

# Macroprudential Regulation Versus Mopping Up After the Crash\*

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## Abstract

How should macroprudential policy be designed when policymakers also have access to ex-post crisis management tools? We show in a tractable model that there are three factors at play: First, ex-post policy measures mitigate financial crises which reduces the need for macroprudential policy. Secondly, macroprudential policy needs to consider moral hazard effects if and only if it uses price-based regulatory measures. Third, if macroprudential instruments are unavailable, it is optimal to commit to less generous ex-post policy as a second-best substitute to macroprudential policy. Finally, it is undesirable to accumulate revenue from a macroprudential tax in a bailout fund.

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# 1 Introduction

The 2008-09 global financial crisis has significantly changed our views on the appropriateness of policy interventions to respond to financial booms and busts. The dominant view before the crisis was that the best time to intervene was *ex post*, at the time of crisis, rather than *ex ante*, when fragilities build up in the financial system. This so-called “Greenspan doctrine” held that it was preferable to “mop up” after a financial crisis had materialized, since *ex ante* interventions tended to be too blunt, unpredictable in their effects or too costly.<sup>1</sup> By contrast, there is now wide agreement that policymakers should try to contain the buildup in risks *ex ante* through macroprudential interventions. *Ex-post* crisis interventions have been criticized for being counter-productive in various ways, in particular for creating moral hazard and inducing excessive risk-taking *ex ante*. This shift in the policy debate is reflected in the financial reforms that were implemented in response to the crisis. For example, the Dodd-Frank reform gives the US Federal Reserve new powers in designing prudential capital and liquidity requirements at the same time as it curtails its ability to support individual institutions in a crisis.<sup>2</sup> The pendulum has swung away from *ex-post* interventions towards *ex-ante* interventions.

This policy debate has been accompanied, on the theoretical side, by a new strand of literature that analyzes the desirability of *ex-ante* macroprudential interventions.<sup>3</sup> Another

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<sup>1</sup>See Greenspan (2002, 2011). Some economists, especially at the Bank for International Settlements (BIS), were early defenders of the view that policymakers should intervene *ex ante* (see, e.g., Borio, 2003; Bordo and Jeanne, 2002).

<sup>2</sup>Before Dodd-Frank the Federal Reserve was allowed to lend to a wide range of entities “in unusual and exigent circumstances” by Section 13(3) of the Federal Reserve Act. This disposition was limited in numerous ways by Dodd-Frank, including the fact that Fed loans can no longer be targeted to individual firms. This would have made many of the Fed’s interventions in the 2008-09 crisis impossible.

<sup>3</sup>See for example Gromb and Vayanos (2002), Caballero and Krishnamurthy (2003), Lorenzoni (2008), Jeanne and Korinek (2010), Korinek (2010), Stein (2012), He and Kondor (2016) and Dávila and Korinek (2017) for papers that motivate macroprudential intervention on the basis of pecuniary externalities, or Farhi and Werning (2012, 2016) and Korinek and Simsek (2016) for a motivation on the basis of aggregate demand externalities. There is also a related quantitative literature – see Korinek and Mendoza (2014) for an overview.

line of literature has focused on ex-post interventions.<sup>4</sup> However there is little work that systematically studies how to design ex-ante macroprudential regulation when policymakers also have tools to respond to financial crises ex-post.<sup>5</sup> The objective of our paper is to fill this gap.

We provide a tractable model of ex-ante and ex-post crisis interventions that allows us to obtain powerful analytic results. Our model features a simple collateral constraint that depends on asset prices, which may lead to financial amplification and systemic risk ex post and to overborrowing ex ante, following the logic of fire-sale models (Shleifer and Vishny, 1992; Lorenzoni, 2008; Dávila and Korinek, 2017). We consider two ex-ante policies: a debt cap as in quantity-based regulation, such as maximum loan-to-value ratios, and a tax on borrowing which could be interpreted either as financial regulation or restrictive monetary policy. We consider two ex-post policies: a subsidy on the debt accumulated ex ante, which can be interpreted as "debt bailouts", and a subsidy on new borrowing, which can be interpreted as "monetary stimulus" as in Farhi and Tirole (2012). One of our contributions is to clarify that debt bailouts and monetary stimulus are equivalent from an ex-post perspective in models of binding financial constraints but differ in their ex-ante incentive effects.

Our main contribution is to show that there are three factors through which ex-post policies influence the optimal design of ex-ante macroprudential policy measures:

First, ex-post policy measures mitigate financial crises, which makes it optimal for private agents to take on more risk and reduces the need for macroprudential policy. This factor

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<sup>4</sup>Acharya and Yorulmazer (2008) and Philippon and Schnabl (2012) compare the efficiency of different types of ex-post policy measures.

<sup>5</sup>Some papers focus on specific aspects of the interaction: Benigno et al. (2012, 2016) show that ex-post interventions reduce the excessive borrowing that arises from pecuniary externalities. Caballero and Lorenzoni (2014) compare ex-ante and ex-post intervention in a setting in which persistent appreciations drain the net worth of the traded sector in an emerging economy, and Bornstein and Lorenzoni (2016) show that ex-post measures to manage aggregate demand do not necessarily generate ex-ante moral hazard. Dogra (2015) designs an optimal mechanism for macroprudential policy and debt bailouts in the presence of private information.

runs against the popular narrative that ex-post policy interventions always call for tighter macroprudential regulation because they increase risk-taking. We show that the “Greenspan doctrine,” which suggested that crises interventions should be conducted only in the form of ex-post policies, is a limit case of this result that holds if (i) ex-post interventions fully resolve crises and (ii) they are conducted exclusively in the form of monetary stimulus. Otherwise, it is always optimal to combine ex-ante and ex-post interventions.

Second, some types of ex-post measures – in our model, debt bailouts – distort ex-ante incentives. We show that this necessitates a tightening in macroprudential policy in order to offset the adverse incentive effects if and only if macroprudential policy is conducted in the form of price-based regulations, such as taxes on borrowing. By contrast, if macroprudential policy is conducted in the form of quantity-based regulations such as debt caps, macroprudential policy is immune to these adverse incentive effects – the optimal debt cap remains unchanged, even if ex-post intervention takes the form of debt bailouts. This finding runs counter to the popular narrative that incentive-distorting crisis management measures always call for tighter macroprudential policy. Moreover, our finding suggests a novel dimension in which quantity-based regulations are more robust and may be easier to implement than price-based macroprudential regulations.

Third, if macroprudential policy can be set optimally, it resolves any time consistency problems that may arise from the use of ex-post policy instruments – no matter if they are provided in the form of monetary stimulus or debt bailouts. Optimal ex-ante interventions ensure that the ex-ante borrowing incentives of private agents are corrected given the anticipated ex-post intervention. As a result, there is no benefit to commitment, and it is optimal to exercise complete discretion in the use of ex-post interventions. This finding also runs against a popular narrative that ex-post policy intervention always introduces time consistency issues. However, if explicit macroprudential instruments are restricted or unavailable, e.g. because of circumvention or shortcomings in the legal mandate for macroprudential policy, we show that time consistency is back in the game: it is optimal to commit to less

ex-post intervention as a second-best macroprudential policy, and to intervene exclusively in the form of monetary stimulus rather than debt relief. By committing to less generous ex-post intervention, the planner can achieve her macroprudential objective of reducing the overborrowing of private agents. In other words, commitment is a substitute – although a second-best one – for explicit macroprudential regulation.

A further contribution of our paper is to analyze the common policy proposal of imposing macroprudential taxes and accumulating the tax revenue in a bailout fund that is used for the debt bailouts in the event of a systemic crisis. We find this proposal undesirable – it is preferable to inject fresh resources coming from outside of the borrowing sector in the event of a crisis, even if these resources are obtained through a distortionary tax. The reason is a form of Ricardian equivalence: borrowers respond to the creation of the bailout fund in good times by borrowing more. In addition, a bailout fund [replaces: However, such a fund] may inefficiently limit the size of stimulus policies in the event of a severe crisis.

The remainder of this paper is structured as follows. In the following section, we introduce the baseline model, characterize the first best and introduce the financial constraint that lies at the heart of our analysis. Section 3 introduces the ex-ante and ex-post policy instruments at the disposal of the social planner. Section 4 analyzes the optimal policy mix. Section 5 presents a numerical illustration and section 6 concludes.

## 2 Model

### 2.1 Assumptions

We consider an economy with three time periods  $t = 0, 1, 2$ , and one homogeneous good. There are two classes of atomistic agents in the economy: borrowers and lenders. For simplicity we assume that there is a mass 1 of each type of agents. Periods 0 and 1 are the lending periods and repayment takes place in period 2.

The utility of the representative borrower and of the representative lender in period 0 are respectively given by,

$$U^b = E_0 [u(c_0) + u(c_1) + c_2], \quad (1)$$

$$U^\ell = E_0 [c_2^\ell], \quad (2)$$

where  $c_t$  is the agent's level of consumption in period  $t$  (with the consumption of lenders superscripted by  $\ell$ ) and needs to be non-negative. We assume that the utility function  $u(\cdot)$  is increasing, strictly concave, and satisfies the Inada conditions. Our baseline analysis considers the case,

$$u(c) = \log c,$$

to allow for well-behaved closed-form solutions. More general preferences are considered in Appendix A.1.

The borrowers borrow in period 0 because they have no income in that period. They receive a stochastic income  $\rho$  in period 1. The exogenous stochastic parameter  $\rho$  is the only source of uncertainty in our model. In the initial period borrowers are endowed with  $a = 1$  unit of an asset that pays off 1 unit of good in the final period 2. In period 1, they trade the asset at market price  $p$  and carry  $a'$  units into the next period. The borrowers also issue one-period debt. We denote by  $d$  and  $d'$  the debts issued by the borrowers in periods 0 and 1 respectively. The resulting budget constraints for borrowers are,

$$\begin{aligned} c_0 &= d, \\ c_1 + d &= \rho + d' + p(1 - a'), \\ c_2 + d' &= a'. \end{aligned}$$

Lenders are endowed with  $y > 1$  units of consumption good in periods  $t = 0, 1$ . They can

lend these to the borrowers or save in a storage technology with gross return 1, which pins down the interest rate at which lenders are willing to lend. We assume that lenders do not have a technology to make productive use of the asset. The asset must be held by borrowers to yield a payoff in period 2 lest it loses all its value. As a result, lenders will not trade the asset at positive prices and borrowers may trade assets at a fire-sale discount, giving rise to what the literature has called “systemic risk.” This captures in a simple manner that borrowers are more productive in operating assets – an assumption that underlies much of the literature on fire sales following Shleifer and Vishny (1992) and Kiyotaki and Moore (1997). In equilibrium, we will therefore find that  $a \equiv a'$  and the term  $(1 - a')p$  cancels from the budget constraint of borrowers.

**Remark 1 (Debt Contracts)** We assume one-period debt contracts in our model since these constitute the simplest financial instrument possible. Our results are unchanged if borrowers can issue two-period debt in period 0 as long as this gives rise to the same incentives to renege as one-period debt (see, for example, our earlier working paper version, Jeanne and Korinek, 2013). Furthermore, our baseline analysis considers the case of uncontingent debt for simplicity of exposition. Our analysis nests the case of perfect foresight. For completeness, we show in Appendix A.2 that our main results are unchanged if a full set of Arrow securities is available. The financial imperfection that matters for our analysis is introduced in the next subsection.

## 2.2 Financial Imperfections

In period 1 the borrowers want to borrow more when their income  $\rho$  is low. We assume that their borrowing is constrained by a financial friction

$$d' \leq \phi p, \tag{3}$$

where  $p$  is the period-1 price of the borrowers' asset and  $\phi$  is a parameter between zero and one. Constraints of this type have been used in the recent literature on systemic risk and can be microfounded as follows by limited commitment. Assume that a borrower can make a take-it-or-leave-it offer to reduce the value of his debt at any time. If creditors reject this offer, they can seize  $\phi$  units of the borrower's assets which they can then sell at price  $p$ , the competitive price that other borrowers are ready to pay for the asset. The creditor, thus, will accept the borrower's offer as long as the offered repayment is at least  $\phi p$ , the amount that she would obtain by foreclosing on the capital. Without loss of generality we assume that debt is default-free, i.e., it is not renegotiated in equilibrium. At the end of period 1, the threat of renegotiation implies that the debt outstanding must be lower or equal to the value of the seizable collateral.

We assume that the debt can be renegotiated right after it is issued in period 1. As a result the collateral constraint involves the current price of the asset, as in the literature on fire sales. The debt could also be renegotiated at the time of repayment (period 2). However it is easy to see that if the constraint (3) is satisfied in period 1, the borrower will not renegotiate in period 2 since the price of the asset never decreases between period 1 and period 2. As we will see, the period-1 price satisfies  $p \leq 1$  whereas the period-2 price of the asset is 1. There could also be a renegotiation over the debt issued in period 0,  $d$ , but we assume that the resulting constraints are never binding to simplify the analysis. (The formal condition for this is given in Appendix A.3, which also analyzes the general case where that constraint may be binding.)

### 2.3 First-Best Allocation

We characterize first-best allocations without financial imperfections as a benchmark for the ensuing analysis.

We define a first-best allocation as a set of allocations  $(c_0)$  and functions

$(c_1(\rho), c_1(\rho), c_2^\ell(\rho))$ , with the latter depending on the realization of the productivity shock  $\rho$ , that maximize aggregate surplus  $E_0 [U^b + U^\ell]$  and satisfy the resource constraints of the economy. It is easy to see that all first-best allocations satisfy  $u'(c_0) = u'(c_1) = 1$ . We denote by  $c^{FB}$  the level of borrower consumption satisfying this condition –  $c^{FB}$  is equal to 1 with logarithmic utility. The total welfare of the representative borrower and lender in a first-best allocation is then given by<sup>6</sup>

$$E [U^{b,FB} + U^{\ell,FB}] = E_0 [\rho] + 2y + 1.$$

### 3 Policies

A systemic financial crisis is an equilibrium in which a low realization of the liquidity shock  $\rho$  leads to a binding financial constraint. This section introduces the policy instruments that a social planner can use to mitigate the welfare cost of a systemic financial crisis. The main distinction that we focus on in this paper is between ex-ante interventions and ex-post interventions. Broadly speaking, the purpose of ex-ante interventions is to mitigate over-borrowing in period 0 whereas ex-post interventions mitigate financial amplification if there is a crisis in period 1. These interventions can take various forms in practice, but, for the purpose of our analysis, it is convenient to model them as taxes and subsidies on borrowing. We discuss in the following how these interventions can be interpreted in terms of macroprudential policy, monetary policy, fiscal policy, or financial safety nets.

#### 3.1 Ex-ante Interventions

The first category of policy instruments target the decision variables of borrowers in period 0, before binding financial constraints materialize. In our simple framework, there is just a

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<sup>6</sup>The allocation of period-2 consumption between the two agents is indeterminate in the first-best since both value consumption equally.

single decision margin for borrowers in period 0, which is how much to borrow and consume. Policy can affect this decision variable using price-based or quantity-based intervention. The first category would be a macroprudential tax on period-0 borrowing. Assume that the social planner makes each borrower  $i$  pay  $\tau$  for every unit of debt issued in period 0 and leaves him the net proceeds  $(1 - \tau)d^i$ , with the tax revenue rebated to all borrowers so that  $c_0^i = (1 - \tau)d^i + \tau d = d$  in a symmetric equilibrium. Such a macroprudential tax modifies the Euler equation of the representative borrower to

$$(1 - \tau) u'(d) = E[u'(c_1)].$$

One interpretation of this macroprudential tax on borrowing, following the spirit of Stein (2012), is contractionary monetary policy, which makes it more expensive for borrowers to take on debt.<sup>7</sup>

Alternatively, the planner could introduce macroprudential quantity restrictions by imposing a ceiling on borrowing such that

$$d \leq \bar{d}.$$

In our framework, a given debt allocation  $d$  can be equivalently implemented using ex-ante price and quantity interventions. Most real-world macroprudential policy interventions involve quantity restrictions, such as minimum requirements for bank capital or maximum loan-to-value ratios for bank lending. However, one reason why we may be interested in the optimal price intervention  $\tau$  that corresponds to a given quantity intervention  $\bar{d}$  is that it reflects the wedge introduced in the optimality condition of borrowers, which is a good indicator for the regulatory burden and for the incentive to circumvent regulation.

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<sup>7</sup>Note, however, that raising the interest rate would also entail a redistribution from borrowers to lenders.

## 3.2 Ex-post Interventions

During a financial crisis, policymakers in the real world have a variety of policies at their disposal, ranging from from monetary relaxation to fiscal bailouts and debt relief. We show in this section that these three interventions can all be modeled as subsidies on borrowing. It matters, however, whether the subsidy is on new borrowing or on old outstanding debt. We interpret a subsidy on new borrowing as a "monetary stimulus" and a subsidy on outstanding debt as "fiscal bailouts".

First, the social planner can pay each borrower  $\sigma$  for every unit of outstanding debt  $d$ . This is a natural assumption if the ex-post intervention takes the form of a fiscal bailout or of debt relief that is proportional to the outstanding stock of debt, as would be the case for example under the typical financial safety nets. In the following, we will refer to this type of policy as a "fiscal bailout."

Second, the social planner could pay each borrower  $\sigma'$  for each unit of new debt  $d'$  issued in period 1. The difference with the previous kind of intervention is that it involves a subsidy on new borrowing rather than outstanding debt. Such an intervention could be interpreted as a monetary relaxation that lowers the real interest rate as in Farhi and Tirole (2012). A subsidy on the collateral asset has the same effect as lowering the interest rate. Finally, the subsidy could also be interpreted as a fiscal transfer that is proportional to new borrowing, for example in the form of an investment tax credit. Because the most natural interpretation of the interest rate subsidy is in terms of monetary policy, we will refer to this policy as a "monetary stimulus" in the following.

The period-1 budget constraint for borrowers under the two subsidy measures is

$$c_1^i + (1 - \sigma)d^i = \rho + (1 + \sigma')d'^i.$$

There is one important difference with the case of ex-ante interventions: we can no longer assume that the subsidy is financed by a tax on borrowers since transferring the borrowers'

resources to themselves does not relax their credit constraints. Hence we assume that the subsidy is financed by a tax on other agents in the economy, which means, in our simple model, on lenders. The total amount of the tax is,

$$s = \sigma d + \sigma' d'. \quad (4)$$

We assume that imposing such a tax on lenders may introduce distortions into the economy, which we formally describe as a deadweight cost  $g(s)$  that satisfies  $g(0) = 0$ ,  $g'(s) \geq 0$  and  $g''(s) \geq 0$ ,  $\forall s > 0$ . It is not difficult to provide microfoundations for this reduced form.<sup>8</sup> The reduced form can also be generalized without affecting the essence of our results. If ex-post interventions take the form of a monetary stimulus, the distortionary cost can be interpreted as the cost of setting the interest rate "too low" from the point of view of the macroeconomic objectives of monetary policy (see Farhi and Tirole, 2012, for an elaboration of this point). We assume that the deadweight cost of taxation is borne by the lenders but this assumption is not important for our analysis.<sup>9</sup>

The period-1 budget constraint of the representative lender then takes the form,

$$c_1^\ell + d' = y + d - s - g(s).$$

An alternative interpretation of the deadweight cost  $g(s)$  is that the social planner cares about the distributive effects of bailouts: if the planner evaluates the expected consumption of lenders according to a concave social welfare function  $w(U^\ell)$  that satisfies  $w'(2y) = 1$  and

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<sup>8</sup>For example, assume that the period-1 income of lenders comes from an activity that allows lenders to produce a quantity of good  $q$  at cost  $C(q)$  where  $C(\cdot)$  is an increasing and convex function, and define net income as  $y(q) = q - C(q)$ . Assume that the bailout is financed by a tax  $\theta$  on this activity,  $s = \theta q$ . Then the deadweight cost  $g(s)$  is implicitly defined by,  $g(s) = y - y(q)$  where  $y$  is the maximum level of net income and  $y'(q)q = s$ . It is easy to see that  $g(s)$  increases with  $s$  and  $g'(0) = 0$ .

<sup>9</sup>In an earlier version of this paper (Jeanne and Korinek, 2013), for example, we assumed that the borrowers combined capital with labor provided by the lenders to produce output in period 2. In that model the deadweight cost of taxation was born by the borrowers (through higher wages). These features complicated the model but did not affect the results in any essential way.

$w'' < 0$ , then we can define  $g(s) = w(U^\ell) - s - w(U^\ell - s)$  to capture the planner's losses arising from redistribution.

### 3.3 Equilibrium

In our baseline analysis, we consider Markov perfect equilibria in which the social planner maximizes welfare in a time-consistent way. The social planner's period-1 policies are contingent on the state  $(\rho, d)$  and consist of two functions  $\sigma(\rho, d)$  and  $\sigma'(\rho, d)$ . A competitive equilibrium consists of

- (i) a set of real allocations  $(c_0, c_1(\rho, d), c_2(\rho, d))$ ;
- (ii) financial allocations  $d$  and  $d'(\rho, d)$  and asset price  $p(\rho, d)$ ;
- (iii) ex-ante policies  $\tau$  (or  $\bar{d}$ ) and ex-post policies  $\sigma(\rho, d)$  and  $\sigma'(\rho, d)$ ;

such that in both periods  $t = 0, 1$  borrowers maximize their utility subject to their budget and financial constraints and the social planner maximizes welfare.

## 4 Optimal Policy Mix

The core question of this paper is to characterize the optimal mix of ex-ante versus ex-post policy interventions that would be chosen by a benevolent social planner to maximize welfare in the economy. We will first focus on the optimal policy problem of a discretionary planner, since excessive discretion in the use of ex-post policy interventions is frequently cited as a reason to engage in macroprudential policies. Then we will compare the optimal policy mix under discretion with the solution under commitment. Finally, we will analyze the complementarity or substitutability between the different types of policy intervention.

## 4.1 Ex post policy

We start with an analysis of the equilibrium in period 1 after all uncertainty has been realized. Borrowers are identical and make the same decision in equilibrium. However, it will be important in some of our derivations to differentiate between variables related to an individual atomistic borrower and variables related to the representative borrower. We denote the variables related to an individual borrower with a superscript  $i$  when this is necessary for clarity.

**Period-1 Problem of Borrowers** An individual borrower  $i$  enters period 1 with a debt level  $d^i$  from the previous period and obtains the endowment income  $\rho$  that depends on the shock realization, resulting in an amount of liquid net worth  $m^i = \rho - d^i$ . Furthermore, he obtains the subsidy rates  $\sigma$  and  $\sigma'$  on his old and new debt levels, providing subsidy revenue  $s^i = \sigma d^i + \sigma' d'^i$ .

As we will show formally below, the planner will only provide positive subsidies when the constraint on borrowers is binding. The borrower's consumption is thus determined by his private liquid net worth  $m^i$ , his subsidy income  $s^i$ , and his borrowing capacity,

$$c_1^i(m^i + s^i; p) = \min \{c^{FB}, m^i + s^i + \phi p\}, \quad (5)$$

In general equilibrium, the asset price is such that the marginal disutility of sacrificing  $p$  units of period-1 consumption of the representative borrower to purchase one unit of the asset equals the marginal gain from receiving a unit payoff from the asset in period 2, that is,<sup>10</sup>

$$pu'(c_1) = 1 \quad \text{or, equivalently,} \quad p = c_1. \quad (6)$$

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<sup>10</sup>This first-order condition holds whether or not borrowers are financially constrained. Note that a borrower does not relax his credit constraint by purchasing the asset because creditors can seize a fixed quantity  $\phi$  of asset in a default (rather than a fraction of the borrower's assets). This is why the asset price in equation (6) does not involve any collateral premium. This setup simplifies our analysis without affecting our qualitative results.

In equilibrium, no borrower defaults and no collateral asset is sold, but all agents in the economy know that the price of collateral is determined by this equation conditional on a default.

Using equation (6) to substitute out  $p$  from (5), we obtain a fixed-point equation for the period-1 consumption of the representative borrower,

$$c_1 = \min \{c^{FB}, m + s + \phi c_1\} = \min \left\{ c^{FB}, \frac{m + s}{1 - \phi} \right\}, \quad (7)$$

where we have solved for  $c_1$  to derive the second equality. This equation defines equilibrium consumption  $c_1(m + s)$  as an increasing function of the representative borrower's total liquid wealth. If liquid wealth is above the threshold  $m + s \geq \hat{m} = c^{FB} - \phi$ , then consumption is at its first-best level. For  $m + s < \hat{m}$ , equilibrium consumption is constrained and the asset price declines, leading to financial amplification. This mechanism is well-known from the literature. However, what is important to emphasize is that subsidies to constrained borrowers lead to a virtuous circle: Suppose that net liquidity  $m + s < \hat{m}$  so the financial constraint is binding, and assume that the policymaker provides a marginal unit extra liquidity subsidies. The impact on consumption can be obtained from implicitly differentiating (7),

$$c'_1(m + s) = \frac{1}{1 - \phi} > 1.$$

Intuitively, the amplification arises because borrowers push up the price of collateral when they have more liquidity, which relaxes the financial constraint and allows them to obtain further liquidity from borrowing. The term  $1/(1 - \phi)$  can be viewed as the sum of the geometric series  $1 + \phi + \phi^2 + \dots$  that captures the initial liquidity injection plus round after round of relaxation of the financial constraint.

Overall, the period-1 utility of borrowers (dropping constant terms) is given by

$$V(m+s) = m+s + u(c_1(m+s)) - c_1(m+s), \quad (8)$$

and is strictly increasing in  $m+s$ . Its derivative  $V'(m+s)$  is strictly decreasing for  $m+s < \hat{m}$  and satisfies  $V' = 1$  for  $m \geq \hat{m}$ .

**Period-1 Problem of Policymaker** For given private liquid net worth  $m$ , the policymaker chooses subsidies  $\sigma$  and  $\sigma'$  to maximize aggregate welfare in the economy. From the perspective of period 1, both subsidy measures enter the expressions for welfare of the two agents always through the sum  $s = \sigma d + \sigma' d'$ . Therefore, the optimization problem of the planner (in which we drop constant terms) can be expressed as

$$W_1(m) = \max_s u(c_1(m+s)) + m - c_1(m+s) - g(s). \quad (9)$$

The planner's objective function strictly increases with  $s$  for  $s = 0$  and strictly decreases with  $s$  for  $s \geq \hat{m} - m$ . Thus we know that it is maximized for an interior solution  $0 < s(m) < \hat{m} - m$  which satisfies the first-order condition,

$$[u'(c_1) - 1] c_1'(m+s) = g'(s). \quad (10)$$

Intuitively, the planner equates the social marginal cost of liquidity and the social marginal benefit of liquidity for borrowers, which is to increase their period-1 consumption by  $c_1' = 1/(1-\phi)$  and bridge the gap between marginal utility in periods 1 and 2,  $[u'(c_1) - 1]$ . The optimality condition defines an optimal subsidy  $s(m)$  that is positive for  $m < \hat{m}$  and is zero for  $m \geq \hat{m}$ . If raising fiscal revenue is distortionary ( $g'(s) > 0$ ), then the optimal subsidy is declining in  $m$  at rate  $-1 < s'(m) < 0$ ; if raising fiscal revenue is costless ( $g'(s) = 0$ ), then the optimal subsidy for  $m < \hat{m}$  is  $s(m) = \hat{m} - m$ .

We summarize our results on the optimal ex-post intervention in the following proposition.

**Proposition 1 (Ex-Post Interventions)** *Assume that ex-post interventions are distortionary ( $g(\cdot) > 0$ ). Then:*

- (i) The planner provides stimulus  $s > 0$  to borrowers if and only if their liquidity  $m$  is strictly below the threshold  $\hat{m}$  at which the credit constraint becomes binding.*
- (ii) The planner mitigates the credit constraint only partially.*
- (iii) It does not matter for period-1 allocations and period-1 welfare whether the stimulus is provided in the form of a debt bailout or a monetary stimulus.*

**Proof.** For point (i), the result follows immediately from equation (10): the equilibrium  $s$  is strictly positive if and only if the l.h.s. is strictly larger than the r.h.s. for  $s = 0$ , that is, if and only if  $m$  is strictly lower than  $\hat{m}$ . To prove point (ii), observe that if the subsidy were to completely relax the credit constraint when  $m < \hat{m}$ , then the l.h.s of equation (10) would be equal to zero whereas the r.h.s. would be strictly positive, a contradiction. To prove point (iii), observe that the two subsidies  $\sigma$ ,  $\sigma'$  enter condition (10) only via  $s$ . Therefore any combination of  $\sigma$ ,  $\sigma'$  that satisfies  $s = \sigma d + \sigma' d'$  implements the optimal allocation from the perspective of period 1. ■

## 4.2 Ex ante policy

We start with the optimal period-0 policy problem when the planner's instrument is a macroprudential tax  $\tau$  on borrowing; then we focus on how to implement the same allocation using a debt cap  $\bar{d}$ .

**Macroprudential tax** The problem of an individual borrower  $i$  who faces a macroprudential tax  $\tau$  and anticipates the period-1 interventions  $\sigma$  and  $\sigma'$  is to choose  $d^i$  to maximize

expected utility. Since the macroprudential tax is rebated to borrowers,  $c_0 = (1 - \tau) d^i + \tau d$  and the borrower solves

$$U^i = \max_{d^i} u((1 - \tau) d^i + \tau d) + E [V^i (\rho - (1 - \sigma) d^i + \sigma' \phi p) ]. \quad (11)$$

The simplification  $\sigma' \phi p = \sigma' d^i$  follows from Proposition 1 since the subsidy  $\sigma'$  is nonzero only when the borrowing constraint is binding.

Using the envelope condition  $V^{i'}(\cdot) = u'(c_1^i)$ , the optimality condition of individual borrowers is,

$$(1 - \tau) u' (d^i) = E [u' (c_1^i) (1 - \sigma)]. \quad (12)$$

By contrast, a social planner sets  $d$  to maximize social welfare in period 0,

$$\max_d u(d) + E [W_1 (\rho - d) ],$$

with optimality condition

$$u' (d) = E [W_1' (m)] = 1 + \frac{E [u' (c_1) - 1]}{1 - \phi}. \quad (13)$$

This condition equates the marginal benefit of consumption in period 0 to the marginal benefit of funds in period 1, which includes the amplification effects captured by the derivative  $c_1' = 1/(1 - \phi)$ . Comparing the first-order conditions (12) and (13) shows that there is overborrowing under laissez-faire. In the absence of macroprudential intervention, equation (12) implies that private agents would pick a higher level of debt than the planner since the right-hand side of the Euler equation of private agents satisfies

$$E[u' (c_1^i) (1 - \sigma)] \leq E[u' (c_1^i)] = 1 + E [u' (c_1) - 1] \leq 1 + \frac{E [u' (c_1) - 1]}{1 - \phi}, \quad (14)$$

which equals the right-hand side of the planner's Euler equation.

Using (10) and (13) to substitute out  $u'(c_1)$  and  $u'(d)$  from the private Euler equation (12), we obtain the optimal macroprudential tax rate,

$$\tau = \frac{E[\sigma u'(c_1)] + \phi E[g'(s)]}{1 + E[g'(s)]}. \quad (15)$$

The two terms in the numerator reflect the two causes of overborrowing in this model. The first term reflects the overborrowing induced by the expectation of the subsidy whereas the second term reflects that private agents do not internalize their contribution to financial amplification in a crisis.

**Debt cap** The planner can equivalently implement the optimal allocation by imposing a debt cap  $\bar{d}$  that prevents borrowers from issuing more than the optimal level of debt. The first-order condition for the optimal debt cap is equation (13). Observe that the equation only depends on  $s$  not the components  $(\sigma, \sigma')$ , therefore the debt cap is independent of the composition  $(\sigma, \sigma')$ .

We summarize our findings in the following proposition:

**Proposition 2 (Ex-Ante Interventions)** *The planner implements the optimal policy mix by following the optimal ex-post policy described in Proposition 1 and imposing*

(i) *either a debt cap  $d^i \leq \bar{d}$  as defined by equation (13), which is independent of the composition of ex-post policy interventions  $(\sigma, \sigma')$  and more relaxed than in the absence of ex-post interventions,*

(ii) *or a macroprudential tax on borrowing given by (15). The optimal macroprudential tax is higher the more of the ex-post intervention is provided in the form of fiscal bailouts  $\sigma$  rather than monetary stimulus  $\sigma'$ .*

**Proof.** See discussion above. ■

The planner’s optimal debt level  $\bar{d}$  is independent of the composition  $(\sigma, \sigma')$  in which ex-post interventions are provided for a given total level of  $s$ . By contrast, the macroprudential tax has to account for any distortions to borrowing incentives created by the subsidy  $\sigma$ . This points to an important practical benefit of debt caps: they are more robust since they need to be less responsive to the incentive effects of ex-post stimulus interventions. The optimal debt cap does not depend on private sector expectations (whether rational or not) about the form of future stimulus. As we will analyze in further detail below, this also provides the policymaker with the freedom to choose an ex-post policy instrument at her discretion without affecting the level of borrowing.

One of the motivations of this paper was to evaluate the conditions under which the “Greenspan doctrine” holds, according to which policymakers should intervene only ex post and not ex ante. The Greenspan doctrine is not true in general in our model, but it is interesting to delineate the assumptions that are necessary and sufficient to make it true.

**Proposition 3 (Greenspan Doctrine)** *Macroprudential regulation is superfluous in the following two cases and only in these two cases:*

- (i) *the ex-post intervention does not involve a fiscal stimulus ( $E[\sigma u'(c_1)] = 0$ ) and has no distortionary cost ( $E[g'(s)] = 0$ ), or*
- (ii) *the ex-post intervention does not involve a fiscal stimulus ( $E[\sigma u'(c_1)] = 0$ ) and there is no financial amplification ( $\phi = 0$ ).*

**Proof.** Using equation (15) it is easy to see that  $\tau = 0$  if (i) or (ii) is true. Conversely if  $\tau = 0$  it must be that the two terms in the numerator are equal to zero, which requires (i) or (ii).<sup>11</sup> ■

A necessary condition for the Greenspan doctrine to hold and macroprudential regulation to be superfluous is that the ex-post subsidy be exclusively on new borrowing—otherwise

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<sup>11</sup>This also encompasses the trivial case that the probability for the constraint to bind is zero.

the expectation of ex-post intervention is sufficient to generate overborrowing which must be offset by ex-ante interventions.

Conditional on this, there are two different scenarios under which the Greenspan doctrine holds. Case (i) represents an economy in which ex-post policy interventions are costless – therefore the planner relies 100% on “mopping up after the crash.” The economy never experiences binding constraints and so there is no systemic risk—and therefore there is no need to impose costly macroprudential regulation. This argument has been developed in greater detail in Benigno et al. (2012).

Case (ii) captures an economy in which government revenue is costly but there is no other distortion such as systemic risk in financial markets. The planner finds it optimal to distribute resources to constrained borrowers in period 1 until their marginal valuation of wealth equals the resource cost plus the deadweight cost of transferring. However, this transfer is efficient, and since there is no systemic risk and amplification, there is no reason for macroprudential intervention in period 0.

Summarizing Propositions 1 and 2, except in the knife-edge cases of Proposition 3, the social planner uses both ex-ante and ex-post interventions because neither type of intervention fully alleviates the financial friction. Ex-ante intervention reduces the risk and severity of financial crises, but crises still occur. When they do, it is optimal for the social planner to resort to ex-post interventions. This result is consistent with the theory of second-best taxation. Both macroprudential regulation and the bailout introduce a second-order distortion into the economy but achieve a first-order benefit from mitigating binding constraints through two alternative channels.

### **4.3 Commitment vs. discretion**

One question that arises when studying the optimal policy mix is that of commitment—whether or not the social planner can commit in period 0 to her future policy interventions.

An important theme in the literature on financial crises is that policymakers tend to be excessively interventionist ex post because they ignore the implications of their policies for ex-ante private risk taking. A related theme is that it is important to set ex-ante limits and constraints on the use of ex post interventions.

In our initial analysis, we stacked the deck against ex-post interventions by assuming that the social planner cannot commit. We now compare the optimal policy mix under discretion to the one under commitment.

**Proposition 4 (Commitment Vs. Discretion)** *The optimal allocation obtained under discretion coincides with the optimal allocation under commitment.*

**Proof.** The behavior of private agents is described by their period-0 Euler equation (12) and the consumption rule (7). Given this and omitting constant terms, a planner under commitment in period 0 chooses a debt level  $d$  and state-contingent subsidy  $s(\rho)$  to solve

$$\max_{d, s(\rho)} u(d) + E [u(c_1(\rho - d + s(\rho))) + \rho - d - c_1(\rho - d + s(\rho)) - g(s(\rho))].$$

The optimality conditions are identical to equations (10) and (13) of the problem under discretion. As a result, the planner chooses the same allocation under commitment as under discretion. ■

It turns out that commitment does not allow the planner to improve on the allocation obtained under discretion. By implication, the optimal policy mix of Proposition 2 implements the constrained efficient allocation of the economy. Intuitively, the benefit of committing to a lower level of bailouts in models of financial constraints is that it induces borrowers to borrow less. In our framework, macroprudential policy already reduces borrowing directly without ancillary distortions. This enables the planner to provide the socially efficient level of bailouts when necessary ex post. In other words, macroprudential policy enables the planner not to worry about “moral hazard” in providing ex-post policy interventions such

as bailouts. In particular, Proposition 4 holds even if the period-1 intervention is provided in the form of distortionary debt relief  $\sigma > 0$  – in that case, the optimal macroprudential tax (15) rises, but the real allocation in the economy is unchanged and remains optimal. As a result, the planner is indifferent about which ex-post policy instruments is used.

One corollary of the proposition is that – if the tasks of imposing optimal macroprudential regulation and of conducting optimal ex-post interventions are performed by separate entities of government – the entities performing ex-post interventions do not need to take into account their effects on ex-ante borrowing incentives and the related time consistency problems – they can simply focus on implementing an optimal stimulus policy  $s(m)$  as described in Proposition 2. The institution conducting macroprudential policy could simply take  $s(m)$  as given and would face the task of imposing optimal ex-ante regulation as described in Proposition 2.

In the next section, we will investigate the importance of macroprudential policy being at its optimal level for the result of Proposition 4 by considering the case in which macroprudential interventions are suboptimal.

#### **4.4 Suboptimal Macroprudential Policy under Commitment**

In practice, macroprudential policy may not implement the optimal allocation described above. One reason is that policymakers have only recently started to explicitly consider macroprudential motives in setting financial regulation and that many financial regulators even lack a macroprudential mandate. Another reason is that financial regulation in general gives rise to circumvention by the private sector. This section considers macroprudential policy that is restricted. For example, a social planner may be restricted to impose a debt cap that is larger than the optimal level or a tax that is smaller than the optimal level.

Restrictions on macroprudential policy create a role for commitment that was absent when macroprudential policy is optimal as in Proposition 4. We study the difference be-

tween commitment and discretion by introducing the following notations. Let us denote by  $s^c(m)$  the stimulus policy under commitment, when the social planner can decide her period-1 interventions in period 0, and by  $s^d(m)$  the stimulus policy under discretion. The stimulus policy under discretion was described in Proposition 1 and we now denote it with the subscript  $d$  for clarity.

We then have the following result:

**Proposition 5** *Consider an economy in which the macroprudential tax is below the optimal level. Then a planner who has the power to commit will commit to*

- (i) *provide a lower stimulus  $s^c(m) < s^d(m)$  for given  $m$  compared to discretion,*
- (ii) *use only monetary stimulus ( $\sigma' > 0$ ) rather than fiscal bailouts (so  $\sigma = 0$ ).*

**Proof.** Given the tax  $\tau$ , the planner chooses  $d$  and the state-contingent subsidies  $s^c$  and  $\sigma^c$  to maximize the Lagrangian

$$u(d) + E \{ u(c(m + s^c)) + m - c(m + s^c) - g(s^c) - \chi [(1 - \tau) u'(d) - E u'(c(m + s^c)) (1 - \sigma^c)] + \zeta \sigma^c \},$$

where  $m = \rho - d$  and  $s^c = \sigma^c d + \sigma'^c d'$  is satisfied. We denote by  $\chi$  the shadow cost on the implementability constraint (12), which reflects the choice of debt by private agents, and by  $\zeta$  the shadow price on the non-negativity constraint on  $\sigma^c$ . The optimality conditions with respect to  $d$ ,  $s^c$  and  $\sigma^c$  are

$$\begin{aligned} FOC(d) & : u'(d) = 1 + E \{ [u'(c_1) - 1] c'_1 \} + \chi [(1 - \tau) u''(d) + E u''(c_1) (1 - \sigma^c) c'_1], \\ FOC(s^c) & : g'(s^c) = [u'(c_1) - 1] c'_1 + \chi u''(c_1) (1 - \sigma^c) c'_1, \\ FOC(\sigma^c) & : \chi u'(c_1) = \zeta. \end{aligned}$$

According to the first condition, the shadow price  $\chi$  satisfies

$$\chi = \frac{u'(d) - 1 - E\{[u'(c_1) - 1]c'_1\}}{(1 - \tau)u''(d) + Eu''(c_1)(1 - \sigma^e)c'_1}.$$

Comparing the numerator with (13), the shadow price is positive  $\chi > 0$  if there is overborrowing, i.e., if the tax rate is too low. The second optimality condition then reveals that the planner reduces the subsidy  $s^c$  compared to the optimal policy mix described in Proposition 1, proving point (i).

The shadow price  $\zeta$  is positive by the third optimality condition, proving point (ii). ■

If the macroprudential tax is too low, then ex-post policy interventions are excessive under discretion because they do not take into account their impact on the incentives to borrow ex ante. At the margin, a small reduction in the size of the “mopping up” interventions has a second-order welfare cost ex post but a first-order welfare gain by reducing borrowing ex ante. Suboptimal macroprudential policy thus makes it optimal to commit to ex-post interventions that are less generous than under discretion ( $s^c < s^d$ ).<sup>12</sup>

Commitment is an inferior substitute to optimal macroprudential policy: if it is set to its optimal level, macroprudential policy can take care of the overborrowing problem that commitment is trying to solve without having to distort the optimal ex-post intervention. Macroprudential policy is strictly superior to commitment—when macroprudential policy is at its optimal level, we know from Proposition 4 that there is no residual role left for commitment. Ex-post interventions can then be used with complete discretion. Although one might have expected macroprudential policy to be caught in a trade-off between two objectives (mitigating the pecuniary externality and mitigating the time-consistency problem) there is actually no tension between these two objectives. This is because both concerns are fully addressed by setting borrowing,  $d$ , at the appropriate level. It is more efficient

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<sup>12</sup>Under additional regularity conditions, it can also be shown that the optimal subsidy under commitment increases the closer the macroprudential instrument is to its optimum level.

to change  $d$  through macroprudential policy than by committing to inefficiently stringent ex-post policies.<sup>13</sup>

Point (ii) of the proposition highlights that, since fiscal bailouts  $\sigma$  increase borrowing incentives in period 0, the planner finds it desirable to commit to monetary stimulus when his macroprudential toolkit is imperfect. This overturns the indifference result under the optimal policy mix, i.e., that the planner does not care which ex-post instrument he uses when macroprudential policy can optimally correct ex-ante incentives.

Proposition 5 looks into the implications of having a macroprudential tax that is too low. If instead the problem is a debt cap that is too high, then the result depends on the level of the cap. For example, if the debt cap is sufficiently high that it is non-binding under the optimal stimulus policy, then it is equivalent to a zero macroprudential tax and the results of Proposition 5 apply, i.e. the planner commits to lower stimulus that is provided in monetary form. By contrast, if the debt cap is sufficiently close to the optimal level, then the costs of committing to a lower level of stimulus outweigh the benefits. Borrowing is determined by the binding cap, and policymakers do not need to be concerned with the incentive effects of stimulus policy. This points to an important practical benefit of sufficiently tight debt caps: they solve the time consistency problem that arises under suboptimal macroprudential taxation.

## 4.5 Suboptimal Ex-Post Interventions

It is natural to analyze restrictions on ex-post stimulus measures next. Just like in the case of macroprudential instruments, policymakers frequently face restrictions on the set of ex-post

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<sup>13</sup>This result reflects a more general insight about time consistency in optimal policy problems: time consistency problems reflect a lack of policy instruments and can be solved if a planner has sufficient (unrestricted) instruments available. Time inconsistency arises when the expectation of a planner's optimal actions affects the behavior of private agents in earlier periods in an undesirable way. In our setup, if the planner can control borrowing in period 0 directly via a macroprudential policy instrument  $\tau$ , then there is no more reason to deviate from the ex-post optimal level of debt, and the time consistency problem disappears.

instruments  $\sigma$  and  $\sigma'$  that prevent them from implementing the optimal policy mix. In our setting, we capture such restrictions by assuming that there is an upper limit  $\bar{s}$  such that the total stimulus provided satisfies

$$s(m) \leq \bar{s}. \quad (16)$$

If  $\bar{s}$  is equal to zero, the economy has no access to ex-post interventions (e.g. no fiscal space and no independent monetary policy), whereas it has unrestricted access to such interventions if  $\bar{s}$  is large enough. When constraint (16) is binding in some states, an increase in  $\bar{s}$  can be interpreted as a marginal extension of the fiscal or monetary policy space. Macroprudential policy is unrestricted.

**Proposition 6 (Restricted Ex-Post Interventions)** *A policymaker who faces restrictions on ex-post policy measures  $s \leq \bar{s}$  will*

- (i) *relax the debt cap the less restricted the ex-post measures, i.e. the higher  $\bar{s}$ ,*
- (ii) *lower the macroprudential tax the less restricted the ex-post measures (assuming ex-post measures take the form of monetary stimulus  $\sigma'$ ).*

**Proof.** For part (i) of the proof, replace the stimulus  $s$  on the right-hand side of the planner's Euler equation (13) by  $\min\{s, \bar{s}\}$  and the result follows. For part (ii) of the proof, observe that for  $\sigma = 0$ , the optimal tax formula (15) implies

$$\tau = 1 - \frac{1 - \phi}{1 - \phi/E[u'(c_1(\rho - d + s))]}.$$

Better ex-post measures reduce marginal utility and therefore lower the optimal tax rate. ■

The results of the Proposition run counter to the intuition of some policymakers who worry about greater moral hazard when more stimulus is available. However, the increase in borrowing in response to greater availability of ex-post stimulus is efficient – the stimulus

measures reduce the social costs of systemic debt crises ex post.<sup>14</sup> Conversely, the more difficult it is to provide bailouts, the less the planner mitigates the financial constraints and the more macroprudential regulation is indicated.

Observe, however, that result (ii) no longer holds if the ex-post stimulus takes the form of distortionary fiscal bailouts – in that case, the numerator of the optimal tax formula (15) implies that the macroprudential tax has to be increased to offset the distortions created by the expectation of fiscal stimulus  $E[\sigma u'(c_1)]$ . This case seems to be the presumption of policymakers who argue that more reliance on ex-post interventions warrants higher macroprudential regulation.

## 4.6 Bailout Fund

Since it is optimal to impose macroprudential restrictions in period 0 and to provide stimulus transfers in period 1, one might be tempted to combine the two policy measures and use the proceeds of a macroprudential tax to finance the stimulus. This can be done by accumulating the macroprudential tax proceeds in a “bailout fund” that is distributed in the future if borrowers experience binding financial constraints.<sup>15</sup> It would seem much preferable to finance the bailouts with a corrective tax than by a tax that introduces new distortions in the economy.

We analyze this policy proposal by assuming that the planner stores the proceeds of the macroprudential tax  $\tau \int d^i di = \tau d$  in a bailout fund. The fund is used to make transfers to constrained borrowers in period 1. Using the bailout fund does not entail any deadweight cost ex post, so that the funds are transferred to constrained borrowers until the constraint is completely alleviated or the bailout fund is exhausted. The funds that are left after borrowers are no longer financially constrained, if any, are rebated in a lump-sum way to borrowers

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<sup>14</sup>Jeanne and Zettelmeyer (2005) emphasize that an increase in borrowing in response to better financial safety nets does not in general reflect true moral hazard. Stavrakeva (2015) finds that better financial safety nets – as enabled by greater fiscal capacity – reduce the need for macroprudential regulation.

<sup>15</sup>This is common practice for most deposit insurance systems (see Garcia, 1999).

or lenders (it does not matter which for total welfare). We assume, without restriction of generality, that the excess funds are redistributed to the borrowers so that we do not have to look at the welfare of the lenders, and that the funds are distributed as a subsidy to new borrowing  $\sigma'$ , which does not distort borrowing in period 0.

The welfare properties of a bailout fund are described in the following.

**Proposition 7 (Bailout Fund)** *A bailout fund financed by a macroprudential tax (i) leads to the same allocation as if no ex-post stimulus is available  $\bar{s} = 0$  and (ii) unambiguously reduces welfare relative to the equilibrium where ex-post interventions are financed by a distortive tax on lenders.*

**Proof.** The ex-ante welfare of borrower  $i$  is given by maximizing

$$u((1 - \tau)d^i) + E \left\{ \max_{c_1^i} [u(c_1^i) + 1 + \rho - d^i - c_1^i + \tau d + \lambda^i (\rho - d^i + \tau d + \phi p - c_1^i)] \right\}.$$

where  $\lambda^i$  is the shadow price on the financial constraint. The first-order condition is,

$$(1 - \tau)u'((1 - \tau)d^i) = E [u'(c_1^i)] \quad \text{where} \quad c_1^i = \min(c^{FB}, \rho - d^i + \phi p + \tau d).$$

Without a bailout fund – if the macroprudential tax proceeds  $\tau d$  are rebated in period 0 or if a debt cap is used instead – and with ex-post stimulus restricted to  $s = 0$ , the first-order condition is,

$$(1 - \tau)u'((1 - \tau)d^i + \tau d) = E [u'(c_1)] \quad \text{where} \quad c_1^i = \min(c^{FB}, \rho - d^i + \phi p).$$

In both cases the first-order condition can be written in terms of the consumption levels as,

$$(1 - \tau)u'(c_0) = E [u'(c_1)] \quad \text{where} \quad c_1 = \min \left( c^{FB}, \frac{\rho - c_0}{1 - \phi} \right).$$

This problem is solved by a unique allocation  $(c_0, c_1(\rho))$ . Thus the allocation is the same whether there is a bailout fund or not. The real allocation is unchanged because the representative consumer borrows more to offset the saving invested by the social planner in the bailout fund. Hence welfare is the same as without bailout fund when ex-post stimulus policies are restricted  $\bar{s} = 0$ , proving point (i) of the proposition. If we remove the restriction  $s = 0$ , Proposition 1 implies that welfare is unambiguously increased whenever there are states in which borrowers are constrained, proving point (ii) of the proposition. ■

This results reflects a form of Ricardian equivalence. Introducing a bailout fund does not yield any efficiency benefits — the planner has no comparative advantage in holding precautionary savings against systemic risk compared to borrowers, as long as she can determine the correct level of private savings via macroprudential regulation. The tax that is used to finance the bailout fund achieves its intended macroprudential benefit. The bailout fund, however, not does not achieve the same gains as a bailout policy that is financed by ex-post taxation because private borrowers finance their contribution to the fund by issuing more debt.

Our result that a bailout fund is undesirable as a precautionary instrument against aggregate risk contrasts with the desirability of funds that are used to share uninsurable idiosyncratic risk: if a planner can pool the idiosyncratic risks of *heterogeneous* borrowers in a common fund, then she can reduce the total amount of savings held and thereby improve efficiency. We conclude that accumulating bailout funds only helps with idiosyncratic risk, not aggregate or systemic risk.

## 5 Numerical Illustration

We now present a numerical illustration of our results to provide additional intuition. Given the logarithmic utility  $u(c) = \log c$ , the first-best level of consumption is  $c^{FB} = 1$  and the

threshold for binding constraints is  $\hat{m} = 1 - \phi$ . The consumption and asset price functions are,

$$c_1(m + s) = p(m + s) = \min\left(1, \frac{m + s}{1 - \phi}\right). \quad (17)$$

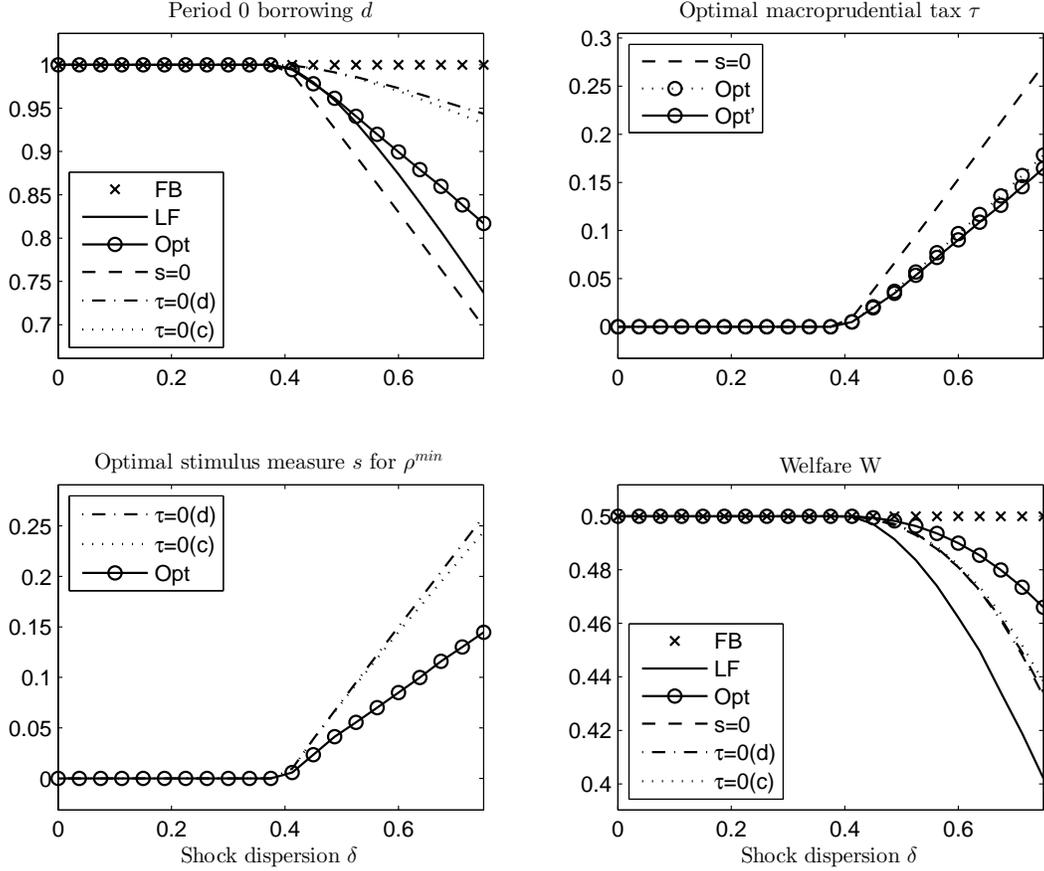
We assume  $g(s) = \gamma s^2/2$ . Furthermore, we assume that  $\rho$  is distributed uniformly over an interval  $[\bar{\rho} - \delta, \bar{\rho} + \delta]$ . The numerical values of the parameters are given below in Table 1.

$\phi$	$\gamma$	$\bar{\rho}$	$\delta$
.9	20	1.5	.75

**Table 1:** Parameter values for numerical illustration

In Figure 1 we vary the dispersion of the shock  $\rho$  from 0 to  $\delta$  and illustrate the effects on the equilibrium across five different policy regimes: the first-best (*FB*), the laissez faire equilibrium without intervention (*LF*), the optimal policy mix (*Opt*), optimal macroprudential policy without ex-post intervention ( $s = 0$ ), and optimal ex-post stimulus policy without macroprudential policy ( $\tau = 0$ ). As the shock dispersion  $\delta$  increases beyond  $\hat{\delta} = \bar{\rho} + \phi - 2 = .4$ , the economy starts to experience binding constraints for low realizations of  $\rho$ , and the probability of binding constraints rises in  $\delta$ .

The top-left panel shows initial consumption  $c_0$ . Since  $c_0 = d$ , the level of consumption is the level of the debt cap that a planner imposes if this instrument is available. Consumption and the debt cap are a declining function of the dispersion of the shock. The planner implements the tightest debt level when ex-post stimulus measures are restricted ( $s = 0$ ). Under the optimal policy mix (*Opt*), the debt level is lower than under laissez-faire (*LF*) for small shock dispersion but higher than laissez-faire for a larger dispersion of shocks. This illustrates that equilibrium debt is affected by two opposing forces – macroprudential policy reduces debt but ex-post stimulus measures make it optimal to increase debt compared to laissez-faire, as emphasized by Benigno et al. (2012). When macroprudential policy is not available ( $\tau = 0$ ), debt is naturally higher since ex-post stimulus measures (which are



**Figure 1:** Numerical illustration of policy regimes

provided as monetary stimulus when  $\tau = 0$ ) increase borrowing incentives compared to laissez-faire. Under commitment ( $\tau = 0(c)$ ), stimulus is lower and therefore borrowing is reduced compared to discretion ( $\tau = 0(d)$ ).

The top-right panel shows the macroprudential tax  $\tau$  required to achieve the optimal level of borrowing in panel 1. When ex-post stimulus measures are unavailable ( $s = 0$ ), the planner uses a greater macroprudential tax than in the optimal policy mix since financial crises are more costly without stimulus measures. The panel also illustrates that a higher macroprudential tax is necessary to implement the same real allocation if ex-post stimulus measures in the optimal policy mix take the form of distortionary fiscal bailouts ( $Opt$ ) rather

than monetary stimulus ( $Opt'$ ).

The bottom-left panel illustrates the optimal stimulus measure  $s$  for the lowest return realization  $\rho^{\min}$ . Under the optimal policy mix ( $Opt$ ), the planner relies less on ex-post stimulus measures than when macroprudential policy is restricted ( $\tau = 0$ ). Furthermore, under restricted macroprudential policy and commitment ( $\tau = 0(c)$ ), the planner reduces her ex-post stimulus efforts compared to discretion ( $\tau = 0(d)$ ) in order to provide incentives for lower borrowing, partially making up for the absence of macroprudential instruments.

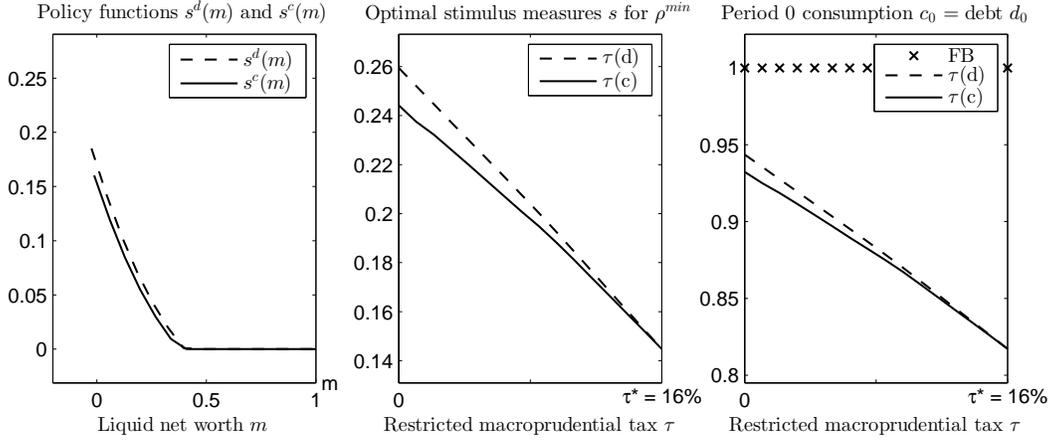
Finally, the bottom-right panel shows the impact of uncertainty on welfare under the different regimes. Under all five regimes, welfare is a strictly declining function of the shock dispersion  $\delta$ . The welfare losses are minimized under the optimal policy ( $Opt$ ) mix but maximal under laissez-faire ( $LF$ ). In our simulation, about one third of the welfare gains of optimal policy come from macroprudential regulation whether or not ex-post stimulus policies are used. The remaining two-thirds come from ex-post policy intervention. When macroprudential policy is not used, it can be seen that committing to less generous ex-post policies provides moderate welfare gains.<sup>16</sup>

Figure 2 shows optimal ex-post stimulus measures and the resulting debt levels under commitment and discretion if macroprudential policy is subject to varying degrees of restrictions. The figure considers the case with shock dispersion  $\delta = 0.75$ . The left panel shows the policy functions  $s^c(m)$  and  $s^d(m)$  if macroprudential policy is absent, i.e.  $\tau = 0$  or equivalently  $\bar{d} = \infty$ . For a given level of period-1 liquid net worth, the planner provides less ex-post stimulus under commitment than under discretion in order to induce greater precautionary behavior and less borrowing in period 0.

The middle and right panels show the effects of improving macroprudential policy, i.e. of increasing the macroprudential tax  $\tau$  along the horizontal axis from zero to its optimal level

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<sup>16</sup>If macroprudential policy is absent and ex-post stimulus is provided in the form of distortionary fiscal bailouts ( $\sigma > 0$ ), then welfare may well be lower than in the laissez-faire equilibrium, since fiscal bailouts create moral hazard. In our simulations, we found that this may occur if the probability of being constrained is high and amplification effects are strong, i.e.,  $\phi$  is close to 1.



**Figure 2:** Ex-post stimulus under commitment versus discretion

( $\tau^* = 16\%$  under the given parameterization). The middle panel depicts the optimal ex-post stimulus measure for the lowest realization of the shock  $\rho^{\min}$ . If macroprudential policy is restricted to  $\tau < \tau^*$ , borrowers carry more debt and experience more severe financial crises in the low state  $\rho^{\min}$ . The optimal ex-post stimulus is thus higher than under the optimal policy mix and is a declining function of the macroprudential tax  $\tau$ . Under discretion, the ex-post stimulus is even higher than under commitment since the planner does not internalize the adverse effects of the ex-post stimulus on ex-ante borrowing incentives. When the macroprudential tax  $\tau$  reaches its optimal level  $\tau^*$  at the right end of the panel, the stimulus under both discretion and commitment converges to the level in the optimal policy mix. The right panel shows the effects on period-0 consumption and debt: as long as macroprudential policy is restricted ( $\tau < \tau^*$ ), debt is above the level in the optimal policy mix, and more so under discretion than under commitment.

## 6 Conclusions

This paper develops a simple framework to analyze optimal policies in an environment where collateral-dependent borrowing constraints lead to financial amplification. Except in knife-

edge cases, all policies fall into the category of second-best interventions, i.e., they achieve first-order welfare gains by mitigating binding borrowing constraints in the economy, but at the expense of introducing second-order distortions. In accordance with the theory of the second-best (see Lipsey and Lancaster, 1956), it is optimal to use all second-best instruments available in such a setting. In particular, we show that it is optimal to both restrict borrowing ex-ante via macroprudential regulation and to relax borrowing constraints ex-post by providing bailouts or other transfers. This implies that policymakers should both “lean against the wind” and “mop up after the crash.”

The two policies are substitutes for each other since they address the same goal from different angles, implying that in equilibrium, macroprudential policy is used more when stimulus measures are restricted and vice versa. We also show that there is no time consistency problem if the optimal mix of ex-ante macroprudential regulation and ex-post stimulus measures is implemented. However, if macroprudential policy is restricted, then committing to less ex-post policy and to providing it exclusively in the form of monetary stimulus serves as a second-best device for reducing excessive borrowing.

There are a number of important questions that we leave for future analysis. First, financial policies such as macroprudential regulation and stimulus have distributive implications. Although we noted that the cost of raising funds  $g(s)$  can be interpreted as the planner’s penalty for redistributing funds, we have not paid attention to the political economy aspects of the choice between macroprudential regulation and mopping up after the crash. It is clear in our setup that borrowers will dislike the former and greatly value the latter. This creates an important role for special interests and lobbying. Secondly, we have modeled ex-post stimulus policies in reduced form, but the administration of stimulus policies in practice gives rise to a number of difficulties that arise from asymmetric information or budgetary limitations (see e.g. Philippon and Schnabl, 2013). This calls for more detailed analysis in an integrated framework of macroprudential regulation and ex-post interventions. Third, it would be interesting to investigate the role for monetary policy in a more comprehen-

sive New Keynesian framework of fire sale externalities with nominal stickiness. Finally, it is desirable to consider the interactions with further externalities that are relevant during financial crises and that create a role for macroprudential regulation, in addition to the pecuniary externalities and bailout distortions that we consider in the current paper. Examples include irrationality and aggregate demand externalities (see e.g. Farhi and Werning, 2013, and Korinek and Simsek, 2016).

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## **A Further Results [Online Appendix]**

### **A.1 General utility function**

This appendix considers more general utility functions and shows that our results continue to hold under appropriate assumptions. First, observe that the asset pricing equation (6) implies

$$p = \frac{1}{u'(c_1)}$$

Substituting this into the consumption function of individual agents yields a fixed-point equation for the period-1 consumption of the representative borrower,

$$c_1 = \min \left\{ c^{FB}, m + s + \frac{\phi}{u'(c_1)} \right\}. \quad (18)$$

In order to ensure that this equation has a unique solution for  $c_1$  we impose the following restriction.<sup>17</sup>

**Assumption 1** *The utility function satisfies  $-\phi u''(c)/u'(c)^2 < 1$  for  $0 \leq c \leq c^{FB}$ .*

This assumption is satisfied for example for CRRA utility functions with coefficient of relative risk aversion  $\theta$  if the pledgeability parameter satisfies  $\phi < 1/\theta$ .

The assumption ensures that the right-hand side of equation (18) rises more slowly than the left-hand side, guaranteeing a unique intersection of the two. The equation then implicitly defines equilibrium consumption  $c_1(m + s)$  as an increasing function of the representative borrower's total liquid wealth,  $m + s$ . The derivative of the function is given by

$$c'_1(m + s) = 1 / \left[ 1 - \phi \left( -\frac{u''(c_1)}{u'(c_1)^2} \right) \right] > 1.$$

The period 1 utility of borrowers and welfare from the perspective of the policymaker continue to be given by (8) and (9), and the optimal ex-post stimulus is determined as the solution to equation (10), which has an interior solution that satisfies  $0 < s(m) < \hat{n} - m$ . The results of Propositions 1, 2, 4 and Proposition 3 readily follow.

If condition (10) has a unique solution, then it is not only necessary but also sufficient for the optimum. However uniqueness is not guaranteed in general because function  $c_1(\cdot)$  could be convex. Thus there is no guarantee that  $s(\cdot)$  is a continuous function of  $m$ , and the sign of the variations

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<sup>17</sup>If this regularity condition was violated, then the left-hand side and the right-hand side of equation (18) may intersect in several points, leading to multiple equilibria: an unconstrained equilibrium with a high collateral price and a high level of consumption may coexist with a constrained equilibrium where both variables are lower. Equilibrium multiplicity raises issues that are not essentially related to the core question in this paper. See for example the appendix of Jeanne and Korinek (2010b).

of  $s$  with  $m$  is a priori ambiguous, as can be seen from implicitly differentiating (10),

$$s'(m) = -\frac{[u''(c_1) - 1] c_1' (m + s)^2 + [u'(c_1) - 1] c_1'' (m + s)}{[u''(c_1) - 1] c_1' (m + s)^2 + [u'(c_1) - 1] c_1'' (m + s) - g''(s)}.$$

One can ensure that function  $s(\cdot)$  is well-behaved by imposing conditions that guarantee that the numerator of this expression is negative. For CRRA utility functions, the consumption function satisfies  $c_1''(m + s) = \frac{\phi\theta(\theta-1)c^{\theta-2}}{(1-\phi\theta c^{\theta-1})^3}$  and is non-positive if  $\theta \leq 1$ . This is a sufficient condition to guarantee that  $s(m)$  is continuous and decreasing with  $s'(m) \in [0, 1)$ . Given these properties of the function  $s(m)$ , Propositions 5 and 6 readily follow.

## A.2 Complete Markets

This appendix replicates the analysis of the period-0 problem for the case of complete markets. The period-1 problem is unchanged from our earlier analysis in Section 4.2 and continues to be described by the same value functions  $V(\cdot)$  and  $W(\cdot)$ . For clarity of notation, we add a superscript denoting the state of nature  $\omega \in \Omega$  to all state-contingent variables. In period 0, borrowers are free to issue different amounts of securities  $d^\omega$  contingent on the state of nature  $\omega$ . Given the risk-neutrality of lenders, the budget constraint of borrowers is  $c_0 = E[d^\omega]$ , and the optimization problem of private borrowers as well as the planner is,

$$\max_{d^\omega} u(E[d^\omega]) + E[V(\rho^\omega - d^\omega)] \quad \text{and} \quad \max_{d^\omega} u(E[d^\omega]) + E[W(\rho^\omega - d^\omega)].$$

The optimality conditions for the security issuance in each state of nature are

$$u'(c_0) = V'(m^\omega) \quad \forall \omega \quad \text{and} \quad u'(c_0) = W'(m^\omega) \quad \forall \omega.$$

The term  $u'(c_0)$  is constant so borrowers and the planner choose to fully insure the period-1 shock and enter period 1 with the same liquid net worth  $m^\omega = E[\rho^\omega] - c_0 \forall \omega$  in all states of nature. This is unsurprising since lenders are risk-neutral and offer actuarially fair insurance. If  $m^\omega \geq \hat{m}$ , then borrowers can insure away the binding constraints and attain a first-best allocation in which  $c_0 = c^{FB}$  and there is no role for policy allocation. Otherwise, constraints are equally binding in

all states of nature. The optimal cap on security issuance in state  $\omega$  is  $\bar{d}^\omega = c_0 + \rho^\omega - E[\rho^\omega]$  where  $c_0$  is given by an Euler equation analogous to equation (13),

$$u'(c_0) = 1 + \frac{u'(c_1) - 1}{1 - \phi}.$$

Period-1 consumption  $c_1$  is the same in all states of nature so we omitted the expectations operator. The optimal tax on securities issued against any state of nature  $\omega \in \Omega$  is constant and is analogous to equation (15),

$$\tau^\omega = \frac{\sigma u'(c_1) + \phi g'(s)}{1 + g'(s)}$$

where we omitted expectations and the superscript  $\omega$  for period 1 variables since they are constant across states of nature. It is straightforward to obtain analogs to Propositions 3, 2, 4 and 5 as well as our remaining results.

In the described case, borrowers fully insure their shock with lenders since lenders are risk-neutral. It would be straightforward to extend our analysis by considering risk-averse lenders. In that case, borrowers would no longer insure fully and the constraint would be binding to different degrees in different states of nature, as in our baseline analysis with uncontingent debt contracts.

### A.3 Collateral constraint in Period 0

This appendix analyzes the conditions under which the borrower will be tempted to renegotiate the debt issued in period 0. We assume that borrowers can make a take-it-or-leave-it offer to renegotiate their debt at any time. If creditors reject this offer, they can seize  $\phi$  units of the borrower's assets and sell them to other borrowers at the prevailing market price. The incentive compatibility constraints that induce borrowers to refrain from renegeing at all time periods are as follows:

(C#) Period	Debt	Collateral
(C1) end of period 0	$d \leq$	$\phi p_0 = \phi c_0 E \left[ 1 + \frac{\rho}{c_1} \right]$
(C2) beginning of period 1	$d \leq$	$\phi(\rho + p_1) = \phi(\rho + c_1)$
(C3) end of period 1	$d' \leq$	$\phi p_1 = \phi c_1$
(C4) beginning of period 2	$d' \leq$	$\phi$

In the main text we have considered the collateral constraints for the debt issued in period 1, (C3) and (C4). Here we focus on the collateral constraints for the debt issued in period 0, (C1) and (C2).

For constraint (C1), observe that we can substitute  $c_0 = d$  and simplify the constraint to

$$\phi E \left[ \frac{\rho}{c_1} + 1 \right] \geq 1.$$

This constraint is satisfied as long as both  $\phi$  and the weighted average shock realization  $\rho$  are sufficiently high. The numerical illustration in Section 5 provides an example.

Constraint (C2) requires that  $d \leq \phi(\rho + c_1)$ . If the constraint (C3) is loose then  $c_1 = 1$  and (C2) can be transformed to

$$d \leq \phi(1 + \rho).$$

Otherwise and assuming the laissez faire equilibrium with  $s = 0$ , observe that  $c_1 = (d - \rho)/(1 - \phi)$ , and (C2) can be transformed to

$$d \leq \phi(2 - \phi)\rho.$$

In both cases, the constraint (C2) is satisfied for all states of nature as long as both  $\phi$  and the lowest shock realization  $\rho^{\min}$  are sufficiently high. Again, the numerical illustration in Section 5 provides an example.

If one of the constraints (C1) or (C2) was binding, then debt issuance in period 0 would be a corner solution that is determined by the binding constraint, and there is nothing the social planner can do using ex-ante interventions. The period-1 decisions would be unchanged from our analysis in Section 4.1 and Proposition 1.