similarly, if bankers reduce their payments b_1^ω to households in low states so as to insure themselves and increase b_1^ω in high states of nature to make up for it, the total interest bill rises, since households are risk-averse and would prefer a constant payment across all states of nature. Bankers pick their portfolios such that the relative costs of the two forms of raising liquidity are equal from their private perspective, as described by the optimality condition (13).

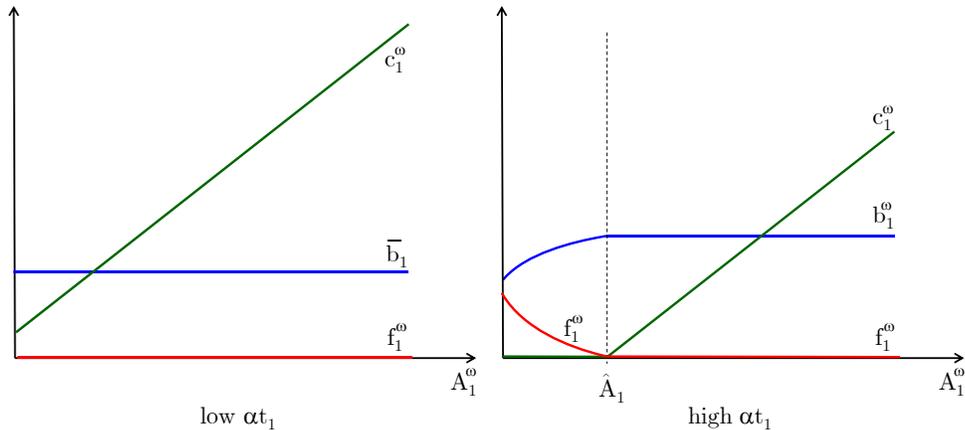


Figure 3: Contingent repayment b_1^ω , fire sales f_1^ω and consumption c_1^ω

4 Welfare Analysis

In this section, we compare the allocations of the decentralized equilibrium with those chosen by a constrained social planner. Our social planning framework is the one developed by Stiglitz (1982) and Geanakoplos and Polemarchakis (1986) who study an economy in which asset markets are open in period 0 and allow agents to trade securities with different payoffs across different states of nature; in period 1 a state of nature is realized, agents receive the payoffs of their security holdings, and a spot market opens in which they allocate their liquid net worth across different commodities.

If the period 0 security market is incomplete, Stiglitz (1982) and Geanakoplos and Polemarchakis (1986) show that the decentralized equilibrium in such economies is generically inefficient, i.e. a planner can choose a reallocation in period 0 markets that leads to a Pareto improvement, given that all prices and allocations in the associated period 1 spot markets adjust to maintain equilibrium. The reason is that a constrained planner internalizes that reallocating the security portfolios of private agents in period 0 leads to price changes in period 1 spot markets which have redistributive effects. Under complete security markets, there are no benefits to such redistributions since agents are already optimally insured. However, under incomplete markets, the marginal rates of substitution of different agents across states of nature generally differ, and the redistributions stemming from price changes in period 1 spot markets have first-order welfare effects. In other words, reallocations in period 0 security markets allow the planner to trigger redistributions in period 1 spot markets that improve risk-sharing and therefore mitigate the market incompleteness.

In the context of our model, a constrained planner can manipulate the period 0 insurance decisions of bankers so as to affect the extent of fire sales of capital assets in the

period 1 spot market. By reducing fire sales, the planner can push up the price of capital assets in states of nature when bankers are financially constrained and when their valuation of wealth is relatively higher than that of households. Such a redistribution has first-order welfare benefits. However, if we limit our analysis to one-way transfers from households to bankers, the resulting equilibrium would clearly not constitute a Pareto improvement. As in Stiglitz (1982) and Geanakoplos and Polemarchakis (1986), a Pareto improvement requires that the planner also distributes resources in the opposite direction, i.e. from bankers to households, in states of nature in which bankers are unconstrained. The constrained planner in Stiglitz (1982) and Geanakoplos and Polemarchakis (1986) performs such redistributions by changing the allocations of different agents in the period 0 security market while ignoring budget constraints, i.e. under the assumption that the planner can simply decree the security holdings of different agents.

In a market setting, affecting the portfolio allocations of private agents in the desired directions requires changing their incentives via tax or subsidy measures while simultaneously engaging in compensatory transfers.⁸ The taxes and subsidies optimally target the marginal incentives of agents and the transfers ensure that efficiency gains in the economy are spread among agents such that a Pareto improvement is achieved.

Before proceeding, let us also note that the allocations of a constrained planner differ from the allocations in a first-best world, in which a planner has the ability to arbitrarily redistribute funds between agents in the economy. In a first-best world, bankers would hold all productive assets in the economy since they have the superior production technology. Financial constraints would be irrelevant and the solution would be trivial. Focusing on a constrained planning setup is useful because it corresponds more closely to the situation that policymakers and regulators face in the real world: they often have to take the existence of financial constraints and other market imperfections as given and attempt to maximize societal welfare subject to those constraints.

In the following subsections, we first illustrate how a marginal reallocation of security issuance in period 0 aimed at reducing fire sales can lead to a Pareto improvement. Then we solve the full constrained planning problem in the described economy.

4.1 Effects of Marginal Reduction in Fire Sales

Let us first analyze the scope for Pareto improvements in the economy by considering the welfare effects of a marginal reallocation of security issuance that aims at reducing fire sales. Suppose the economy is in a decentralized equilibrium with occasionally binding constraints (as described in proposition 1). Assume there are two states of nature

⁸This setup is also used e.g. in Lorenzoni (2008).

$\omega, \psi \in \Omega$ of equal probability density where the period 1 equilibrium in state ω exhibits binding constraints and in state ψ loose constraints. Consider a planner in period 0 who reduces security issuance of bankers in the constrained state ω by an infinitesimal amount db_1^ω in period 0 while holding the prices of Arrow securities constant. In order to satisfy the period 0 budget constraint of bankers, the planner increases security issuance conditional on the unconstrained state ψ by $db_1^\psi = -db_1^\omega \frac{m_1^\omega}{m_1^\psi} > 0$. By the envelope theorem, the change in utility of first-generation households is second-order because they were previously at their optimum.⁹

In period 1, bankers have db_1^ω more liquid resources in state ω . An atomistic agent who takes asset prices as given would anticipate that this allows him to reduce fire sales by

$$\frac{\partial f_1}{\partial b_1^\omega} = \frac{1}{q_1}$$

However, in general equilibrium, the reduction in fire sales pushes up asset prices and leads to an amplified decline in fire sales by

$$\frac{df_1}{db_1^\omega} = \frac{1}{q_1} \cdot \frac{1}{1 - \eta_{qf}}$$

as we captured in equation (11). Employing these assets in production allows bankers to consume $\frac{A_2}{q_1^\omega} \cdot \frac{db_1^\omega}{1 - \eta_{qf}} = \frac{\mu^\omega db_1^\omega}{1 - \eta_{qf}}$ more in period 2 of state ω .

Similarly, in state ψ the increase in the promised repayment requires bankers to reduce period 1 consumption by $db_1^\psi = -\frac{m_1^\omega}{m_1^\psi} db_1^\omega = -\mu^\omega db_1^\omega$, since bankers in the decentralized equilibrium choose repayments such that $\frac{m_1^\omega}{m_1^\psi} = \frac{\mu^\omega}{\mu^\psi}$ and since $\mu^\psi = 1$ in unconstrained states. The net change in the banker's state utility from the planner's reallocation over states ω and ψ is

$$\frac{d(V^\omega + V^\psi)}{db_1^\omega} = \frac{\mu^\omega}{1 - \eta_{qf}} - \mu^\omega = \frac{\eta_{qf}}{1 - \eta_{qf}} \cdot \mu^\omega$$

Second-generation households are unaffected in state ψ , but pay $dq_1^\omega = \frac{\partial q}{\partial f} df_1^\omega = \frac{1}{f_1^\omega} \cdot \frac{\eta_{qf}}{1 - \eta_{qf}} db_1^\omega$ more per unit of asset purchased, implying a change in their utility in state ω of

$$\frac{dW^\omega}{db_1^\omega} = -f_1^\omega \cdot \frac{dq_1^\omega}{db_1^\omega} = -\frac{\eta_{qf}}{1 - \eta_{qf}}$$

(Since second-generation households purchase assets up to the point where $F'(f_1^\omega) = q_1^\omega$, the welfare effects of the reduction in the quantity of assets used in production are second order.)

⁹If we allow the prices m_1^ω and m_1^ψ of Arrow securities to adjust, then there is a redistribution of welfare from bankers to first-generation households, which could be undone by a compensatory transfer from households to bankers, as we will show in the following subsection.

Since $\mu^\omega > 1$, the planner could transfer $\frac{\eta_{af}}{1-\eta_{af}} db_1^\omega$ from bankers to second-generation households in the unconstrained state ψ to compensate them for the reallocation in state ω . This leaves households indifferent and achieves a first order welfare gain for bankers. The described reallocation therefore constitutes a Pareto improvement.

4.2 Constrained Planning Problem

We introduce a constrained social planner who determines the financing and risk-taking decisions in the period 0 security market of the economy, as captured by the variable b_1^ω , while taking as given that private agents behave in an optimizing fashion in the period 1 market for productive assets. We formulate the planner's problem as implementing the constrained equilibrium with specific taxes τ^ω on the security issuance b_1^ω of bankers which are rebated lump-sum.

As in Geanakoplos and Polemarchakis (1986) and Lorenzoni (2008), the planner also makes compensatory transfers between bankers and households in period 0 to ensure that her reallocation of resources leads to a Pareto improvement. We denote by T_0 a transfer from bankers to first-generation households in period 0. Furthermore, we denote by T_1^ω a transfer from bankers to second-generation households. We can think of this transfer as an allocation of Arrow securities issued by bankers that is set aside in period 0 and paid out to second-generation households when they are born in period 1. We impose the constraint $T_1^\omega \geq 0$. A natural interpretation for this is that the planner cannot compel unborn households in period 0 to make state-contingent payments to bankers. The constraint ensures that the planner can use such transfers to compensate households but not to circumvent the financial constraint of bankers by providing them with resources when they are constrained.¹⁰ After the planner has determined allocations and transfers in period 0, private agents follow their optimal strategies in periods 1 and 2.

We formalize the optimization problem of the planner as maximizing the welfare of bankers subject to the constraint that first- and second-generation households are at

¹⁰We study an alternative to the constraint above, that the planner is limited to an uncontingent transfer \bar{T}_1 to second-generation households, in appendix B.2. If the planner's ability to engage in transfers was completely unconstrained, then financial constraints would become irrelevant and the first-best equilibrium could be implemented in which bankers operate all productive assets and no fire sales occur.

least as well off as in the decentralized equilibrium:

$$\begin{aligned}
& \max_{\{c_{1b}^\omega, c_{2b}^\omega, b_1^\omega, f_1^\omega, \tau^\omega, \nu, T_0, T_1\}} E [c_{1b}^\omega + c_{2b}^\omega] & (17) \\
& \text{s.t.} \quad \alpha t_1 = E[m_1^\omega b_1^\omega] - T_0 \\
& \quad c_{1b}^\omega = A_1^\omega t_1^\omega - b_1^\omega + q_1^\omega f_1^\omega - T_1^\omega \geq 0, \quad T_1^\omega \geq 0 \\
& \quad c_{2b}^\omega = \bar{A}_2 (t_1 - f_1^\omega) \\
& \quad \frac{\bar{A}_2}{q_1^\omega} = \nu^{DE} (m_1^\omega + \tau^\omega) \\
& \quad m_1^\omega = \frac{u'(e + b_1^\omega)}{u'(e - \alpha t_1)} \\
& \quad q_1^\omega = F'(f_1^\omega) \\
& \quad U = u(e - E[m_1^\omega b_1^\omega] + T_0) + E[u(e + b_1^\omega)] \geq U^{DE} \\
& \quad W = E[(e - q_1^\omega f_1^\omega + T_1) + (e + F(f_1^\omega))] \geq W^{DE}
\end{aligned}$$

The first three constraints capture the budget constraints of bankers in periods 0, 1 and 2 as well as the non-negativity constraint on consumption c_{1b}^ω and on transfers T_1^ω . The next three constraints denote the relevant optimality conditions of bankers, first-, and second-generation households where ν^{DE} is a suitable constant. The final two constraints ensure that households are at least as well off as in the decentralized equilibrium U^{DE} and W^{DE} . (Throughout this section, we refer to variables in the decentralized equilibrium with the superscript ‘ DE ’ and – in case of ambiguity – to variables in the social planner’s allocation by the superscript ‘ SP ’.)

We start our analysis by observing that the problem of implementing the constrained planner’s allocation via taxes and transfers, as captured by the maximization problem (17), is equivalent to the problem of a planner who directly chooses a period 0 asset allocation and a transfer T_1^ω .

Lemma 1 *The constrained social planner’s problem (17) can equivalently be stated as*

$$\begin{aligned}
& \max_{\{c_{1b}^\omega, b_1^\omega, f_1^\omega, T_1\}} E [c_{1b}^\omega + \bar{A}_2 (t_1 - f_1^\omega)] & (18) \\
& \text{s.t.} \quad c_{1b}^\omega = A_1^\omega t_1^\omega - b_1^\omega + f_1^\omega F'(f_1^\omega) - T_1^\omega \geq 0, \quad T_1^\omega \geq 0 \\
& \quad U = u(e - \alpha t_1) + E[u(e + b_1^\omega)] \geq U^{DE} \\
& \quad W = E[(e - f_1^\omega F'(f_1^\omega) + T_1) + (e + F(f_1^\omega))] \geq W^{DE}
\end{aligned}$$

Proof. In problem (18), we have substituted the period 0 budget constraint of bankers in the definition of U to obtain the aggregate resource constraint $c_{1h} = e - \alpha t_1$. We have substituted the period 2 budget constraint of bankers directly into the objective of the

maximization problem. Since the planner can freely choose b_1^ω by setting appropriate state-contingent taxes τ^ω , we observe that the related optimality condition of bankers does not impose a constraint on the problem. Since the effects of changes in the prices m_1^ω can be undone by the transfer T_0 , the optimality condition of first-generation households can be omitted as well. Finally, we substitute $q_1^\omega = F'(f_1^\omega)$ for the asset price. The resulting problem is the one given in (18). ■

We assign the multipliers μ^ω , λ^ω and κ^ω to the period 1 budget constraint of bankers, and the consumption and transfer non-negativity constraints, and the multipliers ν and ψ to the constraints on household utility of the two generations. For ease of comparison of our results with the decentralized equilibrium, we divide the constraint on U by the constant $u'(e - \alpha t_1)$. The resulting first-order conditions of the problem are

$$\begin{aligned} FOC(c_{1b}^\omega) &: \mu^\omega = 1 + \lambda^\omega \\ FOC(b_1^\omega) &: \mu^\omega = \nu \cdot \frac{u'(e + b_1^\omega)}{u'(e - \alpha t_1)} \end{aligned} \quad (19)$$

$$FOC(f_1^\omega) : \bar{A}_2 = \mu^\omega [F' + f_1^\omega F''] - \psi f_1^\omega F'' \quad (20)$$

$$FOC(T_1^\omega) : \mu^\omega = \psi + \kappa^\omega \quad (21)$$

In the following, we focus on equilibria in which bankers have sufficient resources in unconstrained states of nature to compensate second-generation households without engaging in fire sales. (This is generally the case when fire sales are an infrequent phenomenon. Appendix B.1 provides results for the more general case.) Since bankers and second-generation households have linear utility, the precise allocation of transfers T_1^ω across unconstrained states of nature is indeterminate. However, any set of such transfers has to satisfy

$$E[T_1^\omega] = E \left[F(f_1^{\omega,DE}) - q_1^{\omega,DE} f_1^{\omega,DE} - F(f_1^{\omega,SP}) + q_1^{\omega,SP} f_1^{\omega,SP} \right] \quad (22)$$

$$\text{and } 0 \leq T_1^\omega \leq \max\{0, a^\omega\}$$

In expectation, second-generation households receive in transfers what they lose from reduced fire sales (line 1), but the planner provides transfers to them only when bankers have sufficient liquidity (line 2). In such allocations, observe that $\mu^\omega = \psi = 1$ whenever $a^\omega > 0$, i.e. the valuation of liquidity of bankers and second-generation households coincides. On the other hand, when $a^\omega < 0$ no transfer takes place and $\mu^\omega > \psi = 1$.

Recall that we interpret μ^ω as the valuation of period 1 liquidity in the banking sector. For the set of equilibria under consideration, the first-order condition (20) implies

$$\mu^{\omega,SP} = \frac{\bar{A}_2/F' - \eta_{qf}}{1 - \eta_{qf}} \quad (23)$$

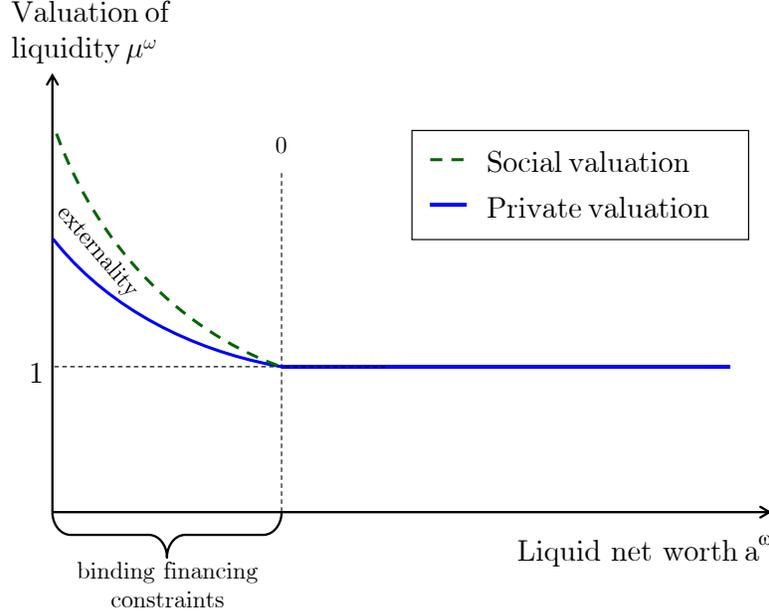


Figure 4: Private and Social Valuation of Liquidity

where $\eta_{qf} = -f_1^\omega F''/F'$. Note that the denominator is positive by assumption A.2. By comparing the constrained planner's valuation of liquidity in the banking sector (23) for a given period 1 allocation with that of decentralized bankers (10) we find

Lemma 2 (Valuation of Liquidity) *When the financial constraint on bankers is loose, the planner and decentralized agents value liquidity equally at $\mu^{\omega,SP} = \mu^{\omega,DE} = 1$.*

When the financial constraint on bankers is binding, the constrained planner values liquidity in the banking sector more than decentralized agents $\mu^{\omega,SP} > \mu^{\omega,DE}$.

Proof. When the financial constraint on bankers is loose, there are no fire sales ($f_1^\omega = 0$) and $F' = \bar{A}_2$ so both expressions are unity.

When the constraint is binding, equation (10) implies $\mu^{\omega,DE} = \bar{A}_2/F' > 1$ and comparison with equation (23) directly yields the result. ■

When there are positive fire sales $f_1^\omega > 0$, a planner internalizes that reducing fire-sales keeps asset prices higher and leads to a wealth transfer from households to constrained bankers, which is beneficial since $\mu^{\omega,DE} > \psi$. Put differently, the planner internalizes that mitigating the asset price decline tempers the financial amplification effects since it raises the amount of liquidity that bankers obtain from the sale of each unit of their assets. This misvaluation is the basis of the inefficiency result in our paper.

Figure 4 schematically depicts the valuation of liquidity of decentralized agents and the planner across different states of nature as a function of bankers' liquid net worth

a^ω . In normal times, i.e. for $a^\omega \geq 0$, the financial constraints of bankers are loose and the two valuations of liquidity coincide and equal 1. When financing constraints are binding and the valuation of liquidity is above average, the planner internalizes that higher liquidity in the banking sector would mitigate the downward spiral in asset prices and production.

Optimal Period 0 Financing Decisions

When the social planner makes her optimal period 0 financing decisions, she recognizes the social valuation of liquidity as illustrated in figure 4 rather than the private valuation perceived by bankers because she internalizes the pecuniary externalities arising from fire sales. This difference in the valuation of liquidity is directly reflected in their period 0 allocations of Arrow securities:

Proposition 2 (Excessive Systemic Risk-Taking) *1. If $\alpha \in [0, \hat{\alpha}]$, the equilibrium exhibits loose constraints in all states of nature. The allocation chosen by a social planner coincides with the decentralized equilibrium.*

2. If $\alpha > \hat{\alpha}$, the social planner's allocation exhibits occasionally binding constraints and is characterized by a productivity threshold for binding constraints that is higher than in the decentralized equilibrium $\hat{A}_1^{SP} > \hat{A}_1^{DE}$ such that:

- *For $A_1^\omega \geq \hat{A}_1^{SP}$, the planner contracts a higher fixed payment $\bar{b}_1^{SP} = \hat{A}_1^{SP} t_1 > \bar{b}_1^{DE}$ than decentralized bankers and there are no fire sales.*
- *For $A_1^\omega < \hat{A}_1^{SP}$, the planner chooses payments below $\hat{A}_1^{SP} t_1$ and provides a reduced repayment $b_1^\omega < \bar{b}_1^{SP}$ to households, engages in positive fire sales $f_1^\omega > 0$ and sets the consumption of bankers to zero $c_1^\omega = 0$.*

Proof. The proof is given in the appendix. ■

Figure 5 illustrates the differences between the repayments contracted by decentralized bankers and by the planner. For low initial investment αt_1 , bankers make a fixed repayment \bar{b}_1 and do not experience binding constraints – and this coincides with the planner's equilibrium. For high initial investment the planner repays more in unconstrained states and less in the most constrained states of nature than decentralized bankers – she “reshuffles” payments from strongly constrained states to unconstrained states of nature in order to reduce socially inefficient fire sales and output declines. In other words, the planner purchases more insurance against low output states that exhibit binding constraints.

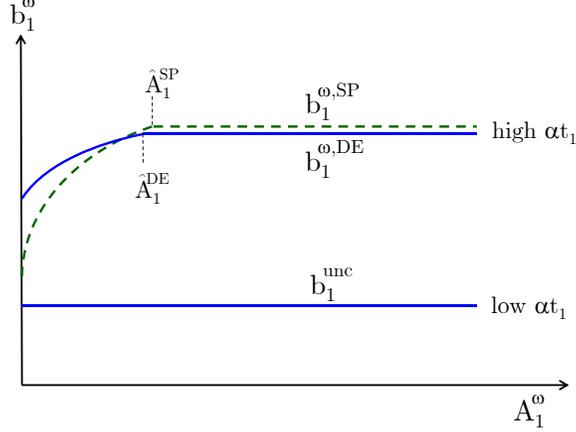


Figure 5: Comparison of repayment b_1^ω for decentralized bankers and planner

In practice, the planner's intervention could be interpreted as substituting uncollateralized debt finance for other, more contingent forms of finance, e.g. preferred stock or reverse convertible bonds.¹¹

Since the planner promises a higher repayment $\bar{b}_1^{SP} > \bar{b}_1^{DE}$ in good states of nature, it is natural that the threshold for binding constraints \hat{A}_1^{SP} and the probability of incurring a binding constraint rises in the planner's allocation: the income of bankers is not sufficient to meet the higher payment \bar{b}_1^{SP} at the old threshold level \hat{A}_1^{DE} without engaging in fire-sales. Therefore the planner reduces the repayment and engages in fire sales already starting at a higher threshold $\hat{A}_1^{SP} > \hat{A}_1^{DE}$.

4.3 Incidence of Systemic Risk

In the described economy, the aggregate productivity shock A_1^ω constitutes *systematic* risk. Whenever financial constraints are binding, amplification effects are triggered and the shock triggers *systemic* risk, which bankers would like to insure against. First generation households are risk-averse and require compensation for taking on this risk, and the decentralized equilibrium is therefore characterized by the privately optimal trade-off between the cost of consumption volatility for households and the efficiency cost of fire-sales for bankers. However, since decentralized bankers internalize only part of the social benefit of insuring against fire-sales, they leave themselves exposed to too much systemic risk. As a result, the economy is characterized by excessive financial amplification and excessive declines in asset prices and output in low states of nature.

¹¹Preferred stock promises a fixed dividend but allows the issuer to skip the dividend without triggering a default event if cash flow problems arise. Reverse convertible bonds grant the issuer the right to deliver stock instead of repaying the principal if the stock value of the issuer falls below a threshold. Both instruments therefore have a payoff profile similar to the one depicted in figure 5.

We emphasized in propositions 1 and 2 that systemic risk and socially excessive fire sales arise whenever the initial investment requirement of bankers αt_1 is so high that fire sales in low states of nature occur, as captured by condition (16). Let us discuss the circumstances that determine whether this condition is likely to be satisfied.

Corollary 1 (Incidence of Systemic Risk) *The economy is more vulnerable to systemic risk, i.e. the inequality $\alpha t_1 \leq \hat{\alpha} t_1$ is more likely to be violated,*

- *the higher the initial financing requirement αt_1 of bankers*
- *the lower the endowment of first-generation households*
- *the lower the minimum period 1 return of bankers $A^{\min} t_1$.*

Proof. The proof follows directly from the definition of the threshold $\hat{\alpha} t_1$ in equation (16) and our assumption that $A^{\min} \geq 0$. ■

The threshold $\hat{\alpha} t_1$ above which the economy becomes vulnerable to binding constraints also depends on the degree of risk-aversion of first-generation households, since greater risk aversion (and lower willingness to substitute intertemporally) increases the cost of raising finance and of insuring bankers.

For simplicity, we assume for the remainder of subsection 4.3 that the utility function of first-generation households exhibits constant relative risk aversion θ , i.e. their utility function is $(c^{1-\theta} - 1) / (1 - \theta)$.

Corollary 2 (Risk Aversion and Incidence of Systemic Risk) *If first-generation households have CRRA utility, the threshold $\hat{\alpha} t_1$ is lower the greater the coefficient of relative risk aversion θ . As the degree of risk aversion declines, we observe $\lim_{\theta \rightarrow 0} \hat{\alpha} = A^{\min}$. If households become fully risk-neutral, they insure bankers against binding constraints as long as $E[A_1] \geq \alpha$, and the resulting equilibrium is constrained efficient.*

Proof. For CRRA utility, we find that the threshold satisfies

$$\hat{\alpha} t_1 = A^{\min} t_1 \cdot \left(\frac{e - \hat{\alpha} t_1}{e + A^{\min} t_1} \right)^\theta$$

where the fraction in parentheses is less than one, which implies that $\hat{\alpha} t_1 < A^{\min} t_1$.

Implicitly differentiating this expression yields $\partial \hat{\alpha} t_1 / \partial \theta < 0$. In other words, the threshold for binding constraints declines as households are less and less willing to substitute consumption intertemporally, since the return that they demand from bankers to accept an unsmooth consumption profile rises.

Applying the limit to this equation, we find $\lim_{\theta \rightarrow 0} \hat{a}t_1 = A^{\min}t_1$, i.e. as households become less averse to intertemporal substitution, they are willing to lend up to $\hat{a}t_1 = A^{\min}t_1$ in period 0 against a repayment of $A^{\min}t_1$ in period 1, which bankers can meet without resorting to fire sales.

If households are fully risk-neutral, then $m_1^\omega = 1 \forall \omega$, i.e. households do not care in which states of nature they are repaid in period 1. Bankers raise finance using any bundle of Arrow securities that satisfies $E[b_1] = \alpha t_1$. They can meet these repayments without triggering binding constraints as long as their period 1 income is in expectation sufficient to cover the initial investment, i.e. $E[A_1]t_1 \geq \alpha t_1$. Since this equilibrium exhibits no fire sales, it is constrained efficient. ■

We can conduct a similar comparison for the magnitude of the undervaluation of liquidity by constrained bankers in the decentralized equilibrium:

Corollary 3 (Magnitude of Undervaluation of Liquidity) *For a given liquidity position $a^\omega < 0$, the undervaluation of liquidity by bankers is greater the higher the demand elasticity for fire sales η_{qf} of second-generation households.*

Proof. The result follows from comparing equations (10) for $\mu^{\omega, DE}$ and equation (23) for $\mu^{\omega, SP}$ and noting that $\partial \mu^{\omega, SP} / \partial \eta_{qf} > 1$ whenever $\bar{A}_2/q > 1$. ■

The demand elasticity η_{qf} captures the price impact of asset sales. The more sensitive the price q is to fire sales f , the greater the scope for a planner to redistribute resources to financially constrained bankers by reducing fire sales. In the limit of $\eta_{qf} \rightarrow 0$, i.e. if the production technology of second-generation households is equally efficient as that of bankers, asset sales do not depress asset prices (they no longer constitute “fire sales”), and the resulting equilibrium is socially efficient.

4.4 Macro-prudential Regulation

The constrained planner’s optimal allocation can be implemented in a market setting by imposing a set of taxes on the risk-taking decisions of bankers that bring the private costs of risk-taking in line with the social cost. We may describe this set of taxes as “macro-prudential regulation,” since it closely captures what the Bank for International Settlements defines as the macro-prudential approach to regulation (see e.g. Borio, 2003): it is designed to limit system-wide financial distress that stems from the correlated exposure of financial institutions and to avoid the resulting real output losses in the economy.

Definition 1 (Externality Pricing Kernel) *We define the externality pricing kernel τ^ω of bankers as the difference between the private valuation and the planner's social valuation of period 1 liquidity*

$$\tau^\omega = \mu^{\omega,SP} - \mu^{\omega,DE} \quad (24)$$

This kernel captures the un-internalized social cost of financially constrained bankers making a payment of one dollar in state ω . Following lemma 2, the externality kernel is zero in unconstrained states and positive in constrained states. Since lower realizations of productivity are associated with tighter constraints, we find $\text{Cov}(\tau^\omega, A_1^\omega) < 0$ whenever there are states with binding constraints ($\alpha t_1 < \hat{\alpha} t_1$).

It is instructive to substitute the valuation of liquidity of decentralized bankers and of the planner for a given banker liquidity position a^ω into the definition of the externality kernel (24):

$$\tau^\omega = \frac{\bar{A}_2/F' - \eta_{qf}}{1 - \eta_{qf}} - \frac{\bar{A}_2}{F'} = \eta_{qf} \cdot \lambda^{\omega,SP}$$

The kernel reflects that an additional unit of liquidity in the banking sector reduces fire sales such that the constraint of all other bankers is relaxed by η_{qf} , and the planner values this relaxation of the constraint at the shadow price $\lambda^{\omega,SP}$.

Corollary 4 (Macro-prudential Taxation) *A planner can implement the constrained efficient allocation by imposing a state-contingent specific tax $\nu\tau^\omega$ on the issuance of Arrow securities b_1^ω that is rebated in lump sum fashion. Compensatory transfers T_0 and T_1^ω ensure that the resulting allocation constitutes a Pareto improvement, where T_0 satisfies $T_0 = E[m_1^\omega b_1^\omega] - \alpha t_1$ and T_1^ω is determined by condition (22).*

Proof. First, observe that a tax $\nu\tau^\omega$ on issuing Arrow securities b_1^ω makes the optimality conditions (13) and (19) of decentralized bankers and the constrained planner coincide. The transfer T_0 ensures that bankers can just afford the constrained planner's allocation of b_1^ω and households end up with period 0 consumption $c_{0h} = e - \alpha t_1$. The transfers T_1^ω satisfying (22) ensure by definition that second-generation households are as well off as in the decentralized equilibrium. ■

Remark: It would be equivalent in our model to impose a specific tax τ^ω on the repayments of Arrow securities, or to impose a proportional tax $\tau^\omega/\mu^{\omega,SP}$ on the issuance or payment on Arrow securities. Furthermore, observe that the transfers T_0 and T_1^ω are greatly facilitated by the fact that the planner raises revenue from the taxation of Arrow securities.

Next, consider an atomistic banker in the equilibrium described above who sells a financial claim in period 0 that has a state-contingent payoff profile X^ω in period 1. Such a claim can be viewed as a collection of Arrow securities with weights X^ω . For example, a risk-free bond corresponds to a vector $X^\omega = 1\forall\omega$.

Corollary 5 (Pricing of Systemic Risk) *The externalities imposed by a financial payoff X^ω are $E[\tau^\omega X^\omega]$. The optimal specific period 0 tax that induces a banker to internalize the full social cost of holding security X^ω is*

$$\tau_X = \nu E[\tau^\omega X^\omega] \tag{25}$$

This formulation draws a close parallel between traditional security pricing and the pricing of pecuniary externalities. The vector τ^ω can simply be viewed as a pricing kernel for systemic risk. The optimal proportional tax on payoff X^ω would be $E[\tau^\omega X^\omega] / E[\mu^{\omega,SP} X^\omega]$.

To gain some intuition, let us compare the magnitude of the externalities imposed by a number of securities with different payoff profiles. Figure 6 schematically depicts several examples. First, for an uncontingent bond with a face value of one dollar, the payoffs are $X^\omega = 1$ in all constrained and unconstrained states of nature. The externality of and optimal tax on such a bond is $\nu E[\tau^\omega]$.

Next consider a risky security with an expected payoff $E[X^\omega]$ of one dollar. The externality imposed by such a security is $E[\tau^\omega] + Cov(\tau^\omega, X^\omega)$. If the payoff X^ω of a security and the externality kernel have positive covariance, then the security imposes larger externalities and embodies more systemic risk than an uncontingent bond, and therefore calls for greater macro-prudential taxation. A stark example would be a credit default swap, which is likely to require large payouts precisely in times of financial turmoil, i.e. when economy-wide financial constraints bind and when the externality kernel τ^ω is high.¹²

On the other hand, the more negatively the payoffs X^ω of a security covary with the externality kernel, the more insurance the state-contingent payoff provides, the smaller the externality and the lower the optimal tax. An example would be if bankers sell equity paying dividends that are linear in the state of productivity A_1^ω , which is by construction negatively correlated with τ^ω . In the extreme case that a security only pays out in unconstrained states, the externality would be zero.

The optimal tax τ_X may be negative, i.e. may be a subsidy. If a security offers sufficient systemic insurance benefits in that it provides positive payoffs to bankers

¹²The payoff profile drawn in figure 6 is not based on a specific analytical example but illustrates the assumption that defaults in the economy occur when the banking sector as a whole experience binding constraints.

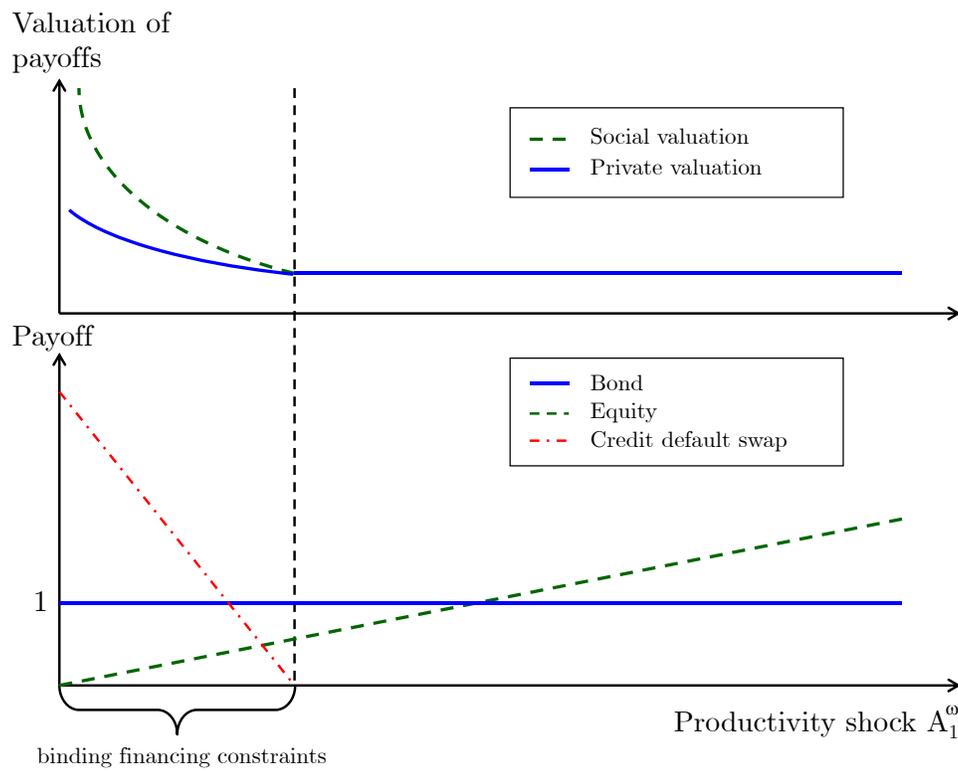


Figure 6: Schematic payoff profile of uncontingent bond, equity and credit default swap

precisely when they are constrained and subject to financial amplification effects, then it imposes a positive externality and should be subject to a subsidy, or a reduction in the capital requirements that banks are subject to. An example would be if bankers buy a credit default swap that shifts systemic risk to agents outside of the financial system who are not subject to financial constraints.

Equivalent Capital Adequacy Requirements While we have formulated our policy measures in terms of taxes, banking regulations typically take the form of capital adequacy requirements, which have tax-like effects since bank capital is costly. If the opportunity cost of holding one dollar of capital is δ for a bank, then a tax τ_X is equivalent to a capital requirement of τ_X/δ .

Macro- vs. Micro-Prudential Regulation Equation (25) captures that what matters for macro-prudential regulation is not the general riskiness inherent in a security, as described e.g. by the variance of its payoff, but rather the correlation with systemic risk, as described by its covariance with the externality kernel τ^ω . This is commonly viewed to be an important feature of macro-prudential regulation (Borio, 2003).

Leverage Leverage multiplies gains or losses by using uncontingent debt to increase the amount invested in a risky security. For example, if a risky investment with payoff X^ω in period 1 is leveraged by a factor $\alpha > 1$, then $(\alpha - 1)$ units are financed by debt and the total payoff is

$$\alpha X^\omega - (\alpha - 1) \frac{E[m_1^\omega X^\omega]}{E[m_1^\omega]}$$

where $E[m_1^\omega X^\omega]$ is the period 0 price of the payoff and $\frac{1}{E[m_1^\omega]}$ is the risk-free interest rate. This amounts to an increase in the dispersion of the total payoff by a factor α , which raises its covariance with the externality kernel in equation (25) equiproportionally and increases the externalities of the investment accordingly.

Reach of Regulation Our theory also offers insights into the question about the reach of regulation: macro-prudential regulation should apply to any financial market participant who might potentially be forced to engage in fire-sales during periods of system-wide amplification effects, since a rational private actor would not internalize the price effects of such sales and the externalities on the financing constraints of other market participants. This includes hedge funds and other actors in the so-called “shadow financial system.”

Socially Risk-Neutral Probabilities Pricing kernels can alternatively be represented as a risk-neutral probability measure that weighs states against which agents are risk-averse more highly. We can apply a similar transformation to the social planner’s pricing kernel. If regulators instruct banks to employ the regulator’s risk-neutral

probabilities in their risk management systems, the externality that is the topic of this paper would be alleviated.

If we denote the probability density function of state ω by $g(\omega)$, then we obtain the socially risk-neutral probability density from the standard formula

$$g_{rn}(\omega) = \frac{g(\omega)\mu^{SP,\omega}}{E[\mu^{SP,\omega}]}$$

where $E[\mu^{SP,\omega}]$ is calculated using the density $g(\omega)$.

We obtain the social value of a payoff X^ω as $E_{rn}[X^\omega]$, where $E_{rn}[\cdot]$ represents the expectations operator under the socially risk-neutral probability measure defined by g_{rn} . This measure weighs states of the world in which amplification occurs more highly than what would be indicated by a traditional ‘privately’ risk-neutral probability measure, which in turn assigns more weight to such states than the objective probability of that state.

Market Discipline It has been argued that transparency requirements in conjunction with the market discipline embodied by pillar 3 of the Basel accord would induce banks to optimally smooth their capital position throughout the business cycle (see e.g. Gordy and Howells, 2006, for a discussion of this argument). In the absence of regulations of systemic externalities, our analysis suggests that markets would actually punish prudent banks that behave socially responsibly and would reward banks that take on socially excessive risks, since maximizing shareholder value involves excessive risk-taking.

5 Extensions

Let us turn our attention to a number of extensions, including the effects of anticipated government bailouts and the suboptimal incentives for bankers to raise equity during episodes of financial amplification.

5.1 Bailout Neutrality

When binding constraints and financial amplification in an economy are triggered, government authorities find it ex post optimal to intervene by providing lump-sum transfers (‘bailouts’) to constrained bankers. This allows them to mitigate the amplification effects and the associated decline in asset prices and output. This section shows that if such bailout transfers are anticipated, decentralized bankers will find it optimal to fully undo them.

Assume that a government commits to a state-contingent period 1 lump-sum transfer Z^ω that provides a bailout $Z^\omega > 0$ to bankers when they experience binding constraints

and levys a fee $Z^\omega < 0$ on them so as to make the policy revenue-neutral in expectation. Assume that the government buys the respective state contingent securities from first generation households at time 0 and distributes the transfers to bankers in period 1 after the productivity shock is realized. The assumption of revenue neutrality implies that the total expenditure on such securities in period 0 is

$$E[m_1^\omega Z^\omega] = 0$$

If we add these transfers to the optimization problems of bankers and first-generation households, their first order conditions are unaffected: decentralized bankers choose their equilibrium allocations on the basis of an optimal tradeoff of risk versus return. If they receive one more dollar in period 1 of a given state ω , they will sell one more Arrow security contingent on that state so as to restore their privately optimal equilibrium.

Proposition 3 (Bailout Neutrality) *An anticipated state-contingent lump sum transfer Z^ω to bankers that satisfies $E[m_1^\omega Z^\omega] = 0$ will be fully undone by optimizing bankers.*

Specifically, the private sales of state-contingent securities of bankers under such a transfer will satisfy

$$b_1^{\omega,Z} = b_1^{\omega,DE} + Z^\omega$$

This implies that – after the transfer has occurred – all other allocations and prices in the economy are identical to those of the decentralized equilibrium. Our finding represents a state-contingent form of Ricardian equivalence (Barro, 1974). Bankers see through the fiscal veil and add up their private budget constraint and the government’s transfers Z^ω when determining their optimal decisions.

Remark 1: The proposition also suggests circumstances under which bailouts may be effective. This may be the case if (a) they are unanticipated or (b) if bankers are prevented from undoing the transfers, either because of regulatory constraints or because the state-contingent markets required for this do not exist.

Remark 2: Transfers in constrained states that were anticipated but that end up not taking place have strongly negative effects, since any exogenous change Δa to the liquidity position of bankers under binding constraints is amplified. The expectation of a bailout leads bankers to take on larger risks than what is privately optimal in the absence of government intervention; their liquidity position after the shock is therefore below what is privately optimal, and by implication even further below what is socially optimal.

Our bailout neutrality result captures a stark version of what is sometimes referred to as the ‘moral hazard’ introduced by the anticipation of government bailouts. In

the described setting, private bankers find it optimal to engage in socially excessive risk-taking if there are some states of nature in which financial constraints are binding. Even if bailouts are lump-sum and do not distort the marginal incentives of bankers as captured by their optimality conditions, they find it optimal to undo them in order to return to their privately optimal allocations. In the given setting, lump sum “bailout” transfers therefore cannot correct for the pecuniary externalities in the decentralized equilibrium once their effects on ex-ante incentives are taken into account. For a more general discussion of the efficiency and incentive effects of bailouts see Korinek (2012).

5.2 Allocation of Endowment Risk

This section illustrates that the incentives for financially constrained bankers towards excessive exposure to aggregate risk is not only a phenomenon of underinsurance against their own productive risk, but also arises if bankers provide insurance to other sectors such as a household sector that are subject to aggregate risk. The general lesson, which was also reflected in our results on bailout neutrality, is that the equilibrium allocation of risk between bankers and first-generation households is driven by preferences and financial constraints, not by the allocation of endowment risk.

We illustrate this by modifying our framework such that all aggregate risk emanates from the endowments of first-generation households and show that bankers will continue to take on excessive exposure to systemic risk. We can interpret this result as bankers providing excessive insurance to households.

Assume an equilibrium with $\alpha t_1 > \hat{\alpha} t_1$ and suppose that we modify the period 1 income and endowments of bankers and first-generation households such that

$$\begin{aligned}\bar{A}_1 &= E[m_1^\omega A_1^\omega] \\ e_{1h}^\omega &= e + (A_1^\omega - \bar{A}_1) t_1\end{aligned}$$

In this modified setup, we have replaced the period 1 asset return of bankers by its certainty equivalent $E[m_1^\omega A_1^\omega]$, and we have instead allocated all risk in the economy to first-generation households. (We could interpret this operation as a swap of the risky stream $A_1^\omega t_1$ against the certainty equivalent $E[m_1^\omega A_1^\omega] t_1 = \bar{A}_1 t_1$ between bankers and households.

Proposition 4 *The equilibrium in the economy where we have reallocated the aggregate risk of bankers $A_1^\omega t_1$ to the endowment of first-generation households e_{1b}^ω is identical to the equilibrium in our benchmark setup.*

Proof. We have performed the reallocation in endowments such that the original equilibrium allocation is still feasible for all agents. Since their optimality conditions

are unchanged, the equilibrium of the original economy is also the equilibrium of the new economy. ■

5.3 Raising New Equity

We extend our model of the previous sections to study the incentives for bankers to raise new equity. Suppose we introduce an audit technology that gives bankers a way around the pledgeability problem for period 2 payoffs. Specifically, assume that second-generation households can take ownership of a fraction γ of bankers' period 2 returns as long as they pay a convex auditing cost $c(\gamma)$ in period 1, where $c(0) = c'(0) = 0$ and $c'(\gamma), c''(\gamma) > 0$ for $\gamma > 0$. Since they are risk-neutral, they are willing to provide $\gamma\bar{A}_2(t_1 - f_1) - c(\gamma)$ in return for their ownership share.

The resulting version of the period 1 problem (9) of a decentralized banker is

$$V(a) = \max_{\{c_{1,b}, f_1, \gamma\}} c_{1,b} + (1-\gamma)\bar{A}_2(t_1 - f_1) - \mu [c_{1,b} - a - q_1 f_1 - \gamma\bar{A}_2(t_1 - f_1) + c(\gamma)] + \lambda c_{1,b}$$

The constrained planner's period 1 surplus can be formulated by modifying the planner's problem (17) analogously. For both decentralized bankers and the constrained planner, the optimality condition with respect to γ is

$$1 = \mu \left[1 - \frac{c'(\gamma)}{\bar{A}_2(t_1 - f_1)} \right]$$

As we observed in lemma 2, $\mu^{DE} = \mu^{SP} = 1$ if the economy is unconstrained, implying that bankers and the planner will not raise new equity in period 1 and $\gamma = 0$. This is because raising equity is not useful in relaxing liquidity constraints, but is costly because of the monitoring technology.

If the economy experiences binding constraints, then $\mu^{SP} > \mu^{DE} > 1$ and the optimality condition implies that $\gamma^{SP} > \gamma^{DE} > 0$. A planner values liquidity more highly than decentralized agents and is therefore more willing than bankers to pay auditing costs to raise new equity. She internalizes that this not only relaxes the constraint of the banker who obtains liquidity but also pushes up the asset price at which all other bankers are fire-selling.

Proposition 5 *A social planner would sell a larger equity stake $\gamma^{SP} > \gamma^{DE}$ than decentralized bankers in states of binding constraints so as to mitigate financial amplification effects.*

Remark : The fundamental difference between fire sales and raising new equity in our example is that fire sales lead to aggregate price declines, which entail pecuniary externalities on other agents, whereas equity issuance entails private costs to bankers that do not have external effects.

6 Conclusions

Financial markets are inherently pro-cyclical – asset prices rise in good times and fall in bad times, the tightness of financial constraints moves in parallel, and this phenomenon may give rise to financial amplification effects: in case of negative aggregate shocks, bankers may experience binding borrowing constraints, requiring them to cut back on their economic activity and sell some of their asset holdings. This depresses asset prices, causes their balance sheets to deteriorate, leads to tighter financing conditions, requires further fire sales etc.

This paper demonstrates that such financial amplification effects give rise to a socially inefficient allocation of aggregate risk in the economy because of a pecuniary externality that leads financially constrained bankers to undervalue liquidity in crisis states. Small agents take asset prices – and the tightness of financing conditions – as given and do not internalize the general equilibrium effects of their actions on prices and constraints. In particular, they do not internalize that fire sales during crises depress asset prices, which trigger amplification effects that hurt other bankers in the economy.

The undervaluation of liquidity in crisis times in turn leads bankers to take on excessive risk and buy insufficient insurance in their financing decisions, and to undervalue the benefits of raising new equity in crises. While we have limited our analysis to the financing decisions of bankers in the initial period, the externality would also lead to excessive real investment in projects that create exposure to systemic risk, as highlighted e.g. by Lorenzoni (2008).

Our paper develops a stylized model that allows us to analytically examine these inefficiencies and investigate related policy measure. In our model, liquidity shortages in period 1 lead to fire sales, but there is no debt carried from period 1 to period 2. There are two directions along which our setup of financial constraints could be extended. First, in infinite horizon models of financial amplification such as Kiyotaki and Moore (1997), falling asset price also reduce the value of collateral and lower the amount of debt that can be carried forward through a ‘dynamic multiplier’ effect. This is explored in Jeanne and Korinek (2010) for the case of uncontingent financial contracts. Second, as emphasized e.g. by Geanakoplos (2009), changes in financial conditions are also reflected in endogenous changes in leverage. Both effects are likely to further strengthen the externalities of financial amplification effects.

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A Mathematical Appendix

A.1 First Generation Households

Assumption A.1 *The utility function of first generation households is such that the elasticity of the pricing kernel m_1^ω with respect to Arrow security sales b_1^ω satisfies*

$$\eta_{mb} < 1$$

The purpose of assumption A.1 is to ensure that the amount raised by selling a marginal unit of an Arrow security is an increasing function of the promised repayment, i.e.

$$\frac{dm_1^\omega b_1^\omega}{db_1^\omega} = m_1^\omega + b_1^\omega \cdot \frac{\partial m_1^\omega}{\partial b_1^\omega} = m_1^\omega \cdot (1 - \eta_{mb}) > 0$$

An alternative way of expressing the assumption is

$$\eta_{mb} = -\frac{b_1^\omega}{m_1^\omega} \cdot \frac{\partial \frac{u'(e+b_1^\omega)}{u'(e-E[m_1^\omega b_1^\omega])}}{\partial b_1^\omega} = -\frac{b_1^\omega}{m_1^\omega} \cdot \frac{u''(c_1^\omega)}{u'(c_0^\omega)} = -b_1^\omega \cdot \frac{u''(c_1^\omega)}{u'(c_1^\omega)} = \frac{b_1^\omega}{c_1^\omega} \cdot R(c_1^\omega) < 1$$

where $R(c_1^\omega)$ denotes the coefficient of relative risk aversion of households. The assumption is satisfied if the product of the debt income/consumption ratio and relative risk aversion is sufficiently low. For the standard value of relative risk aversion used in macroeconomics $R = 2$ the assumption holds as long as the debt income received by households in period 1 makes up less than half of their consumption. This is plausible since approximately two thirds of household income derives from wages.

If the assumption was violated, then the amount of finance raised $m_1^\omega b_1^\omega$ would fall as the promised repayment b_1^ω rises because the pricing kernel of consumers m_1^ω would fall faster than b_1^ω increases. In such a situation, the economy may be subject to multiple equilibria, since different amounts of promised repayments could lead to the same amount of finance raised.

A.2 Second-generation Households

Assumption A.2 *The production function of second-generation households is such that the elasticity of the asset price q_1 with respect to fire sales f_1 satisfies*

$$\eta_{qf} < 1$$

This assumption is parallel to A.1 and guarantees that the revenue raised by fire sales $q_1 f_1$ is an increasing function of the amount of assets sold f_1 , i.e.

$$\frac{dq_1 f_1}{df_1} = q_1 + f_1 F''(f_1) = q_1 \cdot (1 - \eta_{qf}) > 0$$

If this assumption was violated, there would be multiple equilibria in the sense that raising a given amount of liquidity could be accomplished by different levels of fire sales.

A.2.1 Proof of Proposition 1

The period 0 equilibrium for bankers is determined by their period 0 budget constraint $\alpha t_1 = E[m_1^\omega b_1^\omega]$ and their period 0 optimality condition (13). Our proof proceeds in two steps: First we use condition (13) to characterize the optimal payment b_1 as a function of a given productivity shock $A_1 \in [A^{\min}, A^{\max}]$ and of the tightness of the period 0 budget constraint, as captured by the shadow price ν . Second, we will determine the value of ν that makes the period 0 budget constraint hold with equality.

Step 1 Substituting for $q_1 = F'(f_1)$, the period 0 optimality condition (14) of bankers defines the payment b_1 as a function of ν and A_1 via the implicit equation

$$\frac{\bar{A}_2}{F'(f(b_1 - A_1 t_1))} = \nu \cdot \frac{u'(e + b_1)}{u'(e - \alpha t_1)} \quad (\text{A.1})$$

The resulting function $b_1(\nu; A_1)$ is continuous in both parameters and satisfies $\partial b_1 / \partial \nu > 0$ and $\partial b_1 / \partial A_1 \geq 0$, i.e. bankers pay more the tighter the period 0 budget constraint (the higher ν) and the higher the productivity shock A_1 .

For a given ν , we first determine the optimal payment $b_1(\nu; A_1) = \bar{b}_1(\nu)$ of bankers if their financial constraint is loose and there are no fire sales. In that case, the left-hand side of equation (A.1) is one, the variable A_1 drops out of the equation, and we can solve the function explicitly as

$$\bar{b}_1(\nu) = u'^{-1}\left(\frac{u'(e - \alpha t_1)}{\nu}\right) - e \quad (\text{A.2})$$

which is independent of the realization of the productivity shock A_1 . For a given ν , the payment $\bar{b}_1(\nu)$ is feasible without fire sales as long as productivity is above the threshold $A_1 \geq \hat{A}(\nu) = \bar{b}_1(\nu) / t_1$.

If $A_1 < \hat{A}(\nu)$, equation (A.1) only has a solution if we allow for fire sales. Bankers find it optimal to make a period 1 payment that exceeds their asset income $b_1(\nu; A_1) > A_1 t_1$ and engage in fire-sales to second-generation households to meet the shortfall. The left-hand side of (A.1) is strictly increasing in b_1 : for $b_1 = A_1 t_1$ it equals 1, and for $b_1 = A_1 t_1 + s^{\max}$ it is \bar{A}_2 / q^{\min} ; the right-hand side of (A.1) is strictly decreasing in b_1 : as we increase b_1 over the interval $(-e, \infty)$, the right-hand side goes from $+\infty$ to 0 because of the Inada conditions on the utility function of first-generation households. The implicit equation has a solution for all $A_1 \in [A^{\min}, A^{\max}]$ as long as $\nu \leq \nu^{\max} = \frac{\bar{A}_2}{q^{\min}} \cdot \frac{u'(e - \alpha t_1)}{u'(e + A^{\min} t_1 + s^{\max})}$. For higher ν , households cannot make the promised repayment under the productivity shock A^{\min} even if they fire-sold their entire asset holdings. (We have ruled this out in section 2.)

In summary, equation (A.1) defines a continuous function $b_1 : (0, \nu^{\max}] \times [A^{\min}, A^{\max}] \rightarrow (-e, A^{\min} t_1 + s^{\max}]$ that satisfies $\partial b_1 / \partial \nu > 0$ and $\partial b_1 / \partial A_1 \geq 0$. The second inequality is strict, i.e. $\partial b_1 / \partial A_1 > 0$, when there are positive fire sales. (Bankers promise to pay more the tighter the period 0 budget constraint as captured by ν and the higher the productivity shock A_1 .)

Step 2 The period 0 budget constraint of bankers requires

$$\alpha t_1 = E[m_1^\omega b_1^\omega] = E[m(b_1(\nu; A_1^\omega)) b_1(\nu; A_1^\omega)] \quad (\text{A.3})$$

The expectation is taken over all states of nature $\omega \in \Omega$; therefore the right hand side of the equation is a function solely of ν . We observed before that each $b_1(\nu; A_1^\omega)$ is increasing in ν , and by assumption A.1 the product $m_1(b_1(\nu; A_1^\omega)) b_1(\nu; A_1^\omega)$ is strictly increasing in ν . Hence the term on the right-hand side is a function $(0, \nu^{\max}] \rightarrow (-e, \alpha^{\max} t_1]$ that is continuous and strictly increasing, where we define $\alpha^{\max} t_1$ is the level of funds raised for $\nu = \nu^{\max}$. The equation (A.3) therefore pins down a unique $\nu^* \in (0, \nu^{\max}]$. Given the equilibrium ν^* , the optimal borrowing choices of bankers are $b_1(\nu^*; A_1^\omega)$ and the threshold for binding constraints is $\hat{A}_1 = \hat{A}(\nu^*)$. All other variables follow.

A.3 Proof of Proposition 2

We follow similar steps as in our proof of proposition 1: first, we use the period 0 optimality conditions of the planner to characterize the payment function $b_1^{SP}(\nu; A_1)$; then we use the constraint $U \geq U^{DE}$ to determine the optimal level of ν .

Step 1 Combining the optimality conditions (19) and (23) of the constrained social planner, we obtain

$$\frac{\bar{A}_2/F' - \eta_{qf}}{1 - \eta_{qf}} = \nu \cdot \frac{u'(e + b_1^\omega)}{u'(e - \alpha t_1)} \quad (\text{A.4})$$

where $\eta_{qf} = -f_1^\omega F''(f_1^\omega)/F'(f_1^\omega)$. This equation is the planner's analogon to the decentralized optimality condition (A.1). It implicitly defines b_1^{SP} as a function of the parameters ν and A_1 .

If $A_1 \geq \hat{A}(\nu)$ as defined above, the solution to the equation is identical to the decentralized solution $b_1^{SP}(\nu; A_1) = \bar{b}_1(\nu)$. The financial constraint on bankers is loose, there are no fire-sales so $F' = \bar{A}_2$, and the left-hand side of equation (A.4) is one.

If $A_1 < \hat{A}(\nu)$, a solution without fire sales is not feasible; hence $f_1 > 0$. Bankers find it optimal to make a period 1 payment that exceeds their asset income $b_1^{SP}(\nu; A_1) > A_1 t_1$ and engage in fire-sales to second-generation households to meet the shortfall. For any pair (ν, A_1) in this region, the left-hand side of (A.4) is strictly higher than the left-hand side of (A.1) in the decentralized equilibrium, as we observed in lemma 2. Therefore the b_1^{SP} that is the solution to the implicit equation is strictly lower than $b_1^{DE}(\nu, A_1)$.

In summary, the implicit equation (A.4) defines a continuous function $b_1^{SP} : (0, \nu^{\max}] \times [A^{\min}, A^{\max}] \rightarrow (-e, A^{\min} t_1 + s^{\max})$ that satisfies $\partial b_1^{SP}/\partial \nu > 0$ and $\partial b_1^{SP}/\partial A_1 \geq 0$ and $b_1^{SP}(\nu, A_1) \geq b_1^{DE}(\nu, A_1)$. The last two inequalities hold strictly if $A_1 < \hat{A}(\nu)$, i.e. if productivity is so low that bankers are financially constrained and engage in fire-sales.

Step 2 We denote the utility U^{SP} of first generation households under the planner's allocation of Arrow securities $b_1^{SP}(\nu; A_1^\omega)$ as a function of ν

$$U^{SP}(\nu) := u(e - \alpha t_1) + E[u(e + b_1^{SP}(\nu; A_1^\omega))]$$

Since b_1^{SP} is strictly increasing in ν , this utility is strictly increasing in ν and expected profits of bankers are strictly decreasing in ν . The solution to the planner's optimization problem (17) therefore involves the value of ν such that the constraint $U \geq U^{DE}$ is satisfied with equality. If $\alpha t_1 > \hat{\alpha} t_1$, observe that $b_1^{SP}(\nu^{DE}; A_1) \leq b_1^{DE}(\nu^{DE}; A_1)$ with strict inequality for some ω . It follows that $U^{SP}(\nu^{DE}) < U^{DE}$. The planner has to increase the shadow price such that $\nu^{*SP} > \nu^{*DE}$ for the constraint $U \geq U^{DE}$ to hold.

It follows immediately that the threshold for binding constraints satisfies $\hat{A}_1^{SP} = \hat{A}(\nu^{*SP}) > \hat{A}_1^{DE}$. In unconstrained states of nature, the planner pays more to households $b_1^{\omega, SP} = \bar{b}_1(\nu^{*SP}) > \bar{b}_1(\nu^{*DE})$ and in the lowest (most constrained) states of nature the planner pays less to first-generation households than decentralized bankers, $b_1^{SP}(\nu^{DE}; A^{\min}) < b_1^{DE}(\nu^{DE}; A^{\min})$ – otherwise the $U > U^{DE}$ which would be inefficient.

B Generalizations of the Benchmark Model (Online Only)

We constructed our benchmark model to demonstrate the basic insights of this paper in the most transparent way possible. This appendix shows that our results on the private undervaluation of liquidity during fire-sales and therefore on excessive systemic risk-taking continue to hold for more general versions of the model, though they involve additional notational complexity. In particular, the following two sections introduce a generalization in which second-generation households are risk-averse and one in which compensatory transfers are uncontingent.

B.1 Risk-Averse Second-Generation Households

In our benchmark model, the linear utility of both bankers and second-generation households was a convenient analytic tool that made the valuation of compensatory transfers from bankers to households straightforward – the marginal utility of both agents is unity in unconstrained states of nature, as captured by equation (22). This section shows that our results continue to hold with risk-averse second-generation households, but that we need to put additional focus on how the planner values compensatory transfers to households.

Assume that the utility of second-generation households is

$$W = E [w(c_{1,l}^\omega) + w(c_{2,l}^\omega)]$$

where the period utility function $w(\cdot)$ is strictly increasing $w' > 0$ and weakly concave $w'' \leq 0$.¹³ We continue to assume that they buy $f_{1,l}^\omega$ productive assets at price q_1^ω in period 1 and employ them in period 2 production using a production function $F(\cdot)$ that satisfies $F'(0) = \bar{A}_2$ and $F'' \leq 0$, i.e. their marginal productivity is equal to the productivity of bankers at zero, but declines weakly in the amount of assets purchased. We assume that at least one of the two functions $w(\cdot)$ and $F(\cdot)$ is strictly concave.

The resulting optimization problem for second-generation households is

$$\max_{\{f_{1,l}^\omega\}} E [w(e - q_1^\omega f_{1,l}^\omega) + w(e + F(f_{1,l}^\omega))]$$

The first-order condition yields an inverse demand curve for productive assets

$$q(f_{1,l}^\omega) = \frac{w'(c_{2,l}^\omega)}{w'(c_{1,l}^\omega)} \cdot F'(f_{1,l}^\omega)$$

This demand function is downward-sloping, i.e. $dq_1^\omega/df_{1,l}^\omega < 0 \forall \omega$ because of our assumption that either the production technology $F(\cdot)$ exhibits decreasing returns to scale, or the utility function $w(\cdot)$ is concave, or both. As in our benchmark model, we assume

¹³This allows for example for the possibility that the period utility function of second-generation households is identical to that of first generation households $w(c) = u(c)$, or identical to that of bankers $w(c) = c$.

$d(q_1^\omega f_{1,l}^\omega) / df_{1,l}^\omega > 0 \forall \omega$, i.e. $\eta_{qf} < 1$, which allows us to define a function $s(f) = q(f) \cdot f$ that is strictly increasing.

Given the two strictly monotonic functions $q(f)$ and $s(f)$, the description of the decentralized equilibrium proceeds as given above in proposition 1.

B.1.1 Social Planner's Problem

We formulate the analogon to the planner's simplified problem (18) with risk-averse second-generation households as

$$\begin{aligned} \max_{\{T_0, b_1^\omega, c_{1b}^\omega, f_1^\omega, T_1^\omega\}} & E [c_{1b}^\omega + \bar{A}_2(t_1 - f_1^\omega)] \\ \text{s.t.} & c_{1b}^\omega = A_1^\omega t_1^\omega - b_1^\omega + q(f_1^\omega) f_1^\omega - T_1^\omega \geq 0, T_1^\omega \geq 0 \\ & U \geq U^{DE} \\ & W = E[w(e - q(f_1^\omega) f_1^\omega + T_1) + w(e + F(f_1^\omega))] \geq W^{DE} \end{aligned}$$

Using the same convention for the shadow prices as in (18), the optimality conditions of the social planner on c_{1b}^ω and b_1^ω are unchanged, but the following two optimality conditions are modified:

$$\begin{aligned} FOC(f_1^\omega) : & \quad \bar{A}_2 = \mu^\omega [q(f_1^\omega) + f_1^\omega q'(f_1^\omega)] - \psi w'(c_{1l}^\omega) f_1^\omega q'(f_1^\omega) \\ FOC(T_1^\omega) : & \quad \mu^\omega = \kappa^\omega + \psi w'(c_{1l}^\omega) \end{aligned}$$

These two conditions differ in that $F'(f_1^\omega)$ is replaced by the more general $q(f_1^\omega)$ and the transfer to second-generation households is valued at the marginal utility $w'(c_{1l}^\omega)$ of households times the planner's weight ψ .

B.1.2 Valuation of Liquidity

In the following, we determine the valuation of liquidity of the constrained social planner across different states of nature for a given shadow price ψ . Given the planner's valuation of liquidity μ^ω , the solution to the optimization problem proceeds along the same lines as the proof of proposition 2 in A.3.

Each state is characterized by one of three possible regimes, depending on the period 1 liquid wealth of bankers $a^\omega = A_1^\omega t_1 - b_1^\omega$:

1. Unconstrained equilibria for $a^\omega \geq \bar{a}(\psi) \geq 0$: If the net liquid wealth of bankers is above a threshold $\bar{a}(\psi) \geq 0$, then the consumption constraint $c_{1b}^\omega \geq 0$ as well as the constraint on the transfer $T_1^\omega \geq 0$ are loose so $\lambda^\omega = 0$ and $\kappa^\omega = 0$. The marginal valuation of liquid banker wealth is $\mu^\omega = 1$ and there are no fire-sales so that $f_1^\omega = 0$. The threshold $\bar{a}(\psi)$ is the lowest possible liquid wealth level for which the planner can satisfy the $FOC(T_1^\omega)$ without fire-sales. It is determined by the equation

$$1/\psi = w'(e + \bar{a})$$

As long as $a^\omega \geq \bar{a}(\psi)$, the planner transfers $T_1^\omega = \bar{a}(\psi)$ from bankers to second-generation households and lets bankers consume the remainder $c_{1l}^\omega = a^\omega - \bar{a}(\psi)$. The

planner makes these transfers to compensate second-generation households for the loss of utility from reduced fire sales; therefore the shadow price ψ is such that the transfer \bar{a} is in equilibrium positive.

2. Constrained equilibria with positive transfers for $a^\omega \in (\hat{a}(\psi), \bar{a}(\psi))$: Since second-generation households are risk-averse the planner would ideally like to provide them with a constant level of marginal utility $1/\phi$. However, if $a^\omega < \bar{a}(\psi)$, the banker does not have sufficient resources to do so without fire sales. Bankers are then constrained $\lambda^\omega > 0$ and consume $c_{1b}^\omega = 0$ in period 1. The planner finds it optimal to reduce the transfer $T_1^\omega < \bar{a}(\psi)$ but induces bankers to engage in positive fire sales $f_1^\omega > 0$, trading off the utility cost of an unsmooth consumption profile for second-generation households and the efficiency cost of fire-sales.

The planner finds it optimal to provide a positive transfer to second-generation households as long as $a^\omega > \hat{a}(\psi)$. The threshold $\hat{a}(\psi) \leq 0$ is determined by the level of fire-sales $f(-\hat{a}(\psi))$ such that the planner can satisfy the $FOC(T_1^\omega)$ with a transfer of zero that is not constrained by the non-negativity constraint, i.e. that satisfies $\kappa^\omega = 0$.

$$\frac{\bar{A}_2}{q(f(-\hat{a}))} = \psi w'(e + F(f(-\hat{a})))$$

In this regime, the planner transfers $T_1^\omega \in (0, \bar{a}(\psi))$ from bankers to second-generation households and raises liquidity for bankers through fire sales of $f(T_1^\omega - a^\omega)$. The transfer T_1^ω is increasing in a^ω and, for a given a^ω , is determined by the $FOC(T_1^\omega)$,

$$\frac{\bar{A}_2}{q(f(T_1^\omega - a^\omega))} = \psi w'(e + F(f(T_1^\omega - a^\omega)) + T_1^\omega)$$

3. Constrained equilibria without transfers for $a^\omega \in [-s^{\max}, \hat{a}(\psi)]$: If the liquid wealth of bankers is below this level $\hat{a}(\psi)$, then bankers are sufficiently constrained that the planner no longer engages in transfers to second-generation households so $\kappa^\omega > 0$. In this regime, the optimality condition $FOC(f_1^\omega)$ implies that

$$\mu^\omega = \frac{\bar{A}_2 - \kappa^\omega f_1^\omega q'(f_1^\omega)}{q_1^\omega} > \frac{\bar{A}_2}{q_1^\omega} = \mu^{\omega, DE} \quad (\text{A.5})$$

In other words, the planner values liquid banker net worth more highly than decentralized agents. She internalizes that an additional unit of liquidity would not only enable bankers to produce \bar{A}_2/q_1^ω more but would also push up the asset price by $q'(f_1^\omega)/q_1^\omega$, which redistributes $f_1^\omega q'(f_1^\omega)/q_1^\omega$ from households to bankers. Bankers are constrained, and the planner recognizes that they value net worth by $\kappa^\omega = \mu^\omega - \psi w'(c_{1l}^\omega) > 0$ more than second-generation households. By contrast, decentralized bankers take the asset price q_1^ω as given and do not internalize that their asset sales lead to a redistribution of wealth.

Lemma 3 (Undervaluation of Liquidity) *For a given level of required household utility $W \geq W^{DE}$ as captured by the shadow price ψ we find:*

1. If $a^\omega \geq \bar{a}(\psi)$ where $\bar{a}(\psi) \geq 0$, then the constrained planner and decentralized agents value liquidity equally at $\mu^\omega = 1$.

2. If $a^\omega < \bar{a}(\psi)$ then the constrained planner values liquidity more highly than decentralized agents $\mu^{\omega,SP} > \mu^{\omega,DE} \geq 1$, except at $a^\omega = \hat{a}(\psi)$ where the two valuations coincide.

Proof. Part 1. of the lemma holds trivially because there are no fire sales and decentralized agents and the planner both value liquidity at $\mu^\omega = 1$ for $a^\omega \geq \bar{a}(\psi) \geq 0$.

To establish part 2. of the lemma, we focus on two regions for a^ω separately: First, for $a^\omega \in (\hat{a}(\psi), \bar{a}(\psi))$, the planner provides a positive transfer $T_1^\omega > 0$ from bankers to second-generation households, which leaves bankers with a net level of liquidity $a^\omega - T_1^\omega$. Bankers in the planner's allocation engage in fire-sales to raise liquidity of $T_1^\omega - a^\omega > 0$, whereas bankers in the decentralized equilibrium raise liquidity of at most $-a^\omega$. In this region, the valuation of liquidity satisfies $\mu^\omega = \frac{\bar{A}_2}{q(f_1^\omega)}$. Since decentralized bankers fire-sell less than the constrained planner, the asset price in the decentralized equilibrium is higher and the valuation of liquidity satisfies $\mu^{\omega,DE} < \mu^{\omega,SP}$. The planner values liquidity in this region more highly because he uses it to compensate second-generation households.

In the knife edge case where $a^\omega = \hat{a}(\psi)$, the planner ceases her transfers since the constraint $T_1^\omega \geq 0$ is marginally binding and $\kappa^\omega = 0$. At this point, the private and social valuation of liquidity coincide at $\mu^\omega = \frac{\bar{A}_2}{q_1^\omega} > 1$.

For $a^\omega < \hat{a}(\psi)$, the social planner does not make compensatory transfers to second-generation households so $T_1^\omega = 0$ and $\kappa^\omega > 0$. In this region, the planner's valuation of liquidity is determined by equation (A.5). As emphasized above, the planner recognizes the benefits of the wealth transfer that can be achieved via the pecuniary externality $q'(f_1^\omega) < 0$ if she holds additional liquidity. Therefore the planner values liquidity more highly in this region. ■

Figure 7 schematically depicts the valuation of liquidity of decentralized agents and the planner across different states of nature assuming a fixed debt level \bar{b}_1 . For $a^\omega \geq \bar{a}$, financial constraints are loose under the planner and decentralized agents, the planner transfers \bar{a} from bankers to second-generation households, and both the planner and decentralized bankers value liquidity at a marginal value of 1. For $a^\omega \in (\hat{a}, \bar{a})$, bankers are more constrained in the planning solution, the planner transfers some of the resources of bankers to second-generation households, and $\mu^{\omega,SP} > \mu^{\omega,DE}$. For $a^\omega < \hat{a}$, the fire sales of bankers in the planning solution and in the decentralized allocation coincide, but the planner internalizes the pecuniary externalities of fire sales and therefore $\mu^{\omega,SP} > \mu^{\omega,DE}$.

Given the higher valuation of liquidity of the constrained planner, the logic of proposition 2 implies that the planner induces bankers to reduce b_1^ω in highly constrained states and increase b_1^ω in unconstrained states. The proof follows along the same lines as the proof of proposition 2.

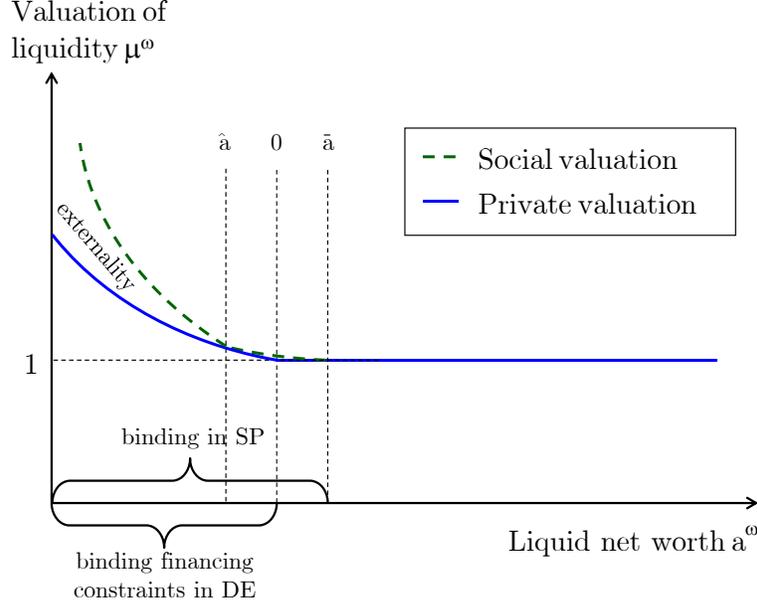


Figure 7: Valuation of Liquidity Under Risk-Averse Second-Generation Households

B.2 Uncontingent Compensatory Transfers

In this section, we modify the constrained planner's problem described in (18) and assume that the compensatory transfer from bankers to second-generation households is uncontingent $T_1^\omega = \bar{T}_1 \forall \omega$. This implies that second-generation households receive a positive transfer even in states of nature in which bankers are financially constrained and have to engage in additional fire-sales to afford the transfer. The equilibrium is therefore less efficient than the one described in our benchmark model. However, this modification may be relevant if the planner cannot condition transfers on the state of nature. Furthermore, it provides an alternative to the constraint $T_1^\omega \geq 0$ that we assumed in our benchmark model.

Analytically, taking the first-order condition of the modified maximization problem yields

$$FOC(\bar{T}_1) : E[\mu^\omega] = \psi \quad (\text{A.6})$$

The planner sets the transfers such that his marginal valuation of funds in the pockets of bankers and of second-generation households is equal. Assuming that $\alpha t_1 > \hat{\alpha} t_1$, there are some states with binding constraints and we conclude $E[\mu^\omega] = \psi > 1$.

The planner's first-order condition on fire sales (20) can be expressed as

$$\mu^\omega = \frac{\bar{A}_2 + \psi f F''}{F' + f F''} \quad (\text{A.7})$$

Comparing this expression to the private valuation of liquidity we find

Lemma 4 (Mis-Valuation of Liquidity) *For a given level of required household utility $W \geq W^{DE}$ as captured by the shadow price ψ we find:*

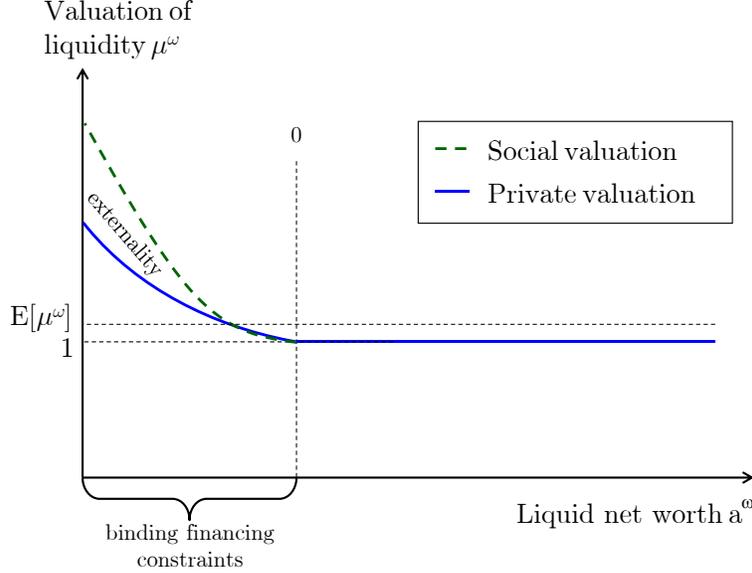


Figure 8: Valuation of Liquidity for Uncontingent Transfer \bar{T}_1

1. If there are no fire-sales, the constrained planner and bankers value liquidity equally at $\mu^\omega = 1$.
2. If $f_1^\omega > 0$, the constrained social planner values liquidity more than bankers $\mu^{\omega,SP} > \mu^{\omega,DE}$ if and only if

$$\frac{\bar{A}_2}{F'} \geq \psi$$

Proof. Part 1. of the lemma follows by substituting $f = 0$ into (A.7). Part 2. follows by comparing $\mu^{\omega,DE} = \bar{A}_2/F'$ with the right-hand side of (A.7) and simplifying the inequality. ■

The results are illustrated graphically in figure 8. Intuitively, decentralized agents and the planner value liquidity equally for $f_1^\omega = 0$ because the planner cannot redistribute between the two agents via asset price manipulations when there are no fire sales. On the other hand, for $f_1^\omega > 0$, the planner recognizes that holding more liquidity mitigates fire-sales and asset price declines, which redistributes between the two agents. The benefit of a marginal redistribution between the two agents is $(\mu^\omega - \psi)$ and is on average zero, as we observed in (A.6), implying that it is positive for states of nature that are more constrained than average and negative for states of nature that are less constrained than average.

The different valuation of liquidity by the constrained planner implies that the planner induces bankers to repay less (reduce b_1^ω) in highly constrained states and repay more (increase b_1^ω) in marginally constrained and in unconstrained states to make up for this. The proof proceeds along the lines of the proof to proposition 2.