Liquidity Trap and Excessive Leverage

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We investigate the role of macroprudential policies in mitigating liquidity traps. When constrained households engage in deleveraging, the interest rate needs to fall to induce unconstrained households to pick up the decline in aggregate demand. If the fall in the interest rate is limited by the zero lower bound, aggregate demand is insufficient and the economy enters a liquidity trap. In this environment, households' ex-ante *leverage and insurance decisions are associated with aggregate demand externalities. Welfare can be improved with macroprudential policies targeted towards reducing leverage. Interest rate policy is inferior to macroprudential policies in dealing with excessive leverage.*

Leverage has been proposed as a key contributing factor to the recent recession and the slow recovery in the US. Figure 1 illustrates the dramatic rise of leverage in the household sector before 2008 as well as the subsequent deleveraging episode. Using county-level data, Mian and Sufi (2014) have argued that household deleveraging is responsible for much of the job losses between 2007 and 2009. This view has recently been formalized in a number of theoretical models, e.g., Guerrieri and Lorenzoni (2011), Hall (2011), and Eggertsson and Krugman (2012). These models have emphasized that deleveraging represents a reduction in aggregate demand. The interest rate needs to fall to induce unconstrained households to make up for the lost aggregate demand. However, the nominal interest rate cannot fall below zero given that hoarding cash provides an alternative to holding bonds—a phenomenon also known as the liquidity trap. When (expected) inflation is sticky, the lower bound on the nominal rate also prevents the real interest rate from declining, plunging the economy into a demand-driven recession. Figure 2 illustrates that the short-term nominal and real interest rates in the US have indeed seemed constrained since December 2008.

An important question concerns the optimal policy response to these types of episodes. The US Treasury and the Federal Reserve have responded to the recent recession by utilizing fiscal stimulus and unconventional monetary policies. These policies are (at least
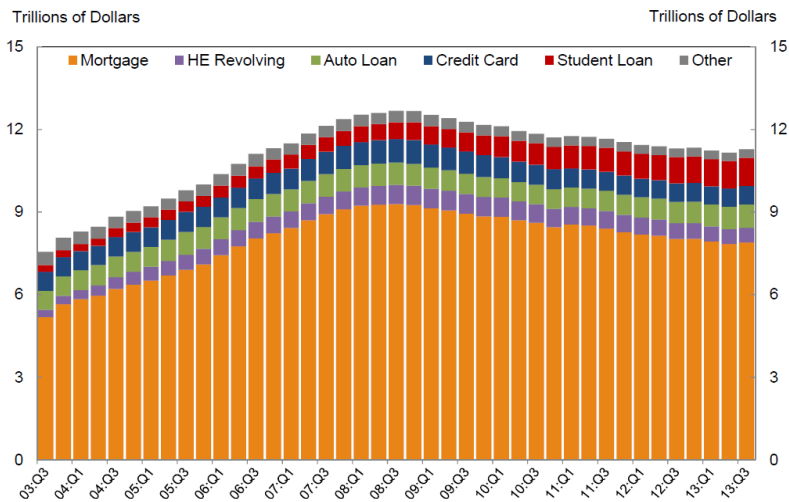
FIGURE 1. HOUSEHOLD DEBT IN THE U.S.

Note: Evolution of household debt in the US over a window of +/– 5 years from its peak in 2008Q3.

Source: Quarterly Report on Household Debt and Credit (August 2013), Federal Reserve Bank of New York.

in part) supported by growing theoretical literature that emphasizes the benefits of stimulating aggregate demand during a liquidity trap. The theoretical contributions have understandably taken an ex-post perspective—characterizing the optimal policy once the economy is in the trap. Perhaps more surprisingly, both the practical and theoretical policy efforts have largely ignored the debt markets, even though the problems are thought to have originated in these markets.[1] In this paper, we analyze the case for ex-ante macroprudential policies in debt markets, such as debt limits and insurance subsidies.

To investigate optimal macroprudential policies, we present a tractable model, in which a tightening of borrowing constraints (e.g., due to a financial shock) leads to deleveraging and may trigger or contribute to a liquidity trap. The distinguishing feature of our model is that some households, which we call borrowers, endogenously accumulate leverage—even though households are aware that borrowing constraints will be tightened in the future. If borrowers have a sufficiently strong motive to borrow, e.g., due to impatience, then the economy features an anticipated deleveraging episode along with a liquidity trap.

Our main result is that it is desirable to slow the accumulation of leverage in these episodes. In the run-up to deleveraging, borrowers who behave rationally individually undertake excessive leverage from a social point of view. Macroprudential policies that restrict leverage (coupled with appropriate ex-ante transfers) could make all households better off. This results whenever deleveraging coincides with a liquidity trap—assuming

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[1] Several papers capture the liquidity trap in a representative household framework that leaves no room for debt-market policies (see Eggertsson and Woodford 2003, Christiano, Eichenbaum and Rebelo 2011, Werning 2012). An exception is Eggertsson and Krugman (2012), which features debt but does not focus on debt-market policies.
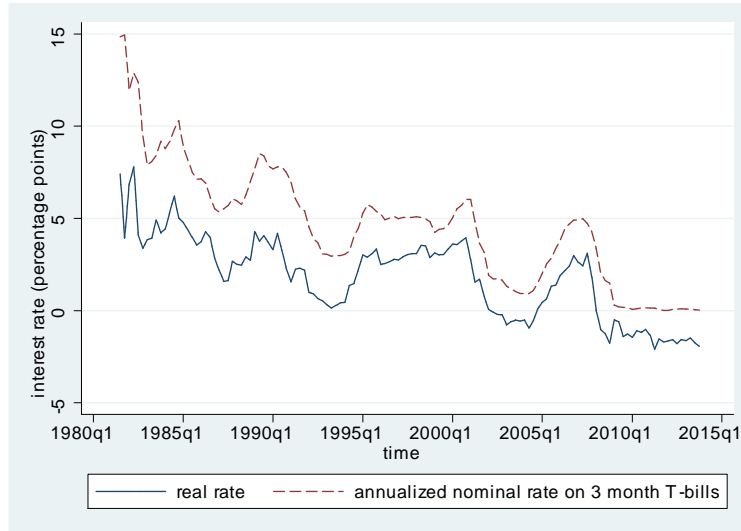
FIGURE 2. NOMINAL AND REAL INTEREST RATES IN THE US.

Note: Nominal and real interest rates on 3-month US Treasury Bills between the third quarter of 1981 and the fourth quarter of 2013. The real interest rate is calculated as the annualized nominal rate minus the expected current-quarter GDP deflator.
Source: Philadelphia Fed's Survey of Professional Forecasters, series TBILL1 and RR1_TBILL_PGDP_2.

that the liquidity trap cannot be fully alleviated by ex-post policies.

The mechanism behind the constrained inefficiency is an aggregate demand externality that applies in environments in which output is influenced by aggregate demand. When this happens, households' decisions that affect aggregate demand also affect aggregate output and therefore other households' income. Households do not take into account these general equilibrium effects, which may lead to inefficiencies. In our economy, the liquidity trap implies that output is influenced by demand and that it is below its (first-best) efficient level. Moreover, greater ex-ante leverage leads to a greater ex-post reduction in aggregate demand and a deeper recession. This happens because deleveraging transfers liquid wealth from borrowers to lenders, but borrowers who are constrained to delever have a much higher marginal propensity to consume (MPC) out of liquid financial wealth than lenders. Borrowers who choose their debt level (and lenders who finance them) do not take into account the negative demand externalities, leading to excessive leverage. In line with this intuition, we also show that the size of the optimal intervention, e.g., the optimal tax on borrowing, depends on the MPC differences between borrowers and lenders.

In practice, deleveraging episodes are often highly uncertain from an ex-ante point of view, as they are often driven by financial shocks such as a decline in collateral values. A natural question is whether households share the risk associated with deleveraging efficiently. Our second main result establishes that borrowers are also underinsured with respect to deleveraging episodes that coincide with a liquidity trap. Macroprudential

policies that incentivize borrowers to take on more insurance can improve welfare. Intuitively, borrowers' insurance purchases transfer financial wealth during deleveraging from lenders (or insurance providers) to borrowers who have a higher MPC. This increases aggregate demand and mitigates the recession. Households do not take into account these aggregate demand externalities, which leads to too little insurance. The size of the optimal intervention, e.g., the optimal insurance subsidy, depends on borrowers' and lenders' MPC differences. An important financial shock in practice is a decline in house prices, which can tighten homeowners' borrowing constraints and trigger deleveraging. In this context, our results support policies that reduce homeowners' exposure to a decline in house prices, such as subsidies for home equity insurance.

While some financial shocks that induce deleveraging can be insured against (in principle), others might be much more difficult to describe and contract upon. We show that these types of environments with incomplete markets also feature excessive leverage in view of aggregate demand externalities. Welfare can be improved with "blanket" macroprudential policies that restrict non-contingent debt, as these provide protection against all deleveraging episodes, including those driven by uninsurable shocks. However, these policies also distort households' consumption in many future states without deleveraging, and thus, their optimal size depends on the ex-ante probability of deleveraging.

We also investigate whether preventive monetary policies could be used to address aggregate demand externalities generated by leverage. A common argument is that a contractionary policy that raised the interest rate in the run-up to the recent subprime crisis could have been beneficial in curbing leverage. Perhaps surprisingly, our model reveals that raising the interest rate during the leverage accumulation phase can have the unintended consequence of increasing leverage. A higher interest rate reduces borrowers' incentives to borrow keeping all else equal—which appears to be the conventional wisdom informed by partial equilibrium reasoning. However, the higher interest rate also creates a temporary recession (or a slowdown in output growth) which increases borrowers' incentives to borrow so as to smooth consumption. In addition, the higher interest rate transfers wealth from borrowers to lenders, which further increases borrowers' incentives to borrow. In our model, the general equilibrium effects can dominate under natural assumptions (e.g., when borrowers and lenders have the same intertemporal elasticity of substitution), and raising the interest rate can have the perverse effect of raising leverage. Our findings may explain why the interest rate hikes by the Fed starting in June 2004 were ineffective in reducing leverage at the time, as illustrated in Figures 1 and 2.

There are versions of our model in which the conventional wisdom holds, and raising the interest rate lowers leverage (as in Cúrdia and Woodford 2009). But even in these cases, the interest rate policy is inferior to macroprudential policies in dealing with excessive leverage. Intuitively, constrained efficiency requires setting a wedge between the relative interest rates faced by borrowers and lenders, whereas interest rate policy creates a different intertemporal wedge that affects the interest rates faced by all households uniformly. As a by-product, interest rate policy also generates an unnecessary slowdown in output growth, which is not a feature of constrained efficient allocations. That said, a different preventive monetary policy, namely raising the inflation target, is supported by

our model as it would reduce the incidence of liquidity traps, and therefore, the relevance of aggregate demand externalities.

Our final analysis concerns endogenizing the debt limit faced by borrowers by assuming that debt is collateralized by financial assets, creating the potential for fire-sale effects. This introduces a new feedback loop into the economy, with two main implications. First, higher leverage lowers asset prices in the deleveraging phase, which in turn lowers borrowers' debt capacity and increases their distress. Hence, higher leverage generates fire-sale externalities that operate in the same direction as aggregate demand externalities. Second, an increase in borrowers' distress induces a more severe deleveraging episode and a deeper recession, which in turn translates into even lower asset prices. Thus, the aggregate demand reduction and the fire sale effects of debt reinforce one another. These observations suggest that macroprudential policies might be particularly desirable in the run-up to deleveraging episodes that involve asset fire sales.

The remainder of this paper is structured as follows. The next subsection discusses the related literature. Section I introduces the key aspects of our environment. Section II characterizes an equilibrium that features an anticipated deleveraging episode that coincides with a liquidity trap. The heart of the paper is Section III, which illustrates aggregate demand externalities, presents our main result about excessive leverage, and derives its policy implications. This section also relates the size of the optimal policy intervention to MPC differences between borrowers and lenders. Section IV generalizes the model to incorporate uncertainty and presents our second main result about under-insurance. This section also generalizes the excessive leverage result to a setting with uncertainty and uninsurable shocks. Section V discusses the role of preventive monetary policies in our environment. Section VI presents the extension with endogenous debt limits and fire-sale externalities, and Section VII concludes. The online appendices contain omitted proofs and derivations as well as some extensions of our baseline model.

A. Related literature

Our paper is related to a long line of economic literature that studies the zero lower bound on nominal interest rates and liquidity traps, starting with Hicks (1937), and more recently emphasized by Krugman (1998) and Eggertsson and Woodford (2003, 2004). A growing strand of recent literature has investigated the optimal fiscal and monetary policy response to liquidity traps (see e.g. Eggertsson 2011, Christiano et al. 2011, Werning 2012, Correia et al. 2013). We contribute to this literature by taking an ex-ante perspective, and focusing on macroprudential policies in debt markets. We view this as an important exercise since the recent experience in a number of advanced economies suggests the set of policy instruments discussed in the cited literature was either restricted or insufficient to allow for a swift exit from the liquidity trap.

Guerrieri and Lorenzoni (2011) and Eggertsson and Krugman (2012) describe how financial market shocks that induce borrowers to delever lead to a decline in interest rates, which in turn can trigger a liquidity trap. Our framework is most closely related to Eggertsson and Krugman because we also model deleveraging between a set of impatient borrowers and patient lenders. They focus on the ex-post implications of deleveraging as

well as the effects of monetary and fiscal policy during these episodes. Our contribution is to add an ex-ante stage and to investigate the role of macroprudential policies. Among other things, our paper calls for novel policy actions in debt markets that are significantly different from the more traditional policy responses to liquidity traps.

Our paper is part of a growing branch of literature that investigates the role of macroprudential policies in mitigating financial crises. This literature has emphasized that agents can take excessive leverage or risks in view of moral hazard (e.g., Farhi and Tirole 2012, Gertler, Kiyotaki and Queralto 2012, Chari and Kehoe 2013), neglected risks (e.g., Gennaioli, Shleifer and Vishny 2013) or pecuniary externalities (e.g., Caballero and Krishnamurthy 2003, Lorenzoni 2008, Bianchi and Mendoza 2010, Jeanne and Korinek 2010ab, and Korinek 2011). We show that aggregate demand externalities can also induce excessive leverage and risk taking, but through a very different channel. In contrast to pecuniary externalities, aggregate demand externalities apply not when prices are volatile, but in the opposite case when a certain price—namely the real interest rate—is fixed. We discuss the differences with pecuniary externalities further in Section III, and illustrate the interaction of our mechanism with fire-sale externalities in Section VI.

The aggregate demand externality that we focus on was first discovered in the context of firms' price setting decisions, e.g., by Mankiw (1985), Akerlof and Yellen (1985) and Blanchard and Kiyotaki (1987). The broad idea is that, when output is not at its efficient level and influenced by aggregate demand, decentralized allocations that affect aggregate demand are socially inefficient. In Blanchard and Kiyotaki, output is not at the efficient level due to monopoly distortions, and firms' price setting affects aggregate demand due to complementarities in firms' demand. In our setting, output is below its efficient level due to the liquidity trap. We also focus on households' debt choices—as opposed to firms' price setting decisions—which affect aggregate demand due to differences in households' marginal propensities to consume.

A number of recent papers, e.g., Schmitt-Grohé and Uribe (2011, 2012ab) and Farhi and Werning (2012ab, 2013), also analyze aggregate demand externalities in contexts similar to ours. Schmitt-Grohé and Uribe analyze economies with fixed exchange rates that exhibit downward rigidity in nominal wages. They identify negative aggregate demand externalities associated with actions that increase wages during good times because these actions lead to greater unemployment during bad times. In Farhi and Werning (2012ab), output responds to aggregate demand because prices are sticky and countries are in a currency union (and thus, under the same monetary policy). They emphasize the inefficiencies in cross-country insurance arrangements. In our model, output is demand-determined because of a liquidity trap, and we emphasize the inefficiencies in household leverage in a closed economy setting.

Farhi and Werning (2013) distill the broader lessons from this emerging literature on aggregate demand externalities in a general framework. They show that financial market allocations in economies with nominal rigidities that cannot be fully undone with monetary policy are generically inefficient. They also provide a number of general results for these environments, including optimal tax formulas to correct aggregate demand externalities. Our paper focuses on the inefficiencies in one specific setting—a liquid-

ity trap driven by deleveraging. We believe that this setting captures one of the most important occurrences of aggregate demand externalities in the US economy in recent decades. Farhi and Werning (2013) also analyze this setting as one out of several examples, which was developed independently and parallel to our work, but they do not provide an in-depth analysis. Our paper's unique analyses include the characterization of macroprudential policies with uncertainty (with and without incomplete markets) and the investigation of the effect of the interest rate policy on leverage.

Finally, our paper is also related to the recent New Keynesian literature that investigates the role of financial frictions and nominal rigidities in the Great Recession (see, for instance, Cúrdia and Woodford 2011, Gertler and Karadi 2011, Christiano, Eichenbaum and Trabandt 2015). We share with that literature the view that financial frictions, combined with high leverage, can induce a demand-driven recession, especially if monetary policy is constrained by the zero lower bound. We differ in our emphasis on household leverage as opposed to financial institutions' (or firms') leverage. We also provide different and complementary remedies for the liquidity trap. While we emphasize macroprudential policies designed to correct externalities, that literature focuses on credit policies (e.g., lending or asset purchases by the central bank) that rely on the government's comparative advantage in financial intermediation (especially during a financial crisis). Both types of policies help to alleviate the liquidity trap, but they do so through different channels. Macroprudential policies prevent leverage from accumulating in the first place, whereas credit policies can be thought of as containing the ex-post damage by slowing deleveraging.

I. Environment and equilibrium

In this section, we introduce the key ingredients of the environment and define the equilibrium, which we characterize in subsequent sections.

HOUSEHOLD DEBT AND THE ANTICIPATED FINANCIAL SHOCK

The economy is set in infinite discrete time, with dates $t \in \{0, 1, ...\}$. There is a single consumption good, which is also the numeraire for real prices. There are two groups of households, borrowers and lenders, denoted by $h \in \{b, l\}$, with equal measure of each group normalized to 1. Households are symmetric except that borrowers have a weakly lower discount factor than lenders, $\beta^b \leq \beta^l < 1$, which will induce borrowers to take on debt in equilibrium. Let d_t^h denote the outstanding debt—or assets, if negative—of household h at date t. Households start with initial debt or asset levels denoted by d_0^h. At each date t, they face the one-period interest rate r_{t+1} and they choose their debt or asset levels for the next period, d_{t+1}^h.

Our first key ingredient is that, from date 1 onwards, households are subject to a borrowing constraint, that is, $d_{t+1}^h \leq \phi$ for each $t \geq 1$. Here, $\phi > 0$ denotes an exogenous debt limit as in Aiyagari (1994), or more recently, Eggertsson and Krugman (2012). The constraint can be thought of as capturing a financial shock in reduced from, e.g., a drop in collateral values or loan-to-value ratios, that would force households to reduce their

leverage. In contrast, we assume that households can choose d_1^h at date 0 without any constraints. The role of these ingredients is to generate household leveraging at date 0 followed by deleveraging at date 1 along the lines of Figure 1. Moreover, to study the efficiency of households' ex-ante decisions, we assume that the deleveraging episode is anticipated at date 0. In our baseline model, we abstract away from uncertainty so that the episode is perfectly anticipated. We will introduce uncertainty in Section IV.A.

Households optimally choose their labor supply, in addition to making a dynamic consumption and saving decision. For the baseline model, we assume households' preferences over consumption $\tilde{c}_t^h$ and labor n_t^h take the particular form $u\left(\tilde{c}_t^h - v\left(n_t^h\right)\right)$. These preferences provide tractability but are not necessary for our main results (see the online Appendix C). As noted in Greenwood, Hercowitz and Huffman (GHH, 1988), the specification implies that there is no income effect on the labor supply. Specifically, households' optimal labor supply solves the static optimization problem:

$$(1) \qquad e_t \equiv \max_{n_t^h} \left(w_t n_t^h - v\left(n_t^h\right)\right) + \int_0^1 \Gamma_t\left(v\right) dv - T_t.$$

Here, e_t is the households' net income, that is, their total income net of labor costs. In addition to their labor income, households also symmetrically receive profits from firms that will be described below, $\int_0^1 \Gamma_t\left(v\right) dv$, and pay lump-sum taxes, T_t. Observe that (due to symmetry and the absence of income effects) households of each type will optimally supply the same amount of labor $n_t \equiv n_t^h$ and receive the same level of net income e_t.

Analogous to net income, we also define households' net consumption as their consumption net of labor costs, $c_t^h = \tilde{c}_t^h - v\left(n_t\right)$. Households' consumption and saving problem can then be written in terms of net variables as:

$$(2) \qquad \max_{\{c_t^h, d_{t+1}^h\}_t} \quad \sum_{t=0}^{\infty} \left(\beta^h\right)^t u\left(c_t^h\right)$$

$$\text{s.t. } c_t^h = e_t - d_t^h + \frac{d_{t+1}^h}{1 + r_{t+1}} \text{ for all } t,$$

$$\text{and } d_{t+1}^h \leq \phi \text{ for each } t \geq 1.$$

Problems (1) and (2) describe the optimal household behavior in our setting. We also make the standard assumptions about preferences: that is $u\left(\cdot\right)$ and $v\left(\cdot\right)$ are both strictly increasing, $u\left(\cdot\right)$ is strictly concave and $v\left(\cdot\right)$ is strictly convex, and they satisfy the conditions $\lim_{c \to 0} u'\left(c\right) = \infty$, $v'\left(0\right) = 0$ and $\lim_{n \to \infty} v'\left(n\right) = \infty$.

LIQUIDITY TRAP AND THE BOUND ON THE NOMINAL RATE

As we will see, household deleveraging will lower aggregate demand and put downward pressure on the interest rate. Our second key ingredient is a lower bound on the nominal interest rate. We assume there is cash (that is, paper money) in the economy that provides households with transaction services. To simplify the notation and the exposi-

tion, however, we consider the limit in which the transaction value of cash approaches zero (as described in Woodford 2003). In the limit, the monetary authority still controls the short term nominal interest rate i_{t+1}. However, the presence of paper money (albeit a vanishingly small amount) sets a lower bound on the nominal interest rate,

(3) $$i_{t+1} \geq 0 \text{ for each } t.$$

Intuitively, the nominal interest rate cannot fall significantly below zero, since households would otherwise hold cash instead of keeping their wealth in interest-bearing (or, more precisely, interest-charging) accounts.[2] A situation in which the nominal interest rate is at its lower bound is known as a liquidity trap. In a liquidity trap, cash and bonds become very close substitutes and households start demanding cash also for saving purposes. As this happens, increasing the money supply in the economy does not lower the nominal interest rate further since the additional money merely substitutes for bonds in households' portfolios.

NOMINAL RIGIDITIES AND THE BOUND ON THE REAL RATE

The bound on the nominal interest rate does not necessarily affect real allocations. Our third key ingredient is nominal rigidities, which turns the bound on the nominal rate into a bound on the real rate, with implications for real variables. We capture this ingredient by utilizing a standard New Keynesian model with an extreme form of price stickiness (see Remarks 1-3 below for a discussion and alternative specifications).

Specifically, suppose labor can be utilized to produce the consumption good via two types of firms. First, a competitive final good sector uses intermediate varieties $v \in [0, 1]$ to produce the consumption good according to the Dixit-Stiglitz technology,

(4) $$y_t = \left(\int_0^1 y_t(v)^{\frac{\varepsilon-1}{\varepsilon}} \, dv \right)^{\varepsilon/(\varepsilon-1)} \quad \text{where } \varepsilon > 1,$$

where y_t denotes aggregate output per household. Second, a unit mass of monopolistic firms labeled by $v \in [0, 1]$ each produce $y_t(v)$ units of intermediate variety v (per household) by employing $n_t(v)$ units of labor according to the linear technology,

(5) $$y_t(v) = n_t(v).$$

Let $P_t(v)$ denote the nominal price level for the monopolist for variety v at time t. Given the Dixit-Stiglitz technology, the nominal price of the consumption good at time t is given by $P_t = \left(\int P_t(v)^{1-\varepsilon} \, dv \right)^{1/(1-\varepsilon)}$.

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[2]Recently, a number of central banks have cut interest rates to levels that are slightly below zero. The most prominent case was the Swiss National Bank (SNB) with a benchmark rate cut to –0.75% in January 2015. The cut was combined with regulations that exempted bank reserves held against small deposits from the negative rates since the SNB feared that small depositors would otherwise shift their holdings into currency. For larger depositors, the costs of holding large amounts of currency were viewed as sufficiently large to discourage significant shifts towards cash. However, policymakers expressed concerns that this may change once the negative rates reach –1%. These events suggest that, even though the lower bound is not exactly zero in practice, it is arguably very close to zero.

In our baseline model, we assume monopolists have preset nominal prices that are equal to each other and that never change, $P_t(v) = P$ for each t. This implies that the final good price is also constant, $P_t = P$ for each t. Combining this with (3) implies that the nominal and the real interest rates are the same. Consequently, the latter is also bounded from below:

(6) $i_{t+1} = r_{t+1} \geq 0$ for each t.

As Figure 2 illustrates, the real interest rate in the US in recent years indeed seemed bounded from below. We normalize inflation to zero so that the lower bound on the real rate is also zero. The online Appendix C shows that our results are qualitatively robust to allowing for a higher yet sticky inflation rate.

DEMAND-DETERMINED OUTPUT AND CONSTRAINED MONETARY POLICY

Our fourth and final ingredient is that, when the interest rate is at its lower bound, the economy experiences a demand-driven recession. To introduce this ingredient, we first describe the efficient allocations in this environment. Given the linear technology in (4) and (5), and the household preferences in (1), the efficient level of net income and labor supply are respectively given by:

(7) $e^* \equiv \max_{n_t} n_t - v(n_t)$ and $n^* \equiv \arg\max_{n_t} n_t - v(n_t)$.

We next describe a frictionless benchmark without price rigidities, which also generates the efficient allocations. Suppose each monopolist resets its price every period. The monopolist faces isoelastic demand for its goods, $y_t p_t(v)^{-\varepsilon}$, where $p_t(v) = P_t(v)/P_t$ denotes its relative price. Thus, her problem can be written (in terms of per household variables) as:

(8)
$$\Gamma_t(v) = \max_{p_t(v), y_t(v), n_t(v)} p_t(v) y_t(v) - w_t [1 - \tau(n_t)] n_t(v) \text{ s.t. } y_t(v) = n_t(v) \leq y_t p_t(v)^{-\varepsilon}.$$

Here, $\tau(n_t)$ captures linear subsidies to employment of each monopolist v, which are financed by lump-sum taxes, that is, $T_t = \tau(n_t) w_t \int_0^1 n_t(v) \, dv$. We assume these subsidies in order to correct the distortions that arise from monopolistic competition and focus our welfare analysis solely on aggregate demand externalities. Specifically, we set $\tau(n_t) = 1/\varepsilon$ if the aggregate employment is below the efficient level, $n_t \leq n^*$, and $\tau(n_t) = 0$ otherwise. The optimality conditions for problems (8) and (1) then imply $e_t = e^*$ for each t. Thus, the subsidies provide us with an efficient benchmark for welfare comparisons, although they are not necessary for any of our results (see the online Appendix C for the case with $\tau = T_t = 0$, as well as an explanation for why we take away the subsidies when $n_t > n^*$).

Set against this frictionless benchmark, monopolistic firms in our setting have the pre-

set nominal price $P_t(v) = P$. Their optimization problem can then be written as

(9)
$$\Gamma_t(v) = \max_{y_t(v),n_t(v)} p_t(v)\, y_t(v) - w_t(1 - \tau(n_t))\, n_t(v) \text{ s.t. } y_t(v) = n_t(v) \leq y_t\, p_t(v)^{-\varepsilon},$$

where $p_t(v) = P_t(v)/P$ denotes the monopolist's fixed relative price, which is equal to 1 by symmetry. That is, the monopolist chooses how much to produce subject to the constraint that its output cannot exceed the demand for its goods. In the equilibria we analyze, the monopolist always meets the demand for its goods, $y_t(v) = n_t(v) = y_t$, since its marginal cost is strictly below its price. By symmetry, this induces an equilibrium level of employment $n_t = y_t$ and net income $e_t = y_t - v(y_t)$.

It follows that the outcomes in this model are ultimately determined by the aggregate demand (per household) for the final consumption good, $y_t = \frac{\tilde{c}_t^b + \tilde{c}_t^l}{2}$. This in turn depends on monetary policy, which controls the nominal and the real interest rate. Since the price level is fixed, we assume that monetary policy focuses on myopic output stabilization (analogous to a Taylor rule) subject to the constraint in (6). In our setting, this amounts to replicating the frictionless benchmark, by setting:

(10)
$$i_{t+1} = r_{t+1} = \max\left(0, r_{t+1}^*\right) \text{ for each } t.$$

Here, r_{t+1}^* is recursively defined as the frictionless interest rate at time t that obtains when households' net income is $e_t = e^*$ and the monetary policy follows the rule in (10) at all future dates $\tilde{t} \geq t+1$. This policy is also constrained efficient in our environment, as long as the monetary authority does not have commitment power (see the online Appendix A).[3]

DEFINITION 1 (Equilibrium): *The equilibrium is a path of real allocations,* $\left\{\left[n_t^h, c_t^h, d_{t+1}^h\right]_h, e_t, y_t, \left[y_t(v), n_t(v)\right]_v\right\}_t$, *and wages, interest rates, profits, and taxes* $\{w_t, r_{t+1}, [\Gamma_t(v)]_v, T_t\}_t$, *such that the households' allocations solve problems* (1) *and* (2), *a competitive final good sector produces according to* (4), *the intermediate good monopolists solve* (9) *for given fixed goods prices, the interest rates are set according to* (10), *and all markets clear.*

REMARK 1 (Interpretation of Price Stickiness): *We interpret our extreme price stickiness assumption as capturing in reduced form an environment in which the aggregate price level is sticky in the upward direction throughout the deleveraging episode. This ensures that the economy cannot have much inflation in the short run, which converts the bound on the nominal interest rate into a bound on the real rate as in* (6). *Our model is consistent with (at least) two forces that might contribute to upward price stickiness in practice: (i) price stickiness at the micro level and (ii) constraints on monetary policy against creating inflation. These forces, which are not mutually exclusive, can be isolated by considering the following two scenarios.*

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[3]A monetary authority with commitment power might find it desirable to deviate from (10) by setting the interest rate below the frictionless benchmark after the economy exits a liquidity trap (see Werning, 2012). We abstract away from these "forward guidance" policies that are not our focus.

First, the prices at the micro level can be effectively very sticky (for reasons emphasized in the New Keynesian literature), as in a literal interpretation of our baseline model. In this case, the aggregate price level will also be very sticky in the short run, even if the monetary policy can flexibly react to the liquidity trap.

Second, prices may be somewhat flexible, but the monetary authority may be constrained to follow an inflation-targeting policy with a predetermined target. The online Appendix C analyzes that case and shows that the equilibrium features the same real allocations as in the baseline model (up to a log-linear approximation) if the inflation target is normalized to zero. Intuitively, even though there is some price flexibility at the micro level, the aggregate price level remains sticky in the upward direction due to the inflation-targeting policy. In practice, many central banks follow policies along these lines, in view of their legal mandates to pursue price stability. Moreover, deviating from these policies so as to create inflation would be dynamically inconsistent. If inflation is costly, then the central bank would optimally revert to an inflation-targeting policy once the economy exits the liquidity trap [see Werning (2012) for a formal analysis].

REMARK 2 (Disinflation): *The online Appendix C also shows that once we introduce limited price flexibility, inflation falls below its target level during the liquidity trap (between dates 0 and 1) in view of the negative output gap. This disinflation could further exacerbate the recession by tightening the bound on the real rate in (6).[4] It is perhaps fortunate that the US economy avoided severe disinflation during the recent macroeconomic slump. A number of papers have argued that the "missing disinflation" represents a puzzle for the standard New Keynesian model and its Phillips curve (e.g., Ball and Mazumder, 2011; Hall, 2013; Coibion and Gorodnichenko, 2015). More recent work, however, has found that the missing disinflation can be reconciled with the New Keynesian model (e.g., Del Negro, Giannoni and Schorfheide 2015), especially after accounting for temporary factors such as the recent productivity slowdown or the financial constraints on firms during the crisis (e.g., Christiano et al. 2015, and Gilchrist et al. 2015).*

REMARK 3 (Alternative Formulations for the Supply Side): *We adopt a New Keynesian model with price rigidities in the goods market for expositional simplicity. However, our results are robust to several alternative specifications for the supply side. The online Appendix C illustrates this point by analyzing a version of our model in which the nominal wages are rigid in the downward direction, as in Eggertsson and Mehrotra (2014) or Schmitt-Grohé and Uribe (2012c), whereas nominal prices are flexible.[5] In this formulation, the demand shortage due to the constraint in (C4) is absorbed by rationing in the labor market—as opposed to rationing (or higher markups) in the goods market*

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[4]This does not happen in our model due to the special feature that deleveraging takes place in a single period (between dates 1 and 2). If we were to split this episode into multiple subperiods, then disinflation would induce a tighter bound on the real rate and a more severe recession during the earlier subperiods (similar to Werning 2012).

[5]Our NBER working paper (Korinek and Simsek 2014) illustrates yet another alternative formulation for the supply side based on an older rationing equilibrium concept, which has also been adopted in some recent work, e.g., Hall (2011), Kocherlakota (2012), and Caballero and Farhi (2013).

which then lowers employment.[6] The appendix shows that this formulation also yields the same real allocations as our baseline model, as long as we continue to assume an inflation-targeting monetary policy.

II. An anticipated deleveraging episode

This section characterizes the decentralized equilibrium and describes an anticipated deleveraging episode that triggers a liquidity trap. The next section analyzes the efficiency properties of this equilibrium. We start with the following lemma, which describes the possibilities for equilibrium within a period.

LEMMA 1: *(i) If $r_{t+1} > 0$, then $e_t = e^*$, (ii) If $r_{t+1} = 0$, then $e_t = \frac{c_t^b + c_t^l}{2} \le e^*$.*

The first part captures the scenario in which the monetary policy in (10) replicates the frictionless outcome. The second part captures a liquidity trap scenario in which the frictionless outcome would call for a negative interest rate. In this case, the interest rate is constrained $r_{t+1} = 0$, and the economy experiences a demand-driven recession. Net income is below its frictionless level e^*, and is determined by net aggregate demand, $\frac{c_t^b + c_t^l}{2}$.

We next combine Lemma 1 with the households' consumption and savings problem (2) to characterize the full equilibrium. Note that the market clearing for debt implies $d_t^l = -d_t^b$. Therefore, we drop superscripts and let $d_t \equiv d_t^b$ denote the aggregate debt level in the economy. We will focus on cases in which borrowers' constraints bind at all dates, that is, $d_{t+1} = \phi$ for each $t \ge 1$. Throughout, we also make the following parametric assumptions.

Assumption (1). *(i)* $\frac{u'(2e^*)}{u'(e^* + \phi(1 - \beta^l))} < \beta^l$, (ii) $d_0 < \tilde{d}_0$ (see the online Appendix A for $\tilde{d}_0$).

The first part allows for the interest rate constraint (6) to bind at date 1, while the second part ensures that it doesn't bind at date 0, simplifying the exposition.

STEADY STATE

We characterize the equilibrium backwards. First consider dates $t \ge 2$, at which the outstanding debt level is already lowered to ϕ. At these dates, the economy is in a steady state. Since borrowers are constrained, the real interest rate is determined by the lenders' discount rate, $r_{t+1} = 1/\beta^l - 1 > 0$. Since the interest rate is positive, the economy features the frictionless outcomes [cf. Lemma 1]. In particular, households' consumption is given by:

$$(11) \qquad c_t^b = e^* - \phi\left(1 - \beta^l\right) \ \text{ and } c_t^l = e^* + \phi\left(1 - \beta^l\right) \ \text{ for } t \ge 2.$$

---

[6]In general, having rationing in the labor market (as opposed to the goods market) could exacerbate the recession as it would make it more difficult for borrowers to pay back their debt by raising their labor supply. This difference does not show up in our setting because of the GHH preferences that shut down labor supply responses (see Eq. (1)).

DELEVERAGING

Next consider date $t = 1$. Borrowers' consumption is given by $c_1^b = e_1 - \left(d_1 - \frac{\phi}{1+r_2}\right)$. Note that the larger the outstanding debt level d_1 is relative to the debt limit, the more borrowers are forced to reduce their net consumption. The resulting slack in aggregate demand needs to be absorbed by an increase in lenders' net consumption:

$$c_1^l = e^* + \left(d_1 - \frac{\phi}{1+r_2}\right).$$

Since lenders are unconstrained, their Euler equation holds $\frac{u'(c_1^l)}{\beta^l u'(c_2^l)} = 1 + r_2$, where $c_2^l = e^* + \phi\left(1 - \beta^l\right)$. Hence, the increase in lenders' consumption at date 1 is mediated through a decrease in the real interest rate, r_2. The key observation is that the lower bound on the real interest rate effectively sets an upper bound on lenders' (or unconstrained agents') consumption in equilibrium, $c_1^l \leq \bar{c}_1^l$, given by the solution to

(12) $$u'\left(\bar{c}_1^l\right) = \beta^l u'\left(e^* + \phi\left(1 - \beta^l\right)\right).$$

The equilibrium at date 1 then depends on the relative size of two terms:

$$d_1 - \phi \lesseqgtr \bar{c}_1^l - e^*.$$

The left-hand side is the amount of deleveraging borrowers are forced into given that the borrowing limit falls to ϕ (and the real rate is at its lower bound). The right-hand side is the maximum amount of demand the unconstrained agents can absorb when the real rate is at its lower bound. If the left side is smaller than the right side, then the equilibrium features $r_2 > 0$ and $e_1 = e^*$. In this case, the effects of deleveraging on aggregate demand are offset by a reduction in the real interest rate. The left side of Figure 3 (the range with $d_1 \leq \bar{d}_1$) illustrates this outcome.

Otherwise, equivalently if the debt level is strictly above a threshold,

(13) $$d_1 > \bar{d}_1 = \phi + \bar{c}_1^l - e^*,$$

then the economy is in a liquidity trap. The real interest rate is at its lower bound, $r_2 = 0$ and the economy experiences a recession driven by low demand. Borrowers' and lenders' net consumption demands are respectively given by $c_1^b = e_1 - d_1 + \phi$ and $c_1^l = \bar{c}_1^l$. By Lemma 1, this implies,

(14) $$e_1 = \frac{c_1^b + c_1^l}{2} = \frac{e_1 - (d_1 - \phi) + \bar{c}_1^l}{2}.$$

After rearranging this expression, the equilibrium level of net income is given by,

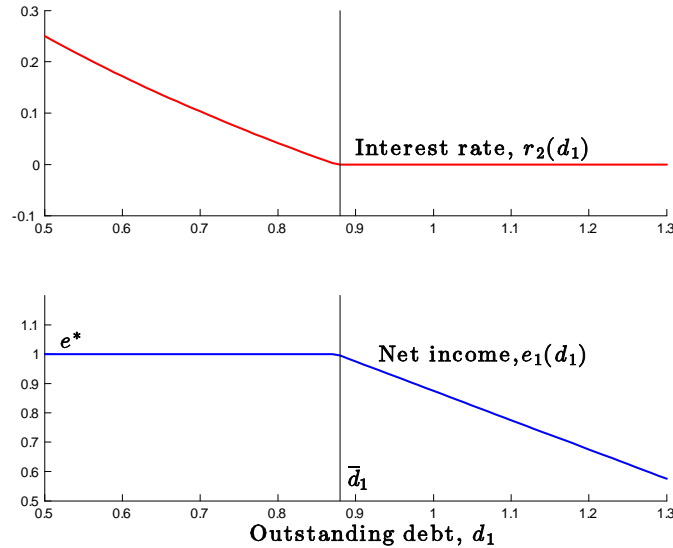(15) $$e_1 = \bar{c}_1^l + \phi - d_1 < e^*.$$

FIGURE 3. INTEREST RATE AND NET INCOME.

Note: Interest rate and net income at date 1 as a functions of outstanding debt d_1.

The right side of Figure 3 (the range with $d_1 \geq \bar{d}_1$) illustrates this outcome.

Eq. (14) illustrates that there is a Keynesian cross and a Keynesian multiplier in our setting. Net income is equal to net aggregate demand as in a typical Keynesian cross. Each additional unit of debt reduces borrowers' net demand by half a unit because the share of borrowers in the population is $1/2$ and their marginal propensity to consume (MPC) out of liquid wealth is 1 since they are constrained. This triggers a Keynesian multiplier: the decline in net demand reduces borrowers' net income by $1/2$ unit, which in turn reduces the net demand further by $1/4$ units, and so on. Eq. (15) puts these effects together and shows that an increase in outstanding debt leads to a deeper recession.

Intuitively, an increase in debt reduces demand and output by transferring wealth from borrowers that have a very high MPC out of liquid wealth to lenders that have a low MPC. The feature that borrowers' MPC is equal to 1 enables us to illustrate our inefficiency results sharply, but it is not necessary. Section III.C shows that net income is declining in outstanding debt, $\frac{\mathrm{d}e_1}{\mathrm{d}d_1} < 0$, as long as borrowers' MPC is greater than lenders' MPC. As we will see, this feature is all we need for aggregate demand externalities to be operational and to generate inefficiencies.

DATE 0 ALLOCATIONS

We next turn to households' financial decisions at date 0. We conjecture an equilibrium in which the net income is at its efficient level, $e_0 = e^*$. Since households are

unconstrained at date 0, their Euler equations hold,

$$(16) \qquad \frac{1}{1+r_1} = \frac{\beta^l u' \left(c_1^l\right)}{u' \left(c_0^l\right)} = \frac{\beta^b u' \left(c_1^b\right)}{u' \left(c_0^b\right)}.$$

The equilibrium debt level, d_1, and the interest rate, r_1, are determined by these equations. We next identify two conditions under which households choose a sufficiently high debt level that triggers a recession at date 1, $d_1 > \bar{d}_1$. The relevant thresholds, $\bar{\beta}^b \left(d_0\right)$ and $\bar{d}_0 \left(\beta^b\right)$, are characterized in the online Appendix A.

PROPOSITION 1: *There is an equilibrium with a deleveraging-induced recession at date 1 if the borrower is sufficiently impatient or sufficiently indebted at date 0. Specifically, for any debt level d_0 there is a threshold level of the discount factor $\bar{\beta}^b \left(d_0\right)$ such that the economy experiences a recession at date 1 if $\beta^b < \bar{\beta}^b \left(d_0\right)$. Conversely, for any level of the discount factor β^b there is a threshold initial debt level $\bar{d}_0 \left(\beta^b\right)$ such that the economy experiences a recession at date 1 if $d_0 > \bar{d}_0 \left(\beta^b\right)$.*

Proposition 1 describes two scenarios that might induce borrowers to carry a high level of debt into date 1, even though they anticipate the deleveraging episode as well as the associated liquidity trap. First, borrowers might have a sufficiently strong motive to borrow at date 0 (due to various spending opportunities) as captured by a low discount factor in our setting. Second, borrowers might also have accumulated a large amount of debt in the past, perhaps at a time at which they did not anticipate the deleveraging episode. We view both scenarios as relevant for macroprudential policy analysis in practice. The first scenario is useful to investigate whether the economy accumulates leverage optimally, and the second scenario is useful to analyze whether the economy can efficiently manage a "smooth landing" to low leverage.[7]

III. Excessive leverage

We next analyze the efficiency properties of the equilibrium characterized in Proposition 1 and present our main result. We first illustrate the aggregate demand externalities in our setting, and contrast them with pecuniary externalities. We then illustrate that the competitive equilibrium is constrained inefficient and that it can be Pareto improved with simple macroprudential policies. The last part quantifies the size of the inefficiency, as well as the optimal intervention, in terms of households' MPC differences.

---

[7]While we emphasize deleveraging as the main cause of the liquidity trap, our welfare analysis is consistent with other forces that might also lower demand at date 1 such as the financial crisis (see, for instance, Gertler and Karadi 2014) or investment overhang (see Rognlie, Shleifer and Simsek 2014). In fact, these forces would be complementary to deleveraging in the sense that they would modify the thresholds in Proposition 1 so as to make a liquidity trap more likely. The important point for our welfare analysis is that deleveraging coincides with a liquidity trap. Recent work, e.g., Summers (2013) and Eggertsson and Mehrotra (2014), has also emphasized long-run forces that could have permanently reduced aggregate demand as well as the safe interest rates. These forces are also complementary to our analysis, and they suggest that deleveraging episodes and liquidity traps might continue to be a serious concern for the world economy in upcoming years.

A. *Aggregate demand externalities*

We consider a constrained planner at date 0 who can affect the aggregate debt level d_1 at date 1 (symmetrically held by borrowers) but cannot interfere thereafter. We focus on constrained efficient allocations with $d_1 \geq \phi$, so that conditional on d_1, the economy behaves as we analyzed in the previous section for date 1 onwards.

Let $V^h \left(d_1^h; d_1 \right)$ denote the utility of household h conditional on entering date 1 with an individual level of debt d_1^h, given an aggregate level of debt d_1. The aggregate debt enters household utility separately because it determines the interest rate or net income at date 1. More specifically, we have:

$$(17) \qquad V^b \left(d_1^b, d_1 \right) = u \left(e_1 (d_1) - d_1^b + \frac{\phi}{1 + r_2 (d_1)} \right) + \sum_{t=2}^{\infty} \left(\beta^b \right)^t u \left(c_t^b \right)$$

$$V^l \left(d_1^l, d_1 \right) = u \left(e_1 (d_1) - d_1^l - \frac{\phi}{1 + r_2 (d_1)} \right) + \sum_{t=2}^{\infty} \left(\beta^l \right)^t u \left(c_t^l \right)$$

where $r_2 (d_1)$ and $e_1 (d_1)$ are characterized in the previous section and the continuation utilities from date 2 onwards do not depend on d_1^h or d_1 [cf. Eq. (11)].

In equilibrium, we have $d_1^b = d_1 = -d_1^l$ in view of symmetry and market clearing. But taking d_1 explicitly into account is useful to illustrate the externalities. Specifically, raising the equilibrium debt level by one unit induces an uninternalized welfare effect $\frac{\partial V^h}{\partial d_1}$ on household h, which we characterize next.

LEMMA 2: *(i) If $d_1 \in [\phi, \bar{d}_1)$, then* $\frac{\partial V^h}{\partial d_1} = \left\{ \begin{array}{l} -\eta u' \left(c_1^h \right) < 0, \text{ if } h = l \\ \eta u' \left(c_1^h \right) > 0, \text{ if } h = b \end{array} \right.$, *where $\eta \in$* (0, 1).
  (ii) If $d_1 > \bar{d}_1$, then

$$(18) \qquad \frac{\partial V^h}{\partial d_1} = \frac{\mathbf{d} e_1}{\mathbf{d} d_1} u' \left(c_1^h \right) = -u' \left(c_1^h \right) < 0, \text{ for each } h \in \{b, l\}.$$

The first part illustrates the usual pecuniary externalities on the interest rate, which apply when the debt level is relatively low. In this case, a higher debt level translates into a lower interest rate r_1—so as to counter the decline in demand—but it does not affect the net income, $e_1 (d_1) = e_1^*$ (see Figure 3). The reduction in the interest rate generates a redistribution from lenders to borrowers captured by η (characterized in Eq. (A4) in the online Appendix A). Consequently, deleveraging imposes positive pecuniary externalities on borrowers but negative pecuniary externalities on lenders. In fact, since markets between date 0 and 1 are complete, these two effects "net out" from an ex-ante point of view: that is, the date 0 equilibrium is constrained Pareto efficient in this region (see Proposition 2).

The second part of the lemma illustrates the novel force in our model, aggregate demand externalities. In this case, the debt level is sufficiently large so that the economy is

in a liquidity trap, which has two implications. First, the interest rate is fixed, $r_2(d_1) = 0$, so that the pecuniary externalities do not apply. Second, net income is decreasing in debt, $\frac{de_1}{dd_1} < 0$, through a reduction in aggregate demand (see Figure 3). Consequently, an increase in aggregate debt reduces households' welfare, which we refer to as an aggregate demand externality.

Lemma 2 also illustrates that, unlike pecuniary externalities, aggregate demand externalities hurt all households, because they operate by lowering incomes. This feature suggests that aggregate demand externalities can be considerably more potent than pecuniary externalities. They also lead to constrained inefficiencies in our setting, which we verify next.

B.   Excessive leverage

We next show that the competitive equilibrium allocation can be Pareto improved by reducing leverage. One way of doing this is ex-post, by writing down borrowers' debt. To see this, suppose the planner reduces borrowers' outstanding debt to lenders from d_1 to the threshold, $\bar{d}_1$, given by Eq. (13). By our earlier analysis, the recession is avoided, and net income increases to its efficient level, e^*. Borrowers' net consumption and welfare naturally increase after this intervention. Less obviously, lenders' net consumption remains the same at the upper bound, $\bar{c}_1^l$. The debt write-down has a direct negative effect on lenders' welfare by reducing their assets, as captured by $\frac{-\partial V^l}{\partial d_1^l} = u'(c_1^l) > 0$. However, the debt write-down also has an indirect positive effect on lenders' welfare through aggregate demand externalities. Lemma 2 shows that the externalities are sufficiently strong to fully counter the direct effect, $\frac{-\partial V^l}{\partial d_1} = u'(c_1^l) > 0$, leading to an ex-post Pareto improvement.

From the lens of our model, debt write-downs are always associated with aggregate demand externalities. However, these externalities are not always sufficiently strong to lead to a Pareto improvement.[8] Furthermore, ex-post debt write-downs are difficult to implement in practice for a variety of reasons, e.g., legal restrictions, concerns about moral hazard, or concerns about the financial health of intermediaries (assuming that some lenders are intermediaries). Therefore we do not analyze our results on ex-post inefficiency further.

An alternative, and arguably more practical way to reduce leverage is to prevent it from accumulating in the first place. This creates a very general scope for Pareto improvements. To investigate ex-ante optimality, suppose the planner can choose households' allocations at date 0, in addition to controlling the equilibrium debt level carried into date 1 (through the policies we will describe). We say that an allocation $\left(\left(c_0^h, n_0^h \right)_h, d_1 \right)$

---

[8]For instance, with separable preferences, $u(c) - v(l)$, analyzed in the online Appendix C, debt write-downs do not generate ex-post Pareto improvement. This is also the case for the extension analyzed in Section III.C with flexible MPC differences.

is constrained efficient if it is optimal according to this planner, that is, if it solves

$$
(19) \qquad \max_{\left(\left(c_0^h, n_0^h \right)_h, d_1 \right)} \sum_h \gamma^h \left[u \left(c_0^h \right) + \beta^h V^h \left(d_1^h, d_1 \right) \right]
$$

$$
\text{such that} \qquad d_1 = d_1^b = -d_1^l \text{ and } \sum_h c_0^h = \sum_h \left[n_0^h - \upsilon \left(n_0^h \right) \right].
$$

Here, $\gamma^h > 0$ captures the relative welfare weight assigned to group h households. We next characterize the constrained efficient allocations over the relevant range.[9]

PROPOSITION 2 (Optimal Leverage): *An allocation* $\left(\left(c_0^h, n_0^h \right)_h, d_1 \right)$, *with* $d_1 \geq \phi$ *and* $u' \left(c_0^l \right) \geq \beta^l u' \left(c_1^l \right)$, *is constrained efficient if and only if net income at date 0 is at its frictionless level, i.e.,* $e_0 = e^*$; *and the consumption and debt allocations satisfy one of the following:*
  (i) $d_1 \leq \bar{d}_1$ *and the Euler equations (16) hold.*
  (ii) $d_1 = \bar{d}_1$ *and the following inequality holds:*

$$
(20) \qquad \frac{\beta^l u' \left(c_1^l \right)}{u' \left(c_0^l \right)} > \frac{\beta^b u' \left(c_1^b \right)}{u' \left(c_0^b \right)}.
$$

The first part illustrates that competitive equilibrium allocations in which $d_1 \leq \bar{d}_1$ are constrained efficient. This part verifies that pecuniary externalities alone do not generate inefficiencies in our setting. The second part, which is our main result, shows that the planner never chooses a debt level above $\bar{d}_1$ that triggers a recession at date 1. Instead, the planner distorts decentralized households' Euler equations according to (20). At these allocations, borrowers would like to increase borrowing—so as to increase their consumption at date 0 and reduce their consumption at date 1—but they are prevented from doing so by the planner. In particular, a competitive equilibrium that features $d_1 > \bar{d}_1$, as well as the Euler equations (16), is constrained inefficient, as we formalize next.

COROLLARY 1 (Excessive Leverage): *The competitive equilibrium allocation* $\left(\left(c_0^{h,eq}, n_0^{h,eq} \right)_h, d_1^{eq} \right)$ *in Proposition 1 is constrained inefficient and is Pareto dominated by the constrained efficient allocation* $\left(c_0^h, n_0^h \right)_h = \left(c_0^{h,eq}, n_0^{h,eq} \right)_h$ *and* $d_1 = \bar{d}_1$.

To understand the intuition for the inefficiency, observe that lowering debt when the economy is in a liquidity trap generates first-order welfare benefits because of aggregate demand externalities, as illustrated in Lemma 2. By contrast, distorting agents' consumption away from their privately optimal levels generates locally second order losses. Thus, starting from an unconstrained equilibrium, it is always socially desirable to lower

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[9]We restrict attention to solutions that satisfy $d_1 \geq \phi$ and $u' \left(c_0^l \right) > \beta^l u' \left(c_1^l \right)$, which is the relevant range of comparison with the competitive equilibrium characterized in Section II. The former condition ensures that the exogenous debt limit also binds for the planner's allocation. The latter condition ensures that the planner's allocation can be implemented without hitting the zero lower bound at date 0, i.e., in the period before the deleveraging. Note that the competitive equilibrium features $r_1 > 0$ and satisfies this condition in view of Assumption (1).

leverage. As this intuition suggests, the ex-ante inefficiency from excessive leverage applies quite generally. For instance, the online Appendix C establishes an analogous result for the case with separable preferences, $u(c) - v(n)$, and Section III.C generalizes the result to the case in which borrowers have lower MPCs.[10]

We next show that the constrained efficient allocations in Proposition 2 and Corollary 1 can be implemented using simple macroprudential policies. We spell out two alternative implementations using quantity and price interventions in households' financial decisions. We allow the planner to use lump-sum transfers at date 0, which enables her to trace the constrained Pareto frontier characterized in Proposition 2.

COROLLARY 2 (Implementing the Optimal Leverage):   *The constrained efficient allocations characterized in Proposition 2 can be implemented alternatively with:*
(i) the debt limit $d_1^h \leq \bar{d}_1$ applied to all households, or
(ii) a tax $\tau_0^b \geq 0$ applied on any positive debt issuance $d_1^h > 0$ (that is rebated lump-sum to households), which satisfies,[11]

$$(21) \qquad \frac{\beta^l u'\left(c_1^l\right)}{u'\left(c_0^l\right)} = \frac{\beta^b u'\left(c_1^b\right)}{u'\left(c_0^b\right)} \frac{1}{1 - \tau_0^b},$$

combined in each case with an appropriate lump-sum transfer T_0^b between borrowers and lenders.

The debt limit policy directly restricts the equilibrium debt level. The tax policy brings about the same outcome by raising borrowers' net-of-tax interest rate, $\frac{1+r_1}{1-\tau_0^b}$, relative to the interest rate received by lenders, $1 + r_1$. Note also that both of these policies are anonymous in the sense that they apply to all households. For lenders, the limit defined in (i) does not bind, and the tax rate in (ii) does not apply, because their debt issuance is negative. However, this feature of the model does not generalize to richer settings. In general, the optimal policy requires targeted interventions for different groups (see, for instance, Sections III.C and IV.A).

The corollary describes restrictions on borrowing, but observe that the same allocations can be implemented by policy measures on saving. A binding quantity limit on wealth accumulation $d_1^h < -\bar{d}_1$ would ensure that lenders will not carry excessive wealth into the deleveraging period. Similarly, a tax on wealth accumulation could achieve the same objective.[12]

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[10]In our baseline setting, the externalities are so strong that the planner fully avoids a recession. This feature is less general. With separable preferences for households or lower MPCs for borrowers, the planner typically alleviates, but does not fully eliminate, the recession.

[11]Specifically, a household who issues $d_1^h > 0$ units of debt at interest rate r_1 at date 0 receives only $\frac{1-\tau_0^b}{1+r_1} d_1^h$ units, whereas its lender needs to provide $\frac{1}{1+r_1} d_1^h$ units. The difference, $\frac{\tau_0^b}{1+r_1} d_1^h$, is government revenue, which is rebated in a lump-sum and equally to all households.

[12]One interesting further question is whether the planner's optimal intervention would change if the deleveraging is anticipated several periods in advance. We find that the optimal interventions are unchanged in that case. For instance, the planner could announce the debt limit for date 1 ($d_1^h \leq \bar{d}_1$) ahead of time and let private agents decide how to optimally smooth consumption in earlier periods.

More broadly, our analysis supports policies that are targeted towards lowering household leverage (as well as corporate and bank leverage, as we discuss in Section VII). This is in contrast with some tax policies in the US, e.g. the mortgage interest tax deduction that incentivizes households to take on debt. Our analysis provides another rationale for revisiting these policies, especially in environments (with already low interest rates) in which deleveraging can induce or exacerbate a liquidity trap.

Our findings also point out that macroeconomic stabilization and financial stabilization are two sides of the same coin in the described setup. Since the recession in our model is driven by deleveraging, macroprudential policies increase both macroeconomic stability (by mitigating recessions) and financial stability (by reducing the size of deleveraging). We employ the label "macro-prudential" for this policy since it constitutes a financial market intervention that delivers the macroeconomic benefit of avoiding output costs, in line with the ultimate objective of macroprudential policy described by Borio (2003).[13]

C. Quantifying the inefficiency with MPC differences

Let MPC_1^h denote the increase in household h's consumption at date 1 in response to a transfer of one unit of liquid wealth at date 1, keeping its wage and interest rates at all dates constant. Our analysis so far had the feature that $MPC_1^b = 1$, that is, borrowers consume all of their additional income. This feature is useful to illustrate our welfare results sharply, but it is rather extreme. We next analyze a version of our model in which borrowers' MPC can be flexibly parameterized. To keep the analysis simple, we assume $u(c) = \log c$ in this section so that we can calculate households' MPCs in closed form.

The main difference is that borrowers are now subject to heterogeneous shocks at date 1 that generate heterogeneity in their MPCs—and lower their MPCs as a group. In practice, there are many shocks that could create heterogeneity along these lines (e.g., income shocks). In our analysis, we find it convenient to introduce this heterogeneity through shocks to constraints. Specifically, all borrowers are identical at date 0 but they realize one of two types starting date 1. A fraction $\alpha \in [0, 1]$ of borrowers, denoted by type b_{con}, are subject to an exogenous borrowing constraint ϕ as before, and thus, they continue to have $MPC_1^{b_{con}} = 1$. The remaining fraction, denoted by type b_{unc}, are unconstrained at all dates, and thus they have a lower MPC. In particular, in view of the log utility, unconstrained borrowers—as well as lenders—consume a small and constant fraction of the additional income they receive. To simplify the expressions, suppose also that all households have the same discount factor starting date 1 denoted by β (as before, borrowers have a lower discount factor at date 0, $\beta^b \leq \beta^l$). This implies:

$$(22) \qquad MPC_1^l = MPC_1^{b_{unc}} = 1 - \beta.$$

---

[13]This also follows the established practice of existing academic literature that motivates macroprudential policy based on alternative market imperfections (see our literature review on page 6 for a detailed list of references). For a more general discussion of the scope of macro-prudential policy, see for example Jeanne and Korinek (2014).

Hence, the MPC of borrowers as a group is given by:

$$(23) \qquad MPC_1^b \equiv \alpha + (1 - \alpha)(1 - \beta).$$

In particular, the parameter α enables us to calibrate the MPC differences between borrowers and lenders.

We simplify the analysis by assuming that borrowers are identical at date 0 and cannot trade assets whose payoffs are contingent on the type of shocks they will receive at date 1. This ensures that each borrower enters date 1 with the same amount of outstanding debt d_1. To obtain slightly more general formulas, we also parameterize the relative mass of borrowers and lenders. Assume that the mass of lenders is given by ω, and that of borrowers by 1 so that $1/(1 + \omega)$ denotes the borrowers' share of aggregate income. The baseline model is the special case with $\alpha = 1$ and $\omega = 1$.

As before, there is a threshold debt level $\bar{d}_1$, such that the equilibrium features a liquidity trap if and only if $d_1 > \bar{d}_1$. The analysis in the online Appendix B further shows that

$$(24) \qquad (1 + \omega)\frac{\mathbf{d}e_1}{\mathbf{d}d_1} = -\frac{MPC_1^b - MPC_1^l}{1 - \overline{MPC}_1},$$

where $\overline{MPC}_1 = \frac{MPC_1^b + \omega MPC_1^l}{1 + \omega}$ denotes the average MPC across all households. Here, the left-hand side illustrates the marginal effect of debt on total net demand, $e_1(1 + \omega)$ (which takes into account the total size of the population). As before, greater debt induces a deeper recession. However, the strength of the effect now depends on the MPC differences between borrowers and lenders. Intuitively, greater debt influences aggregate demand by transferring wealth at date 1 from borrowers to lenders. This transfer affects demand more when there is a greater difference between borrowers' and lenders' MPCs. The effect is further exacerbated by the Keynesian income multiplier as captured by the denominator in (24).

We next characterize the planner's optimality condition as well as the optimal tax rate on borrowing for this case—the analogues of Eqs. (20) and (21). With some abuse of notation, we let $u'\left(c_1^b\right) = \alpha u'\left(c_1^{bcon}\right) + (1 - \alpha)u'\left(c_1^{bcon}\right)$ denote borrowers' expected marginal utility before the realization of their type at date 1. The first order condition for the constrained planning problem stated in the online Appendix B implies,

$$(25) \qquad \frac{\beta^l u'\left(c_1^l\right)}{u'\left(c_0^l\right)} = \frac{\beta^b u'\left(c_1^b\right)}{u'\left(c_0^b\right)} - \frac{\mathbf{d}e_1}{\mathbf{d}d_1}\left(\frac{\beta^b u'\left(c_1^b\right)}{u'\left(c_0^b\right)} + \omega\frac{\beta^l u'\left(c_1^l\right)}{u'\left(c_0^l\right)}\right),$$

for each $d_1 > \bar{d}_1$. Note that the planner takes into account the negative effects of debt on all households' net incomes and welfare. Combining Eq (25) with Eqs. (21) and (24),

we further characterize the optimal tax rate as,

$$(26) \qquad \tau_0^b = \frac{MPC_1^b - MPC_1^l}{1 - \overline{MPC}_1}(1 - \overline{\tau}) = \frac{MPC_1^b - MPC_1^l}{1 - MPC_1^l}.$$

Here, $\overline{\tau} = \frac{1}{1+\omega}\tau_b$ denotes the average tax rate across all households. The first equality says that the optimal tax on borrowing is to a first order determined by the MPC differences between borrowers and lenders, and the Keynesian income multiplier. The correction term, $1 - \overline{\tau}$, accounts for the distortions that are introduced by relatively large tax rates. The second equality simplifies the formula further and shows that it is independent of borrowers' and lenders' relative income shares (captured by ω).

The online Appendix B generalizes these results to a setting with multiple (identifiable) groups of borrowers each of which might have different MPCs at date 1 (due to different α's). The analysis also accommodates multiple groups of lenders, some of which might have higher MPCs at date 1 (perhaps because they have relatively low assets and might become constrained with some probability). The optimal tax rate in (26) continues to apply for each group of borrowers or lenders, once we interpret group l in the formula as fully unconstrained households with $MPC_1^l = 1 - \beta$ [see Eq. (B11)]. However, the implementation with multiple groups features two differences relative to Corollary 2. First, the policies are non-anonymous in the sense that a particular tax rate τ_0^h applies only to group h households (as opposed to all households). Second, the tax rate applies to all debt choices by this group—as opposed to only positive debt issuance. In fact, a tax on negative debt issuance, $d_0^h < 0$, is in effect a subsidy for saving. The planner might use these subsidies to raise the savings of lenders with relatively high MPCs.

The empirical literature finds that the MPCs of households indeed differed greatly depending on their debt or asset position in the recent liquidity trap episode. For example, using data from an Italian household survey conducted in 2010, Jappelli and Pistaferri (2014) find that the MPC out of transitory income shocks for households in the lowest decile of the cash-on-hand distribution was about 62%, whereas the MPC of households in the highest decile was about 36% (Figure 2).[14] Our analysis suggests that the results from this literature can be used to guide optimal macroprudential policy. However, the formula in (26) assumes that the deleveraging episode occurs with probability one. This is useful for expositional simplicity, but would deliver unrealistically high tax rates. To address this, our next step is to introduce uncertainty into our framework.

IV. Uncertainty about the deleveraging episode

Our analysis so far has focused on a special case in which the deleveraging episode is perfectly foreseen. This section extends the model to incorporate uncertainty about deleveraging. We first consider the case that financial markets are complete at date 0 so that households can trade insurance contracts contingent on the deleveraging episode. In

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[14]See also the survey by Jappelli and Pistaferri (2010) and recent papers by Mian, Rao and Sufi (2013), Parker et al. (2013), Baker (2014) and Auclert (2015).

this context, we establish our second main result that borrowers in a competitive equilibrium purchase too little insurance. We then consider the case in which financial markets are incomplete in the sense that households cannot trade contingent contracts and generalize our excessive leverage result to this setting.

A.   Deleveraging driven by insurable shocks

Consider the baseline setting described in Section I, but suppose the economy is in one of two states $s \in \{H, L\}$ from date 1 onwards. The states differ in their debt limits. State L captures a "low leverage" state in which the economy experiences a financial shock and becomes subject to a permanent debt limit, $\phi_{t+1,L} \equiv \phi$ for each $t \geq 1$. State H in contrast captures the "high leverage" state in which households' debt choices remain unconstrained similar to date 0 of the earlier analysis, that is, $\phi_{t+1,H} = \infty$ for each $t \geq 1$. We use π_s^h to denote group h households' belief for state $s \in \{H, L\}$ and $E^h[\cdot]$ to denote their expectation operator over states. We assume $\pi_L^h > 0$ $\forall h$ so that the deleveraging episode is anticipated by all households.

We simplify the analysis by assuming that starting date 1, both types of households have the same discount factor denoted by β.[15] As before, borrowers have a lower discount factor at date 0 denoted by $\beta^b \leq \beta^l$. In addition, we also assume $\pi_L^b \leq \pi_L^l$, so that borrowers assign a weakly lower probability to the deleveraging state compared to the lenders. Neither of these assumptions is necessary, but since impatience/myopia and excessive optimism were viewed as important contributing factors to many deleveraging crises, they enable us to obtain additional interesting results. We also replace the second part of Assumption (1) with the appropriate limit on d_0 for this case so that the interest rate constraint does not bind at date 0.

At date 0, households are allowed to trade two types of securities. First, as before, they choose their debt (or asset) level d_{t+1}^h for the next period. The debt is non-contingent in the sense that it promises the same payment $1 + r_{t+1}$ (per unit) in each state s, where r_{t+1} denotes the safe real interest rate as before. Second, households can also hold an Arrow-Debreu security that pays 1 unit of the consumption good in state L and nothing in the other state. We refer to this asset as an insurance contract, and denote households' position in this asset with m_L^h and the price of the asset with q_L. Households' budget constraint can be written (in net variables) as:

$$c_0^h = e_0 - d_0^h + \frac{d_1^h}{1 + r_1} - m_L^h q_L^h,$$

$$\text{and } c_{1,s}^h = e_{1,s} - d_{1,s}^h + \frac{d_{2,s}^h}{1 + r_1}, \text{ where } \left\{ \begin{array}{c} d_{1,L}^h \equiv d_1^h - m_L^h \\ d_{1,H}^h \equiv d_1^h \end{array} \right. .$$

Here, $d_{1,s}^h$ denotes households' effective debt level in state s. Note that the two securities complete the market in the sense that they enable the households to freely choose their

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[15]This ensures that the equilibrium is non-degenerate in the high state H. Alternatively, we could impose a finite debt limit $\phi_{t+1,H} < \infty$.

effective debt (or asset) levels. Given these changes, the optimization problem of households and the definition of equilibrium generalize to uncertainty in a straightforward way. We also let $d_{1,s} \equiv d_{1,s}^b$ denote the effective aggregate debt level in state s and $m_L \equiv m_L^b$ denote borrowers' aggregate insurance purchase.

The equilibrium in state L is the same as described before. In particular, the interest rate is zero and there is a demand-driven recession as long as the effective debt level exceeds a threshold, $d_{1,L} > \bar{d}_1$. The equilibrium in state H jumps immediately to a steady-state with interest rate $1 + r_{t+1} = 1/\beta > 0$ and consumption $c_{t,H}^h = e^* - (1 - \beta) d_{1,H}^h \; \forall t \geq 1$.

The main difference concerns households' date 0 choices. In this case, households' optimal debt choice implies Euler equations as before,

$$(27) \qquad \frac{1}{1 + r_1} = \frac{\beta^l E^l \left[u' \left(c_1^l \right) \right]}{u' \left(c_0^l \right)} = \frac{\beta^b E^b \left[u' \left(c_1^b \right) \right]}{u' \left(c_0^b \right)},$$

and their optimal insurance choice implies full insurance conditions for state L,

$$(28) \qquad q_{1,L} = \frac{\beta^l \pi_L^l u' \left(c_{1,L}^l \right)}{u' \left(c_0^l \right)} = \frac{\beta^b \pi_L^b u' \left(c_{1,L}^b \right)}{u' \left(c_0^b \right)}.$$

We next describe under which conditions households choose a sufficiently high debt level for state L to trigger a recession, $d_{1,L} > \bar{d}_1$:

PROPOSITION 3: *There is a deleveraging-induced recession in state L of date 1 if the borrower is either (i) sufficiently impatient, or (ii) sufficiently indebted, or (iii) sufficiently optimistic at date 0. Specifically, for any two of the parameters $\left(\beta^b, d_0, \pi_L^b \right)$, we can determine a threshold for the third parameter such that $d_{1,L} > \bar{d}_1$ if the threshold is crossed, i.e. if $\beta^b < \bar{\beta}^b \left(d_0, \pi_L^b \right)$ or $d_0 > \bar{d}_0 \left(\beta^b, \pi_L^b \right)$ or $\pi_L^b < \bar{\pi}_L^b \left(\beta^b, d_0 \right)$.*

The thresholds are characterized in more detail in the online Appendix A. The first two cases are analogous to the cases in Proposition 1: if borrowers have a strong motive to carry debt into date 1, they also choose to hold a large level of effective debt in state L, even though this triggers a recession. The last case identifies a new factor that could exacerbate this outcome. If borrowers assign a sufficiently low probability to state L relative to lenders, then they also naturally choose to hold a large level of effective debt in state L. In each scenario, $d_{1,L} > \bar{d}_1$ and there is a recession in state L of date 1.

To analyze welfare, consider a planner who can choose households' allocations at date 0 and control their effective debt levels at date 1 (via the simple policies we will describe below), but leaves the remaining allocations to the market. The constrained planning

problem can be written as:

$$(29) \qquad \max_{\left((c_0^h, n_0^h)_h, d_{1,H}, d_{1,L}\right)} \sum_h \gamma^h \left[u\left(c_0^h\right) + \beta^h \sum_s \pi_s^h V_s^h \left(d_{1,s}^h, d_{1,s}\right) \right]$$

$$\text{such that } d_{1,s} = d_{1,s}^b = -d_{1,s}^l \text{ for each } s, \text{ and } \sum_h c_0^h = \sum_h \left[n_0^h - \upsilon\left(n_0^h\right) \right].$$

Our next result characterizes the solution to this problem over the relevant range.

PROPOSITION 4 (Optimal Insurance): *An allocation* $\left((c_0^h, n_0^h)_h, d_{1,H}, d_{1,L}\right)$, *with* $d_{1,L} \geq \phi$ *and* $u'\left(c_0^h\right) \geq \beta^l E^l \left[u'\left(c_1^l\right)\right]$, *is constrained efficient if and only if output at date 0 is efficient, i.e.,* $e_0 = e^*$; *households' full insurance condition for state H holds,* $\frac{\beta^l \pi_H^l u'\left(c_{1,H}^l\right)}{u'\left(c_0^l\right)} = \frac{\beta^b \pi_H^b u'\left(c_{1,H}^b\right)}{u'\left(c_0^b\right)}$; *and the remaining consumption and leverage allocations satisfy one of the following:*

  (i) $d_{1,L} \leq \bar{d}_1$ *and the full insurance conditions (28) also hold for state L,*
  (ii) $d_{1,L} = \bar{d}_1$ *and the following inequality holds for state L:*

$$(30) \qquad \frac{\beta^l \pi_L^l u'\left(c_{1,L}^l\right)}{u'\left(c_0^l\right)} > \frac{\beta^b \pi_L^b u'\left(c_{1,L}^b\right)}{u'\left(c_0^b\right)}.$$

The second part illustrates our main result with uncertainty. The planner limits the effective debt level in the deleveraging episode, $d_{1,L} = \bar{d}_1$ and distorts households' insurance conditions according to (30). As this inequality illustrates, borrowers would like to reduce their insurance purchases (which would raise their effective debt in state L) so as to consume more in state 0 and less in state L, but they are prevented from doing so by the planner. In particular, a competitive equilibrium with $d_{1,L} > \bar{d}_1$ is constrained inefficient, as we formalize next.

COROLLARY 3 (Underinsurance): *The competitive equilibrium allocation* $\left((c_0^{h,eq}, n_0^{h,eq})_h, d_{1,H}^{eq}, d_{1,L}^{eq}\right)$ *in Proposition 3 is constrained inefficient, and it is Pareto dominated by the constrained efficient allocation* $(c_0^h, n_0^h)_h = (c_0^{h,eq}, n_0^{h,eq})_h, d_{1,H} = d_{1,H}^{eq}$ *and* $d_{1,L}^{eq} = \bar{d}_1$.

This result identifies a distinct type of inefficiency in our setting: borrowers in a competitive equilibrium buy too little insurance with respect to aggregate deleveraging episodes. Intuitively, they do not take into account the positive aggregate demand externalities their insurance purchases would bring about. Therefore, they end up with financial portfolios that are too risky from a social point of view.

We next show that the constrained efficient allocations can be implemented with macroprudential insurance policies. First, suppose the planner can require households' effective debt level in state L to be bounded from above, that is, $d_{1,L}^h \leq \bar{d}_1$ for each h. This is equivalent to setting a minimum insurance requirement that depends on households' total debt, $m_L^h \geq d_1^h - \bar{d}_1$. Note the planner is setting a tighter requirement for

more indebted households. Second, suppose the planner can also set a linear subsidy (or tax) on borrowers' insurance positions, m_L^b. Specifically, m_L^b units of the insurance contract cost the borrowers $m_L^b q_L \left(1 - \zeta_0^b\right)$ units of the consumption good. Note that this policy corresponds to a subsidy to borrowers when they purchase insurance $m_L^b > 0$, but it would correspond to a tax if they chose to sell insurance $m_L^b < 0$. The policy does not apply to lenders (and thus, is not anonymous) who continue to receive or pay q_L per unit of the insurance contract.[16] The total cost of the subsidy is $m_L \zeta_0^b$, which is financed by lump-sum taxes on all households. As before, the planner can also combine these policies with a transfer of wealth T_0^b from lenders to borrowers.

COROLLARY 4 (Implementing the Optimal Insurance): *The constrained efficient allocations characterized in Proposition 4 can be implemented alternatively with:*
  (i) the minimum insurance requirement, $m_L^h \geq d_1^h - \bar{d}_1$ for each h, or
  (ii) insurance subsidies to borrowers, $\zeta_0^b > 0$, that satisfy:

$$(31) \qquad \frac{\beta^l \pi_L^l u'\left(c_1^l\right)}{u'\left(c_0^l\right)} = \frac{\beta^b \pi_L^b u'\left(c_1^b\right)}{u'\left(c_0^b\right)} \frac{1}{1 - \zeta_0^b},$$

combined with an appropriate ex-ante transfer T_0^b for each case.

The insurance requirement directly restricts borrowers' outstanding debt in state L. The subsidy policy brings about the same outcome by lowering the net-of-tax insurance price that borrowers face relative to lenders. We can also quantify the optimal subsidy in our setting after modifying the model as in Section III.C so as to flexibly parameterize households' MPCs. The online Appendix B specifies the details and obtains the following analogue of Eq. (26),

$$(32) \qquad \zeta_0^b = \frac{MPC_1^b - MPC_1^l}{1 - MPC_1^l}.$$

The optimal subsidy rate—just like the optimal tax rate—is determined by the MPC differences between borrowers and lenders. Like the optimal tax rate, this formula generalizes to a setting with multiple groups of borrowers or lenders, once we interpret group l as fully unconstrained households [see Eq. (B13)].

Our model has many stylized features, departing from that which would naturally affect the optimal subsidy (as well as tax) formulas.[17] Nonetheless, we view the formula in (32) as providing a useful benchmark for understanding the order of magni-

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[16]The planner needs non-anonymous policies in this case because, in view of belief disagreements, borrowers might choose to sell insurance as opposed to buying. If this happens, subsidizing insurance purchases anonymously would create the opposite of the intended effect. In the special case with common beliefs, $\pi_0^l = \pi_0^b$, the equilibrium features insurance purchases by borrowers, $m_L^b > 0$, and the anonymous policy also works.

[17]For instance, a different utility function than the GHH form, $u\left(c - v\left(n\right)\right)$, would typically affect the optimal subsidy and tax formulas. The online Appendix C analyzes separable preferences, $u\left(c\right) - v\left(n\right)$, and shows that the optimal tax rate in that case also depends on the labor wedge, which measures the welfare benefits of raising aggregate output net of the increased costs of labor supply [see Eqs. (C14) and (C15)].

tude for reasonable subsidy policies in practice. Using the MPC estimates from Jappelli and Pistaferri (2014) that we cited at the end of Section III, we obtain $\zeta_0^b \simeq (62\% - 36\%) / (1 - 36\%) = 40.6\%$. Hence, with these estimates, our analysis suggests that the government should subsidize roughly 40% of the insurance bill for the households in the lowest decile of the asset distribution.[18]

An important financial shock in practice is an economy-wide (or widespread) decline in house prices, which can considerably tighten many homeowners' borrowing constraints and induce a demand-driven recession. In this context, Proposition 4 provides a rationale for policies that reduce mortgage borrowers' exposures to house prices. Corollary 4 and Eq. (32) illustrate how this can be implemented by subsidizing a type of home equity insurance that pays homeowners when there is a severe and economy-wide downturn in house prices.[19]

Shiller and Weiss (1999) proposed home equity insurance along these lines to protect homeowners against declines in housing prices, but demand for such insurance has been muted (see Shiller, 2003). In the recent housing boom, one of the reasons households wanted to expose themselves to house price risk was arguably their optimism about house prices increases (see Case, Shiller and Thompson 2012). This optimism is also one of the factors that we capture in Proposition 3. However, our model emphasizes that there are uninternalized social benefits to home equity insurance purchases by highly leveraged borrowers. Aggregate demand externalities provide a rationale for insurance requirements or subsidies, even if policymakers respect households' different beliefs.

B.   *Deleveraging driven by uninsurable shocks*

While certain financial shocks that trigger deleveraging seem possible to insure against (at least in principle), other shocks can be much more difficult to identify and contract upon. Consider, for example, the recent subprime financial crisis that put many financial institutions into distress, which arguably lowered credit to households even after controlling for their collateral values (see Mondragon (2015) for empirical evidence). Caballero and Simsek (2013) describe several reasons that could have made it difficult or costly to purchase ex-ante insurance protection against this event.[20] Perhaps for these (or other) reasons, the recent literature on financial frictions typically assumes that insurance markets are missing for aggregate financial shocks (see Brunnermeier and Sannikov (2014) and the references therein).

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[18]This number (40%) might sound large, but note that this is a subsidy only on the insurance bill of borrowers for severe deleveraging episodes, which is likely to be much smaller than their loan balances.

[19]It is important to emphasize that our model does not support subsidizing insurance to all types of home equity insurance. For instance, insurance with respect to idiosyncratic events that lower the value of a small number of houses (such as a fire or a local flood) are not supported by our policies, since these events are unlikely to influence aggregate demand. In contrast, our analysis supports subsidizing insurance with respect to aggregate shocks to house prices, which could induce widespread deleveraging and influence the aggregate demand. These insurance markets are arguably also resilient to moral hazard or adverse selection, since individual insurance buyers are unlikely to influence the probability of aggregate events or to have private information about the probability of such events.

[20]First, while the problems in the subprime market were anticipated, the exact location or the magnitude of the losses were not understood until well into 2008. Given that the event was difficult to describe, it was also arguably difficult (or very costly) to insure against. Second, the shock was systemic and the potential insurance sellers (which are left out of our analysis) also became distressed, which would have further increased the costs of insurance.

What becomes of our welfare analysis when the underlying financial shock—and therefore, the deleveraging episode—is uninsurable? To investigate this question, consider the model with uncertainty with the only difference being that households cannot trade insurance contracts, so that, $m_L^h = 0$ for each h. To obtain slightly more general and quantifiable formulas, suppose also that the model is modified as in Section III.C (see the online Appendix B for details). The equilibrium at date 0 is characterized by households' Euler equations analogous to Eq. (27). Under appropriate conditions, there is a recession in state L as in Proposition 4.

To analyze constrained efficiency, suppose the planner is also subject to the same market incompleteness, so that she is constrained to choose $d_1 = d_{1,H} = d_{1,L}$. The online Appendix B shows that the constrained efficient allocations in this case satisfy:[21]

$$(33) \quad \frac{\beta^l E^l \left[u' \left(c_1^l \right) \right]}{u' \left(c_0^l \right)} = \frac{\beta^b E^b \left[u' \left(c_1^b \right) \right]}{u' \left(c_0^b \right)} - \frac{\mathbf{d} e_{1,L}}{\mathbf{d} d_1} \left(\pi_L^b \frac{\beta^b u' \left(c_{1,L}^b \right)}{u' \left(c_0^b \right)} + \pi_L^l \omega \frac{\beta^l u' \left(c_{1,L}^l \right)}{u' \left(c_0^l \right)} \right),$$

where $\frac{\mathbf{d} e_{1,L}}{\mathbf{d} d_1} < 0$ as before (see Eq. (24)). Thus, the competitive equilibrium features excessive leverage also in this case. However, unlike in Section III.C, the size of the required intervention also depends on households' perceived probabilities for the deleveraging episode, π_L^b and π_L^l.

In this case, we cannot provide exact formulas for the optimal tax rate on borrowing (since households' marginal utilities are not necessarily equated at the no-tax benchmark). For a back-of-the-envelope calculation, consider the special case in which households agree about the probability of the deleveraging episode, $\pi_L \equiv \pi_L^b = \pi_L^l$. Suppose also that the no-tax allocations roughly satisfy $u' \left(c_{1,L}^h \right) \simeq u' \left(c_{1,H}^h \right)$ for each h. With these simplifications, combining Eqs. (21), (24), and (25) imply,

$$(34) \quad \tau_0^b = \pi_L \cdot \frac{MPC_1^b - MPC_1^l}{1 - \overline{MPC}_1} \left(1 - \overline{\tau} \right) \simeq \pi_L \cdot \frac{MPC_1^b - MPC_1^l}{1 - \overline{MPC}_1}.$$

The first equality says that the optimal tax rate is determined by the tax formula without uncertainty multiplied by the probability of the deleveraging episode [see Eq. (26) for comparison, and Eq. (B14) for the more general version with multiple groups of borrowers or lenders]. The second equality is an approximation that holds when π_L (and thus, $\overline{\tau}$) is small.[22]

As before, although the formula depends on special features of our model, we view it as helpful in understanding the order of magnitude for reasonable tax policies in practice. Using the MPC estimates from Figure 2 of Jappelli and Pistaferri (2014) and a probability

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[21] In general, this type of environment might also feature pecuniary externalities that need not net out because markets are incomplete across states H and L (cf. Lemma 2). In our setting, these pecuniary externalities are mute because the interest rate is constant in both states (over the relevant range), that is, $1 + r_H = 1/\beta$ and $1 + r_L = 1$. Consequently, the constrained optimality condition features only aggregate demand externalities.

[22] The approximate formula is slightly different than the simplified formula in Eq. (26), because the optimal tax rate in this case can depend on households' income shares.

of once every thirty years for a deleveraging crisis (e.g. Reinhart and Rogoff 2009), we obtain $\tau_0^b \simeq 1/30 \cdot (62\% - 36\%) / (1 - 43\%) = 1.5\%$. Hence, with these estimates, our analysis suggests that the government should apply a tax rate in the order of 1.5% on the noncontingent debt of the households in the lowest decile of the asset distribution.

Intuitively, macroprudential policies that restrict debt provide blanket protection with respect to all deleveraging episodes, including those driven by uninsurable ones. These policies bring benefits by raising aggregate demand when the episode is realized. However, they also generate costs by distorting households' consumption in many other future states without deleveraging. Consequently, their desirability depends on the probability of deleveraging. This result also illustrates how the optimal macroprudential regulation is likely to be time (as well as context) dependent. Policies that are optimal for a particular time and environment might cease to be optimal in the future, e.g., if deleveraging becomes less likely or if other unmodeled considerations become relevant. This highlights the importance of continuous monitoring and analysis for optimal macroprudential regulation.

Let us summarize our findings on optimal macroprudential policy interventions in a more realistic environment with uncertainty. If the planner can identify certain states that will induce deleveraging, and if insurance markets exist for those states, then she can improve welfare by subsidizing or encouraging borrowers' insurance purchases as described in Section IV.A. For all other deleveraging states that cannot be clearly identified ex-ante, or that do not have insurance markets associated with them, the planner can improve welfare by taxing or discouraging debt as described in Section IV.A. The common theme is that the policymaker induces the households to internalize the social cost of carrying too much debt into states in which the economy experiences deleveraging.

V. Preventive monetary policies

The analysis so far has focused on macroprudential policies, i.e., interventions in financial markets. A natural question is whether preventive monetary policies could also be desirable to mitigate the inefficiencies in this environment. In this section, we analyze respectively the effect of changing the inflation target and adopting a contractionary monetary policy.

A. Changing the inflation target

Blanchard, Dell'Ariccia and Mauro (BDM, 2010), among others, emphasize that a higher inflation target could be useful to avoid or mitigate the liquidity trap. We illustrate this point using the version of our model with intermediately sticky prices and an inflation-targeting Taylor rule, developed in the online Appendix C. There, the Taylor rule ensures that the aggregate price inflation between dates 1 and 2 is equal to the inflation target, that is, $P_2/P_1 = \Pi$ where Π is the gross inflation target. Combining this with (3), the real interest rate is bounded from below, that is, $1 + r_2 \geq 1/\Pi$. It follows that raising inflation target Π relaxes the bound on the real rate. Consequently, a greater level

of leverage is necessary to plunge the economy into a liquidity trap. Hence, raising the inflation target reduces the incidence of liquidity traps, consistent with BDM (2010), as well as the incidence of aggregate demand externalities. These welfare benefits should be weighed against the various costs of higher steady-state inflation.

B.　Contractionary monetary policy

It has also been discussed that interest rate policy could be used as a preventive measure against financial crises. In fact, a number of economists have argued that the US Federal Reserve should have raised interest rates in the mid-2000s in order to lean against the housing bubble or to reduce leverage (see Woodford (2012) and Rajan (2010) for detailed discussions). We next investigate the effect of contractionary policy at date 0 on household leverage.

To this end, consider the baseline setting with a single type of borrower and no uncertainty. Suppose the conditions in Proposition 1 apply so that there is a liquidity trap at date 1. Suppose the monetary authority sets $r_1 > r_1^*$ at date 0, and follows the rule in (10) thereafter. In this case, the equilibrium at date 0 features a policy-induced recession, that is, households' net income falls to $e_0 < e^*$. Moreover, households' Euler equations are now given by

$$(35) \qquad \frac{1}{1+r_1} = \frac{\beta^l u'\left(e_1 + (d_1 - \phi)\right)}{u'\left(e_0 + d_0 - \frac{d_1}{1+r_1}\right)} = \frac{\beta^b u'\left(e_1 - (d_1 - \phi)\right)}{u'\left(e_0 - \left(d_0 - \frac{d_1}{1+r_1}\right)\right)},$$

where $e_1 = \bar{c}_1^l - (d_1 - \phi) < e^*$ as in (15). This describes two equations in two unknowns, $e_0(r_1)$, $d_1(r_1)$, which can be solved as a function of the policy rate r_1. Our next result characterizes the comparative statics with respect to r_1.

PROPOSITION 5 (Contractionary Monetary Policy): *Consider the equilibrium described above with a liquidity trap at date 1 and the interest rate $r_1 > r_1^*$. Suppose $-u''(x)/u'(x)$ is a weakly decreasing function of x. Suppose also that d_0 is sufficiently large so that $d_0 - \frac{d_1(r_1)}{1+r_1} > 0$. Then, $e_0'(r_1) < 0$ and $d_1'(r_1) > 0$, that is: increasing the interest rate r_1 decreases the current net income and increases the outstanding debt level d_1.*

The proposition considers cases in which the utility function lies in the decreasing absolute risk aversion family—which encompasses the commonly used constant elasticity case—and lenders' initial assets are sufficiently large so that their consumption exceeds borrowers' consumption (see (35)). As expected, raising the interest rate in the run-up to a deleveraging episode creates a recession. However, perhaps surprisingly, raising the interest rate in our setting increases the equilibrium leverage. This in turn leads to a more severe recession at date 1.

To understand this result, suppose $u(c) = \log c$ and $\phi = 0$. In this case, borrowers' and lenders' optimal debt choices have closed form solutions, conditional on the income levels e_0 and e_1, given by

(36)
$$d_1^b \;=\; \frac{1}{1+\beta^b}\left(e_1 - \beta^b\,(1+r_1)\,(e_0 - d_0)\right)$$

$$d_1^l \;=\; \frac{1}{1+\beta^l}\left(e_1 - \beta^l\,(1+r_1)\,(e_0 + d_0)\right).$$

In particular, keeping e_0 and e_1 constant, a higher r_1 reduces both d_1^b and d_1^l. Intuitively, the substitution effect induces borrowers to borrow less but also induces lenders to save more. This creates an excess demand in the asset market (that is, $d_1^b + d_1^l$ falls below 0)— or equivalently, a shortage of demand in the goods market. To equilibrate markets, output falls and households' net income e_0 declines. As this happens, both d_1^b and d_1^l increase: that is, borrowers borrow more and lenders save less so as to smooth their consumption. In our model, these effects are roughly balanced across borrowers and lenders since all households share the same elasticity of intertemporal substitution. In fact, if d_0 were equal to 0, the reduction in e_0 would be (with log utility) just enough to counter the initial effect and the equilibrium debt level $d_1 = d_1^b = -d_1^l$ would remain unchanged (see (36)). When d_0 is sufficiently large, higher r_1 creates an additional wealth transfer from borrowers to lenders. This increases borrowers' debt d_1^b further—while increasing lenders' assets—generating a higher equilibrium debt level $d_1 = d_1^b$. The proof in the online Appendix A uses more subtle arguments to establish the result more generally.

Hence, the conventional wisdom—that raising the interest rate decreases leverage— fails in view of two general equilibrium effects on borrowers' income and wealth. First, the higher interest rate creates a temporary recession, which reduces borrowers' current income and induces them to take on greater debt. Second, the higher interest rate also transfers wealth from borrowers to lenders, which further increases borrowers' debt. The combination of these two effects can dominate the partial equilibrium effect of the higher interest rate on borrowers, leading to greater debt in equilibrium.

We could construct variants of our model in which raising the interest rate decreases the outstanding leverage, d_1. For instance, if borrowers' intertemporal substitution is more elastic than lenders', as in Cúrdia and Woodford (2009), then the equilibrium debt level might decrease due to a stronger substitution effect for borrowers. However, even in these cases, the interest rate policy would not be the optimal instrument to deal with the excessive leverage problem. The following proposition establishes this point by characterizing the jointly optimal monetary and macroprudential policies at date 0.

PROPOSITION 6 (Jointly Optimal Monetary and Macroprudential Policy): *Consider the baseline model with the only difference that borrowers and lenders have heterogeneous utility functions at (only) date 0, $u_0^b(\cdot)$ and $u_0^l(\cdot)$ (so as to allow for richer effects of monetary policy on leverage). Suppose the planner chooses the interest rate r_1 at date 0, in addition to setting the macroprudential policies described in Section III. It is optimal for this planner to set $r_1 = r_1^*$ and implement $e_0 = e^*$.*

That is, once macroprudential policies are in place, it is optimal for monetary policy to simply focus on setting period 0 output to its efficient level. Intuitively, the constrained

efficient allocations characterized in Proposition 2 feature the frictionless output level at date 0. Moreover, those constrained efficient allocations can be implemented with macroprudential policies alone. It follows that raising the interest rate is not desirable because it triggers an inefficient recession at date 0 without providing any benefits over and above macroprudential policies.

These results illustrate that interest rate policy is not the right tool to deal with the excessive leverage problem. The problem is one of inefficient distribution of financial wealth between borrowers and lenders during the liquidity trap episode. Consequently, the constrained efficient allocations require creating a wedge between borrowers' and lenders' interest rates (see Corollary 2 and Eq. (21)). In contrast, monetary policy sets a different "intertemporal" wedge that affects both borrowers' and lenders' interest rates. Given that monetary policy targets "the wrong wedge," it could at best be viewed as a crude solution for dealing with excessive leverage. In contrast, macroprudential policies, e.g., debt limits or insurance subsidies, optimally internalize aggregate demand externalities created by leverage.

It is important to emphasize that contractionary monetary policy could well be desirable for reasons outside the scope of our model. For instance, if macroprudential policies are not available, then raising the interest rate might be useful to mitigate inefficient investment booms and fire-sale externalities as in Lorenzoni (2008), Stein (2012) or Jeanne and Korinek (2013). A higher interest rate might also be useful to lean against asset price bubbles, e.g., by discouraging the "search for yield" phenomenon discussed in Rajan (2010). Our point is that contractionary monetary policy is not the ideal instrument to reduce household leverage, and in fact, might have the unintended consequence of raising leverage.

VI. Aggregate demand and fire-sale externalities

In this section we endogenize the debt limit faced by borrowers by assuming that debt is collateralized by a financial asset, creating the potential for fire-sale effects.[23] We illustrate how this introduces a new feedback loop into the economy, while also creating fire-sale externalities for debt that operate in the same direction as aggregate demand externalities.

We modify our earlier setup by assuming that borrowers hold one unit $a_t = 1$ of a tree from which they obtain a dividend y_t every date. For simplicity, we assume that the tree only pays dividends if it is owned by borrowers so the tree cannot be sold to lenders. The tree trades among borrowers at a market price of p_t. We follow Jeanne and Korinek (2010b) in assuming that borrowers are subject to a moral hazard problem and have the option to abscond with their loans after the market for loans has closed. In order to

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[23]The interaction between asset fire sales and aggregate demand has also been studied in Carlstrom and Fuerst (1997), Bernanke et al. (1999) and Iacoviello (2004). In these papers, monetary policy can mitigate the feedback effects resulting from tightening borrowing constraints. We consider the possibility of a lower bound on interest rates that prevents this, and we add a normative dimension focused on debt market policies. There is also related literature in open economy macroeconomics that analyzes how price-dependent financial constraints capture the dynamics of sudden stops in capital inflows, and how this may interact with monetary policy. See for instance Krugman (1999), Aghion, Bacchetta and Banerjee (2001, 2004), and Aguiar and Gopinath (2005).

alleviate the moral hazard problem, they pledge their trees as collateral to lenders. When a borrower absconds with her loan, lenders can detect this and can seize up to a fraction $\phi_{t+1} < 1$ of the collateral and sell it to other borrowers. The borrowing constraint is therefore endogenous and given by:

$$d_{t+1} / (1 + r_{t+1}) \leq \phi_{t+1} a_{t+1} p_t.$$

Similar to earlier, we assume $\phi_1 = 1$ and $\phi_{t+1} = \phi < 1$ for each $t \geq 1$. Deleveraging may now be driven by two separate forces: a decline in the pledgeability parameter, ϕ_t, and a decline in the price of the collateral asset. We will see shortly that declines in ϕ_t are generally amplified by asset price declines.

In the following, we make two simplifying assumptions. First, starting at date $t = 2$, we assume that the output from the tree is a constant y and there are no further shocks. Second, we let the discount factors of the two households be $\beta^b = \beta^l = \beta$. Together, these two assumptions imply that the economy will be in a steady state starting at date 2 in which debt is constant at $d_t = d_2$ and the asset price and consumption satisfy $p_t = \frac{\beta}{1-\beta} y$, $c_t^b = y + e^* - (1 - \beta) d_2$, $c_t^l = e^* + (1 - \beta) d_2$ for $t \geq 2$ respectively.

We next consider the equilibrium at date 1 at which the asset's dividend is given by some $y_1 \leq y$. As before, if the debt level is sufficiently large, that is, $d_1 > \bar{d}_1$ for some threshold $\bar{d}_1$, then the economy is in a liquidity trap. In particular, borrowers are constrained, $d_2 = \phi p_1$, the interest rate is at zero, $r_2 = 0$, and output is below its efficient level, $e_1 < e^*$. Moreover, the equilibrium is determined by lenders' Euler equation at the zero interest rate:

$$(37) \qquad u' (e_1 + d_1 - \phi p_1) = \beta u' \left(e^* + (1 - \beta) \phi p_1 \right).$$

The difference is that the asset price also enters this equation since higher prices increase the endogenous debt limit, which influences aggregate demand and output. The asset price is in turn characterized by:

$$(38) \qquad p_1 \;=\; MRS \, (e_1, p_1) \cdot p_2 = \frac{u' \left(c_2^b \right)}{(1 - \phi) \, u' \left(c_1^b \right) + \phi \beta u' \left(c_2^b \right)} \cdot \frac{\beta y}{1 - \beta},$$

$$\text{where } \begin{cases} c_2^b = e^* + y - (1 - \beta) \phi p_1 \\ c_1^b = e_1 + y_1 - d_1 + \phi p_1 \end{cases}.$$

This captures that today's asset price is tomorrow's price $p_2 = \frac{\beta y}{1-\beta}$ discounted by the MRS applicable to asset purchases, which in turn reflects that a fraction ϕ of the asset can be purchased with borrowed funds. Since the extent of deleveraging at date 1 is endogenous to p_1, the MRS is itself a function of the asset price p_1. For the implicit asset price equation (38) to have a unique and well-defined solution, it is necessary that the slope of the left-hand side is higher than the slope of the right-hand side, i.e., $p_2 \cdot \partial MRS / \partial p_1 < 1$ (The condition is characterized in terms of fundamental parameters in the online Appendix A). We also observe that $\partial MRS / \partial e_1 > 0$ as higher income

today makes borrowers more willing to buy assets. Therefore the equilibrium asset price defined by the equation is increasing in current income, $dp_1/de_1 > 0$. Furthermore, the asset price is increasing in the exogenous collateral limit, ϕ, which can be understood from a collateral value channel: a higher ϕ implies the asset is more useful in relaxing the borrowing constraint, which raises its price.

The equilibrium is characterized by two equations, (37) and (38), in two unknowns (e_1, p_1). The first equation describes an increasing relation, $e_1^{AD}(p_1)$, that represents the aggregate demand effects of asset prices. Intuitively, a higher price raises the endogenous debt level, which in turn raises aggregate demand and output. The second equation describes the consumer's asset pricing relationship $e_1^{AP}(p_1)$, i.e. it captures the level of income required to support a given asset price. It is also increasing under our earlier assumption on the MRS. Intuitively, higher net income e_1 raises borrower consumption and therefore supports a higher asset price. Any intersection of these two curves, that also satisfies $\partial e_1^{AP}/\partial p_1 > \partial e_1^{AD}/\partial p_1$, is a stable equilibrium.

Consider a marginal increase in the debt level d_1 in this setup. For given p_1, greater deleveraging reduces the output one-for-one as in our earlier analysis, which shifts the aggregate demand curve, $e_1^{AD}(p_1)$, downwards. For given e_1, greater deleveraging also reduces borrowers' consumption and asset prices according to equation (38), which shifts the asset pricing curve, $e_1^{AP}(p_1)$, to the left. Since both curves are upward sloping, the two shifts reinforce one another. In particular, an increase in d_1 strictly reduces output, e_1, and the asset price, p_1. Moreover, e_1 declines more than it would do if the price were to remain constant, and p_1 declines more than it would do if the output were to remain the constant. Thus, the aggregate demand reduction and the fire sale effects of debt exacerbate one another.

To analyze welfare, consider the externalities from leverage, $\frac{\partial V^h}{\partial d_1}$, which can now be written as:

$$
\begin{aligned}
\frac{\partial V^l}{\partial d_1} &= u'\left(c_1^l\right)\frac{\mathbf{d}e_1}{\mathbf{d}d_1}, \\
\frac{\partial V^b}{\partial d_1} &= u'\left(c_1^b\right)\frac{\mathbf{d}e_1}{\mathbf{d}d_1} + \phi\frac{\mathbf{d}p_1}{\mathbf{d}d_1}\left[u'\left(c_1^b\right) - \beta u'\left(c_2^b\right)\right],
\end{aligned}
$$

The expressions for both types of households feature aggregate demand externalities, which are larger than in the absence of fire sale effects. The expression for borrowers additionally features fire-sale externalities. Intuitively, a higher debt level lowers the asset price, which tightens borrowing constraints and reduces borrowers' welfare since $u'\left(c_1^b\right) > \beta u'\left(c_2^b\right)$ in view of the binding borrowing constraints.

It follows that endogenizing the financial constraint as a function of asset prices re-inforces the problems of excessive leverage and underinsurance through two channels. First, it introduces a new adverse feedback loop into the economy between aggregate demand and asset prices. Second, it introduces fire-sale externalities by which greater leverage lowers borrowers' welfare through price effects, in addition to lowering all investors' welfare through aggregate demand effects. These observations suggest that a recession that involves deleveraging and asset fire-sales may be particularly severe and

especially costly from a social point of view. Hence, macroprudential policies might be particularly desirable in the run-up to these episodes.

VII. Conclusion

When borrowers are forced to delever, the interest rate might fail to decline sufficiently to clear the goods market, plunging the economy into a liquidity trap. This paper analyzed the role of preventive policies in the run-up to such episodes. We establish that the competitive equilibrium allocations feature excessive leverage and underinsurance in view of aggregate demand externalities. A planner can improve welfare and implement constrained efficient allocations by using macroprudential policies that restrict debt and incentivize borrowers' insurance. We also show that optimal borrowing taxes and insurance subsidies depend on, among other things, the differences in the MPC out of liquid wealth between borrowers and lenders.

We show that contractionary monetary policy that raises the interest rate cannot implement the constrained efficient allocations in this setting. Moreover, due to general equilibrium effects, this policy can have the unintended consequence of increasing household leverage and exacerbating aggregate demand externalities. That said, a contractionary monetary policy could well be desirable for reasons outside our model. We leave a more complete analysis of preventive monetary policies for future work.

Although we focus on consumption and household leverage, our mechanism also has implications for investment and firms' leverage. Similar to constrained households, firms that are financially constrained have a high propensity to invest (see, for instance, Rauh 2006), especially during a financial crisis (see Campello, Graham and Harvey 2010). Hence, transferring ex-post wealth from borrowing firms to their lenders is likely to decrease investment and aggregate demand. Our main results then suggest that such firms will also borrow too much and purchase too little insurance in the run-up to deleveraging episodes that coincide with a liquidity trap. Just like with households, these inefficiencies can be corrected with macroprudential policies such as debt limits and capital/insurance requirements.

Many macroprudential policies in practice concern banks (or financial institutions) that intermediate funds between ultimate lenders and borrowers. Our analysis can also be extended to provide a justification for some of these policies. A large body of literature in corporate finance has emphasized that banks' net worth affects credit supply, which in turn affects consumption or investment by credit-constrained borrowers. In fact, in some theoretical benchmarks, banks' net worth is interchangeable with borrowers' net worth (see, for instance, Holmstrom and Tirole 1997, or Brunnermeier and Sannikov 2014). An extension of our model with financial intermediation would then suggest that banks, just like borrowers in our current model, would have too much leverage and too little insurance in the run-up to a liquidity trap. There would be some room for macroprudential policies that restrict banks' leverage and risks, precisely because these policies would improve aggregate demand and output during the liquidity trap (see, for example, the discussion in Jeanne and Korinek 2014). We leave a formal analysis of macroprudential regulation of banks in environments with aggregate demand externalities for future work.

A strand of growing literature on financial crises has emphasized various other factors that encourage excessive leverage, including fire-sale externalities, optimism, and moral hazard. Our analysis suggests these distortions are complementary to the aggregate demand externalities that we emphasize. For instance, asset fire sales reduce aggregate demand by tightening borrowing constraints, which in turn exacerbates aggregate demand externalities. Similarly, optimistic beliefs imply households take on excessive leverage and do not want to insure, which makes it more likely that the economy will enter the high-leverage conditions under which aggregate demand externalities matter. An interesting future direction is to investigate further the interaction between various sources of excessive leverage.

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