Foreign Currency Debt, Risk Premia and Macroeconomic Volatility

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Abstract

This paper studies the relationships between foreign currency debt, macroeconomic volatility, and risk premia in a model of a small open emerging market economy. The external value of the local currency is counter-cyclical so that foreign currency debt requires larger repayments than local currency debt in bad states of nature. The level of foreign currency denominated debts therefore affects the volatility of aggregate demand and by extension of the exchange rate. Exchange rate volatility is in turn an important determinant of the risk premium on local currency debt. Finally, this risk premium is a major factor in the choice of local versus foreign currency for emerging market borrowers. The mutual endogeneity of foreign currency debt, risk premia, and macroeconomic volatility creates important feedback effects in the economy: small increases in international risk aversion may entail large amplification effects on macroeconomic volatility since domestic borrowers substitute towards cheaper but riskier foreign currency debt finance.

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1 Introduction

The turmoil in international markets in the aftermath of the Great Financial Crisis of 2008 has served as a stark reminder of the dangers of foreign currency denominated debts. Many emerging economies in Eastern Europe had accumulated large debts in foreign currency during the preceding decade. When boom turned to bust, they experienced severe current account reversals, precipitous declines in economic activity, and strong pressure on their exchange rates.

Foreign currency denominated debts may amplify the response of an economy to macroeconomic shocks because their value is counter-cyclical to the aggregate state of the economy. However, at the same time, macroeconomic volatility is an important factor in determining the portfolio choice of a country. This implies that it is essential for a comprehensive understanding of the macroeconomic dynamics of emerging market economies to account for the mutual endogeneity of their external debt structure and macroeconomic outcomes such as exchange rate volatility and risk premia. Such an understanding is especially important for policymakers who aim to deepen local currency bond markets in order to make their economies less vulnerable to financial instability (Burger et al., 2009).

This paper develops a simple model of the currency denomination of debts in a small open emerging market economy that takes into account the mutual endogeneity of the country’s external debt structure and of macroeconomic outcomes, specifically the volatility of aggregate demand, exchange rates, and the risk premium on local currency debt. We assume the country has access to two internationally-traded assets, a bond denominated in foreign currency and a bond denominated in local currency, capturing the dominance of debt finance in international capital flows.

An integral element of international debt contracts is that one party has to carry exchange rate risk. If the contract is denominated in foreign currency, the emerging market agent carries this risk, which is counter-cyclical and therefore aggravates the impact of aggregate shocks on the agent. If the debt contract is denominated in local currency, international lenders carry the risk and demand a risk premium to compensate them. Naturally, this risk premium is endogenous to the level of macroeconomic volatility.

Figure 1 depicts the average foreign currency exposure of the financial system as well as the standard deviation of output growth for a set of Eastern European countries over the period of 1998 – 2009. While the figure does not imply causality, it suggests

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1See e.g. Devereux and Sutherland (2009a) and Tille and van Wincoop (2010), who develop DSGE models that endogenize countries’ portfolio choices.

2Calvo and Reinhart (2002) document that emerging markets exhibit “fear of floating,” i.e. they attempt to mitigate exchange rate fluctuations so as to reduce the effect of shocks on macroeconomic volatility. However, even if nominal exchange rates are fixed, countries experience fluctuations in their real exchange rates, with important welfare implications. Furthermore, interventions in nominal exchange rates are often subject to limitations in both directions, as e.g. Argentina and China have witnessed over the past decade.

3The foreign currency exposure of the financial system measures the unhedged foreign currency
that higher foreign currency exposure in a country is generally associated with greater output volatility.

The setting in which we analyze this problem is a model of a small open emerging market economy with two goods, tradables and non-tradables, the relative price of which represents the real exchange rate. There are two time periods, labeled 0 and 1. In period 0, a representative domestic agent allocates a pre-determined level of debt held with large international lenders between local currency (linked to the relative price of non-tradable goods) and foreign currency (linked to the price of tradable goods). In period 1, he receives an endowment of tradable and non-tradable goods. An aggregate shock affects the endowment in the tradable sector, and the exchange rate adjusts in order to equilibrate demand and supply.

In case of a negative shock, for example, tradable goods become relatively scarcer and the relative price of tradables to non-tradables rises, i.e. the real exchange rate depreciates. Hence the repayments on local currency debt fall in parallel with aggregate demand, and this provides insurance against consumption risk. The repayments on foreign currency debt, on the other hand, are fixed in terms of tradable goods, i.e. they rise in terms of the composite consumption good when the exchange rate depreciates. If domestic agents have borrowed in foreign currency and a negative shock hits, the large repayments reduce domestic supply of tradables further, leading to an amplification of the initial shock and a further decline in the exchange rate. In case of a positive shock the opposite results apply. Foreign currency debt therefore results in an amplification mechanism that endogenously increases the volatility of domestic consumption and of exposure of all banks operating in the country (i.e. the sum of foreign currency liabilities and foreign currency lending to unhedged domestic borrowers minus foreign currency assets) as a percentage of total bank assets. The data was obtained from Rancière et al. (2010).
the real exchange rate in period 1.

Our analysis proceeds by backward induction: First, we analyze the macroeconomic implications of a given debt structure in period 1. Since exchange rates are countercyclical, foreign currency-denominated debts entail high repayments in low states when exchange rates depreciate, which aggravates the impact of negative economic shocks, and low repayments in high states when exchange rates appreciate, which amplifies the boom that results from positive economic shocks. In short, the greater the fraction of debt denominated in foreign currency, the higher the impact of a given output shock on aggregate demand and the higher macroeconomic volatility. This in turn raises exchange rate volatility and by extension the risk premium on local currency debt that international lenders charge.

Furthermore, these relationships are convex, i.e. the volatility-enhancing effects of foreign currency debt exhibit increasing returns to scale: each additional unit raises the variance of the payoff of the existing stock of debt plus adds a new unit of foreign currency debt. We also show that there exists a natural foreign currency debt limit: as the amount of foreign currency debt in the country’s portfolio approaches that limit, macroeconomic volatility in the economy diverges towards infinity.

Second, we solve for the optimal currency denomination of debts in the economy in period 0, taking into account the implications for period 1 macroeconomic volatility. When individual borrowers choose the currency composition of their debt portfolio, they weigh the expected costs of fluctuations in consumption that result from foreign currency debt against any savings from the interest rate spread between local and foreign currency. This risk-return tradeoff allows us to describe the amount of foreign currency debt held in an emerging market economy as an optimal portfolio decision.

We show that the response of a small open economy to exogenous shocks differs significantly when the feedback channels of our model are taken into account: In response to an increase in international risk aversion, the risk premium that international lenders charge on local currency rises; as a result domestic borrowers substitute from local towards foreign currency debt, raising macroeconomic volatility, which in turn raises the risk premium further, leading to an amplification effect. Small changes in international risk aversion can therefore lead to large changes in emerging market volatility. By contrast, increases in the riskiness of the emerging market lead domestic borrowers to take on more local currency debt so as to be less exposed to shocks. As a result, macroeconomic volatility remains virtually constant, while borrowers’ expected consumption declines mildly because of the larger interest payments on local currency debt.

Our work is closely related to the literature on financial dollarization. Traditionally, this literature has argued that emerging market economies could not borrow in local currency and only have access to foreign currency debt (see e.g. Eichengreen and Hausmann, 2005). Theoretical justifications for this observation are provided e.g. by Burnside et al. (2001), Chamon (2003) and Jeanne (2003). However, Burger et al. (2001) motivate currency mismatches on the basis of moral hazard; Chamon (2003) emphasizes that foreign currency creditors crowd out local currency creditors because they receive...
document that local currency bond markets have developed rapidly over the past decade. This makes a careful analysis of the macroeconomic effects of borrowing in local versus foreign currency an important research and policy question. The three aforementioned theoretical papers assume that international lenders are risk-neutral, and emerging economies borrow in foreign currency debt because of a distortion. By contrast, our model assumes that international lenders charge a risk premium on local currency debt, as documented e.g. in Dodd and Spiegel (2005), which makes emerging market local currency debt more expensive than foreign currency debt. The decision on how much debt to denominate in local versus foreign currency is therefore a trade-off between risk and return.

This makes our work related to the literature on portfolio choice in open economy macroeconomics. Devereux and Sutherland (2009b), for example, study a portfolio model of emerging market economies that captures the experience of many Asian economies that accumulate dollar reserves and experience large FDI inflows. They observe that this combination yields considerable risk-sharing benefits. Coeurdacier and Gourinchas (2009) develop a model of two symmetric developed economies and study the role of bonds in the two different currencies. By contrast, our paper focuses on the role of a small open emerging economy that interacts with large risk-averse international investors. In our model, this risk aversion makes it costly for domestic agents to use local currency for insurance purposes. In a similar vein, Broner et al. (2007) study short-term versus long-term debt in a model of risk-averse international lenders that charge a risk premium on long-term debt.

We capture the exchange rate of an emerging economy in a model of the real exchange, as in Mendoza (2002) or Chamon (2003). Whereas the focus of Mendoza (2002) is on the effect of exchange rate depreciations on credit constraints and the focus of Chamon (2003) is on the implicit violations of creditor seniority during bankruptcies when exchange rates are depreciated, we analyze the portfolio choice of a country between local and foreign currency debt when local currency debt commands a risk premium. Unlike Chamon (2003), we do not explicitly account for the possibility of bankruptcy. While bankruptcy is an important factor in the portfolio dynamics of emerging economies, we show that a strong link between the currency composition of a country’s debts and macroeconomic volatility can be established if bonds are default-free. Bankruptcy can be interpreted as an alternative implicit insurance instrument against the volatility created by foreign currency debt (see e.g. Dubey et al., 2005). Caballero and Krishnamurthy (2001, 2004) present a framework of financial dollarization where local currency debt is interpreted as a claim that can only be pledged to domestic investors. In our framework, by contrast, domestic and foreign currency debt can be held both by domestic and international investors, but the two differ in their return characteristics.

larger payoffs in bankruptcy, which on average occur in crises when exchange rates are depreciated; Jeanne (2003) pinpoints lack of monetary credibility, which may introduce significant noise into the real payoffs of local currency debt, thereby making it undesirable for insurance purposes.
The remainder of this paper is organized as follows. Section 2 describes the model setup. Section 3 describes the equilibrium of the economy in period 1, i.e. the macroeconomic outcomes for a given amount of foreign currency debt, and analyzes the implications for volatility. In section 4 we solve for the optimal amount of local and foreign currency debt, and discuss the response of the economy to a number of exogenous shocks, given optimal portfolio adjustments. Section 5 concludes.

2 Analytical Environment

Let us analyze a small open economy with two time periods denoted \( t = 0 \) and 1.\(^5\) We assume that there are a continuum of mass 1 of domestic agents and a continuum of international lenders, who are large in comparison to the small open economy. Both are risk averse. Their detailed behavior is described in the following two subsections.

Assume that there are two perishable goods, tradables \( T \) and non-tradables \( N \). Tradable goods are the numeraire and can be costlessly moved across borders for international borrowing and lending transactions. Non-tradable goods have to be consumed domestically in the period of their production. We denote the relative price of non-tradables in terms of tradables as \( p_N \), which is a measure for the real exchange rate.\(^6\) Note that the real exchange rate fluctuates even in economies that have pegged or tightly controlled exchange rates. In period 1, the small open economy experiences an aggregate shock \( \omega \in \Omega \), where \( \Omega \) is the set of all possible realizations.

2.1 Domestic Agents

Domestic agents are born in period 0 with a level of external debt \( D \) (denoted in tradable goods). For most applications below, we will assume \( D > 0 \) so as to capture the situation of emerging markets that are net debtors. In that situation, we can interpret \( D \) for example as external borrowing for investment purposes in period 0. However, identical considerations apply for positive levels of wealth, which would correspond to \( D < 0 \).

\(^5\) The interpretation of what constitutes one time period in our model depends on the macroeconomic environment of the country under investigation. For a country that is well integrated in the world economy and never faces borrowing constraints, it makes sense to regard macroeconomic variables such as output or external debt in period 1 as the sum of the discounted stream of these variables over the infinite future. On the other hand, for a country that is liquidity-constrained, the values of output and external debt due in a given period are the relevant variables. Most emerging markets are somewhere in between these two polar cases, though the latter case is the one that best describes the situation of a country facing a financial crisis.

\(^6\) More generally, the real exchange rate \( q \) is defined as the relative price of a home consumption basket in terms of a foreign consumption basket, i.e. \( q = \frac{(p_T)^\ast (p_N)^{1-\sigma}}{(p_T^\ast)^{(p_N)^{1-\sigma}}} \). If purchasing power parity for tradables is assumed, i.e. \( \frac{p_T}{p_T^\ast} = 1 \), the prices of tradable goods cancel out from both baskets. Holding the price of foreign non-tradables constant, the real exchange rate is a monotonic transformation of the price \( p_N \) of home non-tradables relative to tradables.
Domestic agents derive utility from period-1 consumption $C$, which is a composite of tradable consumption $C_T$ and non-tradable consumption $C_N$:

$$U = E\{\hat{u}(C)\} \quad \text{where} \quad C = C_T\sigma C_N^{1-\sigma} \quad (1)$$

$E$ is the expectations operator conditional on information available at period 0. $\hat{u}(\cdot)$ is agents’ utility function, which satisfies the standard conditions $u'' < 0 < u'$. The parameters $\sigma$ and $1 - \sigma$ are the expenditure shares of tradables and non-tradables in the optimal consumption bundle.

In period 1, domestic agents receive an endowment of tradable and non-tradable goods of $(Y^\omega_T, \bar{Y}^\omega_N)$. Tradable output $Y^\omega_T$ is subject to an aggregate productivity shock, which can be thought of as either an external terms-of-trade shock (e.g. a neighboring country’s devaluation, a fall in the world price of exported commodities etc.) or a domestic slowdown in productivity growth in the export sector. Its expected value is $\bar{Y}_T$. The non-tradable endowment is assumed constant at $\bar{Y}_N$.

Domestic agents can borrow or lend abroad. (Since they are all identical and risk averse, there is no scope for a domestic credit market.) We assume that domestic agents have a choice of borrowing from international lenders in either foreign currency debt $F$, which is linked to the price of tradable goods (the numeraire), or in local currency debt $L$, linked to the price of non-tradable goods.

The issuer of one unit of foreign currency debt receives one unit of the tradable good in period $t = 0$, and repays $R_F$ units at $t = 1$. The payoffs of local currency debt are linked to the price of non-tradable goods $p^\omega_N$, but all international borrowing and lending transactions have to occur in tradable goods, since non-tradable goods cannot be moved across borders. We assume that the issuer of one unit of local currency debt obtains $E[p^\omega_N]$ tradable goods (i.e. the equivalent of one unit of the non-tradable good) in period 0 and is obliged to repay $R_LP^\omega_N$ tradable goods (the equivalent of one unit of the non-tradable good plus interest) in period $t = 1$. The gross interest rates $R_F$ and $R_L$ are determined in world markets.

We assume that there are no limits to how much foreign and local currency individual agents can take on, as long as they maintain strictly positive consumption in all states of the world. Furthermore, we assume that the interest rate on local currency debt is greater than on foreign currency debt so as to ensure that the problem has a non-degenerate solution.\(^9\)

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7 Analyzing an endowment economy rather than a production economy allows us to focus our analysis on individuals’ portfolio decisions. This can be justified as a reasonable approximation of short-run dynamics – in the short run, production factors cannot be re-allocated. Instead, relative prices have to adjust, i.e. the real exchange rate has to appreciate or depreciate.

8 Note that this setup implies that there are no endogenous effects of financial conditions on output, as would be observed e.g. in the case of a credit crunch or default.

9 If this assumption was not satisfied, Korinek (2009b) shows that domestic agents would have incentives to take on an infinite amount of local currency debt and infinite holdings of foreign currency reserves so as to obtain full insurance against domestic shocks.
We summarize the optimization program of domestic agents as

\[
\max_{\{C_T^\omega, C_N^\omega, F, L\}} E \left\{ u\left( [C_T^\omega]^{\sigma} [C_N^\omega]^{1-\sigma} \right) \right\}
\]

\[\text{s.t. } F + E[p_N^\omega]L = D \tag{3}\]

\[C_T^\omega + p_N^\omega C_N^\omega = Y_T^\omega + p_N^\omega \bar{Y}_N - R_F F - p_N^\omega R_L L \tag{4}\]

The budget constraint \((3)\) captures that the debt obligation \(D\) has to be covered by new borrowing in foreign and local currency debt \(F\) and \(L\) in period 0. The budget constraint \((4)\) expresses that total consumption equals total endowment income minus debt repayments in foreign and local currency in period 1.

### 2.2 International Lenders

International lenders are large in comparison to the small open economy. By implication equilibrium in international capital markets is exogenous. In the given context, this implies that the economy faces an exogenously given pricing kernel \(M^\omega\) of international lenders.\(^{10}\)

For international capital markets to be in equilibrium, the expected return \(R_i\) of any asset \(i\) with a price distribution \(\{p_i^\omega\}\) must obey the pricing condition

\[R_i E[p_i^\omega M^\omega] = E[p_i^\omega] \tag{5}\]

The expectation on the left-hand side of this expression captures the valuation of the risky payoff \(p_i^\omega\) by international investors. In equilibrium, this valuation has to equal the expected payoff of the payoff \(E[p_i^\omega]\) discounted by the required rate of return \(R_i\).

### 2.3 Definition of Equilibrium

In accordance with the small economy assumption, we take equilibrium in international capital markets as predetermined. Given international lenders’ pricing kernel \(M^\omega\), an equilibrium in the emerging market economy is defined as

- an allocation \((C_T^\omega, C_N^\omega, F, L)\) and
- prices and returns \((p_N^\omega, R_F, R_L)\) for all \(\omega \in \Omega\)
- which are consistent with international lenders’ pricing condition \((5)\)
- which satisfy domestic agents’ optimization problem \((2)\)
- and which clear goods markets in all states \(\omega \in \Omega\), i.e.
  - for tradables in period 0: \(F + E[p_N^\omega]L \geq D\)
  - and period 1: \(C_T^\omega = Y_T^\omega - R_F F - p_N^\omega R_L L\)
  - as well as for non-tradables in period 1: \(C_N^\omega = \bar{Y}_N\)

\(^{10}\)This pricing kernel, or stochastic discount factor, can be derived from any asset pricing model, such as for example a consumption-based asset pricing model.
2.4 Lenders’ Equilibrium Returns

In equilibrium, the return on any internationally traded asset satisfies lenders’ optimality condition (5). This equation pins down the risk-free return that international lenders demand on foreign currency debt as $R_F = 1/E[M^\omega]$. From the point of view of the small open economy, lenders’ supply of foreign currency debt is infinitely elastic at that interest rate.

Let us denote the risk premium that international lenders require for holding local currency debt as $\rho$, which we define so that $(1 - \rho)R_L = R_F$. Substituting this expression into lenders’ asset pricing condition $R_L E[p_N^\omega M^\omega] = E[p_N^\omega]$, we find that

$$\rho = -R_F \text{Cov} \left( \frac{p_N^\omega}{E[p_N^\omega]}, M^\omega \right)$$

If the pricing kernel $M^\omega$ is constant (risk-neutral lenders) or if exchange rate risk is uncorrelated to $M^\omega$, then $\rho = 0$ and international lenders would supply local currency debt at actuarially fair prices $R_L = R_F$. On the other hand, if lenders are risk-averse (i.e. the pricing kernel is a non-degenerate random variable) and if the real exchange rate $p^\omega N$ is negatively correlated with their pricing kernel, then lenders charge a positive risk premium $\rho > 0$ and the required return on local currency debt is higher than that on foreign currency debt. This second case is our focus here.

2.5 Domestic Agents in Equilibrium

In competitive equilibrium, representative agents solve optimization problem (2). We substitute the constraint for tradable consumption $C_T^\omega$ and for foreign currency borrowing $F$ to arrive at the following maximization problem:

$$\max_{C_N^\omega, L} E \left\{ \hat{u} \left( Y_T^\omega + p_N^\omega (\bar{Y}_N - C_N^\omega) - R_F D - R_L L (p_N^\omega - (1 - \rho)E[p_N^\omega]) \right) \right\}$$

Small agents take the level of the real exchange rate $p_N^\omega$ as well as the expected returns $R_F$ and $R_L$ as given. We obtain the following first-order conditions

$$\text{FOC}(C_N^\omega): \hat{u}'(C^\omega) \left( \sigma [C_T^\omega]^{\sigma - 1} (-p_N^\omega) [C_N^\omega]^{1 - \sigma} + [C_T^\omega]^{\sigma} (1 - \sigma) [C_N^\omega]^{-\sigma} \right) = 0$$

$$\text{FOC}(L): \ E \left\{ \hat{u}'(C^\omega) \sigma [C_T^\omega]^{\sigma - 1} [C_N^\omega]^{1 - \sigma} R_L \left[ p_N^\omega - (1 - \rho)E[p_N^\omega] \right] \right\} = 0$$

In the following subsection, we use the expression FOC($C_N^\omega$) to obtain an expression for the real exchange rate in response to the output shock. In section 3 this allows us to characterize equilibrium in the economy in period 1, given a choice $L$ of foreign and local currency debt in period 0, and to discuss the response of a small open economy to macroeconomic shocks as a function of private borrowing choices.
The second condition $\text{FOC}(L)$ describes private agents’ insurance decision, i.e. the optimal tradeoff between costly local currency debt and risky foreign currency debt in period 0, before the output shock is realized. Using this condition we can solve for the optimal $L$ and for the general equilibrium of the economy in subsection 4.

2.6 Exchange Rate Determination

The first order condition on $C^\omega_N$ characterizes equilibrium in the market for non-tradable goods. For the market to clear, non-tradable consumption has to equal the constant non-tradable endowment, $C^\omega_N = \bar{Y}_N$. Changes in aggregate demand thus affect the price rather than the quantity of goods in the non-tradable sector. From the first order condition $\text{FOC}(C^\omega_N)$, we can obtain the equilibrium level of the real exchange rate

$$p^\omega_N = \frac{1 - \sigma}{\sigma} \cdot \frac{C^\sigma_T}{\bar{Y}_N} = \psi C^\sigma_T$$

(8)

The real exchange rate $p^\omega_N$ is thus linear in the economy’s consumption of tradables $C^\sigma_T$, which we can interpret as a measure of aggregate demand. (We define the constant $\psi = \frac{1 - \sigma}{\sigma \bar{Y}_N}$ to save on notation.) Positive shocks to productivity in the tradable sector increase the relative availability of tradable goods, which depresses the relative price of tradables and increases the relative price of non-tradable goods. Opposite results hold for negative shocks to the tradable sector. While the cyclicity of the exchange rate in developed economies is often ambiguous, the pro-cyclicality of $p_N$ in our model reflects the empirical regularity that exchange rates in emerging economies tend to depreciate when the economy experiences negative shocks.

We confirmed that this finding is empirically valid by calculating the correlation between the change in an index of the real exchange rate and the change in GDP for all economies that are classified as emerging by MSCI and for which the relevant IFS data is available. The (unweighted) average correlation between the two variables is +0.11, which is statistically significant, whereas the average correlation for the population of all countries in the world for which data is available is -0.02, which is not statistically distinguishable from zero. Our finding is also consistent with a wide range of theoretical models of exchange rate determination in small open economies (see e.g. Obstfeld and Rogoff 1996).

The linearity of the relationship is a result of the Cobb-Douglas form in which the two goods enter the utility function: Agents spend constant fractions on each good, i.e. $\sigma p^\omega_N C^\omega_N = (1 - \sigma)C^\sigma_T$; if a positive shock increases tradable consumption, but non-tradable consumption is fixed, then the price of non-tradable goods increases such that the value of non-tradable consumption rises in proportion to tradable consumption.

Note that the situation is, however, different in models of large economies in which the exchange rate is modeled as a country’s terms-of-trade. In such models (see e.g. Coeurdacier et al. 2010) a positive shock to one country’s output increases the supply of the country’s output and therefore deteriorates its terms of trade. In our model of a small open economy, world prices and therefore the economy’s terms-of-trade are by definition exogenous.
This simple analytical form allows us to derive a number of results in closed form in the remainder of the paper. However, our main insights are robust to different specifications of the exchange rate, as long as the external value of the local currency is procyclical.

3 Macroeconomic Effects of Foreign Currency Debt

3.1 Amplification of Aggregate Shocks

We solve the equilibrium of the economy using backward induction. Let us start by analyzing the equilibrium in period 1, given a pre-determined amount of foreign and local currency debt, as captured by the variable $L$. The market clearing conditions for tradable goods imply that tradable consumption in period 1 satisfies

$$C_T^\omega = Y_T^\omega - R_F D - R_L L \{ p_N^\omega - (1-\rho) E[p_N^\omega] \}$$

(9)

The term in square brackets can be interpreted as the insurance payoff per unit of local currency. For any given output shock $Y_T^\omega$ and any amount of local currency debt $L$, we solve the two equations (8) and (9) for the two unknowns $C_T^\omega$ and $p_N^\omega$. Recall that we assumed the total level of debt as constant in our analysis, implying that an increase in $L$ represents a shift from foreign to local currency debt within the given stock of debt.\(^{12}\)

We substitute the exchange rate from (8) into the equilibrium for the tradable goods sector (9) to obtain

$$C_T^\omega = Y_T^\omega - R_F D - \psi R_L L \{ C_T^\omega - (1-\rho) E[C_T^\omega] \}$$

(10)

Naturally, in the absence of local currency debt ($L = 0$), consumption equals production plus initial wealth. For non-zero $L$, the term in square brackets captures that local currency debt involves selling the risky consumption stream $C_T^\omega$ in exchange for lenders’ certainty equivalent $(1-\rho) E[C_T^\omega]$. Solving for $C_T^\omega$ we find

$$C_T^\omega = \frac{Y_T^\omega - R_F D + \psi R_L L \cdot (1-\rho) E[C_T^\omega]}{1 + \psi R_L L} \quad \text{where} \quad E[C_T^\omega] = \frac{Y_T - R_F D}{1 + \rho \psi R_L L}$$

(11)

The term $\psi R_L L = \frac{E[p_N^\omega | R_L L]}{E[C_T^\omega]}$ represents the repayments on local currency debt as a proportion of tradable consumption.

**Proposition 1 (Mitigation/amplification of aggregate shocks)** The higher the share of local currency debt $L$ in an economy, the less the response of consumption $C_T^\omega$ to a given output shock $Y_T^\omega$.

\(^{12}\)The level of foreign currency debt is not bounded at zero; a ‘negative amount of foreign currency debt’ would correspond to foreign currency reserve holdings.
The response of consumption to output shocks is given by

\[ \frac{dC_T^\omega}{dY_T^\omega} = \frac{1}{1 + \psi R_L L} \]  \hspace{1cm} (12)

This derivative is decreasing in the amount of local currency debt \( L \). Consequently, a given output shock has a smaller impact on consumption the greater the amount of local currency debt in the economy. If the economy has taken on a positive amount of local currency debt \( (L > 0) \), the effect of output shocks on consumption is mitigated. On the other hand, the more external foreign currency debt the economy has taken on while investing in local currency \( (L < 0) \), the more the effect of output shocks is amplified.

The expression captures one further effect that is worth pointing out: the less open an economy as captured by a low tradable share \( \sigma \) in consumption, the stronger the effects of local currency debt in either direction. Specifically, the constant \( \psi = \frac{1-\sigma}{\sigma Y_N} \) is declining in the share \( \sigma \) of tradable goods in consumption; therefore the effect of local currency debt \( L \) in equation (12) is stronger the less open the economy to tradable goods, as captured by a small value of \( \sigma \). In particular, for positive amounts of local currency debt, the economy will experience more shock mitigation if the economy is open; for external reserve holdings denominated in local currency \( (L < 0) \) the economy will experience more shock amplification if the economy is relatively open than if it is relatively closed.

Equation (11) suggests that we could express consumption \( C_T^\omega \) as an affine combination of the output shock plus initial wealth \( (Y_T^\omega - R_F D) \) and the certainty equivalent of consumption \( (1 - \rho)E[C_T^\omega] \) with weights \( \alpha \) and \( (1 - \alpha) \), where \( \alpha \) is identical to the derivative (12):

\[ C_T^\omega = \alpha \cdot (Y_T^\omega - R_F D) + (1 - \alpha) \cdot (1 - \rho)E[C_T^\omega] \]  \hspace{1cm} (13)

If \( L > 0 \), then some of borrowers’ debts are denominated in local currency, which moves in parallel with aggregate consumption. This implies that \( \alpha \in (0, 1) \) and that consumption is a convex combination of \( (Y_T^\omega - R_F D) \) and \( (1 - \rho)E[C_T^\omega] \); therefore output shocks are attenuated. Full insurance \( (\alpha = 0) \) could only be reached in the limit as \( L \to \infty \) (Korinek, 2009b). Intuitively, the reason is that each additional unit of local currency debt reduces exchange rate volatility, and thus reduces the insurance effect of the existing stock of local currency debt – there are decreasing returns to insurance, and no finite amount of local currency debt can result in full insurance in the given framework.

On the other hand, the case \( L < 0 \) describes emerging market economies that invest in assets that pay off in local currency and borrow in foreign currency \( F > D \) to pay for these. Borrowers effectively leverage their exposure to the domestic output shock: their weight \( \alpha \) on the output shock is greater than one, or \( dC_T^\omega/dY_T^\omega > 1 \). Taking on foreign currency debt therefore amplifies the effects of aggregate fluctuations. Intuitively, a
Figure 2: The figure solves for the equilibrium exchange rate and equilibrium level of tradable consumption as a function of several levels of the productivity shock $Y^L$, $\bar{Y}$, $Y^H$ and of local currency borrowing (i) $L = 0$, (ii) $L > 0$, and (iii) $L < 0$.

A marginal reduction in $L$ raises exchange rate volatility; this raises the destabilizing effect of the existing stock of foreign currency debt while adding an additional unit of such debt. The destabilizing effects of foreign currency debt are thus subject to increasing returns to scale, up to a natural foreign currency debt limit, which will be characterized in proposition 2.

Figure 2 depicts our findings graphically. The left panel illustrates the case without local currency debt. The diagonal $\psi C_T^\omega = p^\omega N$ represents all combinations of tradable consumption and the real exchange rate for which equilibrium in the non-tradable market prevails according to condition (8). The three horizontal lines labeled $Y^L$, $\bar{Y}$, $Y^H$ depict equilibrium condition (9) for the tradable sector for three different levels of the output shock $Y^\omega_T$. In the absence of local currency debt, $L = 0$ and the country’s entire external debt stock is denominated in foreign currency, i.e. $F = D$. The level of tradable consumption in equation (9) is not affected by the exchange rate; therefore the three lines are horizontal. The level of the output shock $Y^\omega_T$ uniquely determines tradable consumption $C_T^\omega$. The real exchange rate adjusts to clear the market for non-tradable goods as given by (8).

The center panel, by contrast, depicts the situation when private agents have taken on a positive amount of local currency debt, i.e. $L > 0$ and $F < D$. When the exchange rate depreciates (low $p^\omega_N$), the repayments on local currency debt are low, implying a higher level of consumption. Conversely, when the exchange rate appreciates, repayments are higher, implying a lower level of consumption. The equilibrium condition (9) for the tradable sector is therefore represented by downward sloping lines. As a result, the impact of productivity shocks (i.e. vertical shifts in this equilibrium condition) on tradable consumption is mitigated: part of the shock is absorbed by the insurance effects of local currency debt; only the remainder shows up in fluctuations in consumption $C_T^\omega$.

Finally, the right panel focuses on the impact of foreign currency debts coupled
with long positions in local currency, i.e. investments that pay off in terms of local currency, implying \( L < 0 \) and \( F > D \). When the exchange rate \( p^*_N \) falls, the value of agents’ local currency earnings falls, but they have to repay uncontingent foreign currency debts. This contracts their consumption. Conversely, when the exchange rate \( p^*_N \) appreciates, their return on local currency holdings rises and they can consume more. The equilibrium condition \([6]\) for the tradable sector is thus represented by upward-sloping lines. In equilibrium, the impact of output shocks \( Y^*_T \) on consumption \( C^*_T \) is amplified by the effects of exchange rate movements on agents’ debt burden.

### 3.2 Natural Foreign Currency Debt Limit

We may also express the equilibrium level of consumption in terms of foreign currency debt \( F \) rather than local currency debt \( L \):

\[
C^*_T = \frac{Y^*_T - RF}{\bar{Y}_T - RF} \cdot E[C^*_T] \quad (14)
\]

The term \( Y^*_T - RF \) captures the actual availability of tradables in the economy after uncontingent debt repayments have been made, whereas \( \bar{Y}_T - RF \) captures the expected level thereof. Consumption is scaled up or down from its expected level \( E[C^*_T] \) in parallel to the fraction of these two variables. The amplification effects of foreign currency debt are then captured by the derivative

\[
\frac{dC^*_T/E[C^*_T]}{dY^*_T/\bar{Y}_T} = \frac{1}{1 - RF/\bar{Y}_T} \quad (15)
\]

For \( F = 0 \), i.e. in the absence of foreign currency debt, the output shock and consumption would always increase/decrease by identical fractions; for \( F < 0 \), i.e. for positive net holdings of foreign currency reserves, the percentage impact of the output shock on consumption is less than one, i.e. the output shock is attenuated; for positive amounts of foreign currency debt \( F > 0 \) the derivative is greater than one, i.e. the percentage deviation of consumption is greater than the percentage deviation of output caused by the underlying shock.

**Proposition 2 (Natural Foreign Currency Debt Limit)** The expected tradable output \( \bar{Y}_T \) forms the natural foreign currency debt of a country. As foreign currency debt approaches this limit \( RF \rightarrow \bar{Y}_T \), consumption volatility diverges towards infinity.

When \( RF \rightarrow \bar{Y}_T \), the country has promised its entire expected tradable production to foreigners in the form of uncontingent foreign currency debt. From derivative \([15]\) it is easy to see that the response of consumption to the output shock goes towards infinity in that case, i.e. borrowers have infinitely leveraged their exposure to the output shock – the country is ‘betting the house.’ This corresponds to a long position in local currency debt of \(-\psi RL \rightarrow 1\), which is equivalent to \(-p^*_N RL \rightarrow C^*_T\). (Recall that negative positions of local currency debt means positive amounts of investments with
payoffs denominated in local currency debt.) In other words, any tradable consumption would be financed by the contingent payoffs of local currency reserves, or equivalently, the valuation of local currency reserves is identical the level of tradable consumption.

In the limiting case of \( R_F = \bar{Y}_T \), a below-average realization of the shock to tradable production \( Y^\omega_T < \bar{Y}_T = R_F F \) would result in a debt crisis: The country’s endowment of tradable goods is insufficient to cover its foreign currency debts; therefore there is a severe shortage of tradable goods in the economy and the relative price of non-tradables plummets; this implies that the value of local currency bonds falls without bounds, and the returns on local currency investments cannot be used to cover the shortfall of tradable goods. The severe declines in the real exchange rate and in tradable consumption and the parallel rise in the country’s current account surplus are reminiscent of the empirical regularities of sudden stops (Calvo et al., 2004). While we do not formally incorporate default in our framework, this situation will in practice likely lead to default.

On the other hand, an above-average shock \( Y^\omega_T > \bar{Y}_T = R_F F \) would imply that, after the country repays foreign currency creditors, there is still a positive amount of tradable production \( Y^\omega_T - \bar{Y}_T > 0 \) left; tradables are abundant in the economy and the real exchange rate would rise sharply, causing a severe increase in the payoffs on local currency investments and resulting in a strong boom that further appreciates the exchange rate. The (degenerate) equilibrium in such an economy would imply that \( p^\omega_N \to \infty, C^\omega_T \to \infty \).

The knife-edge case of \( Y^\omega_T = \bar{Y}_T = R_F F \) would entail that the country’s entire tradable production is precisely sufficient to repay their foreign currency debts, and that the income on local currency reserves can be consumed. However, there would be a continuum of equilibria: any level of the real exchange rate \( p^\omega_N \) and corresponding tradable consumption \( C^\omega_T = \frac{p^\omega_N}{\psi} \) would fulfil the equilibrium conditions. Whenever consumers raise (reduce) consumption, the real exchange rate appreciates (appreciates) by the same factor, and the payoffs on local currency investments adjust so as to precisely cover their expenditure on tradable goods.

In panel (ii) of figure 2, the case \( \psi R L L \to -1 \) would correspond to the three lines representing equilibria in the market for tradable goods for \( Y^\omega_T = Y^H, \bar{Y}, Y^L \) having a positive slope of \( \psi \) – identical to the slope of the equilibrium condition for the non-tradable market. It is easy to see that for \( Y^\omega_T = \bar{Y}_T \) this results in a continuum of equilibria, whereas for \( Y^\omega_T \neq \bar{Y}_T \) no intersection between the two equilibrium conditions exists for finite levels of \( p^\omega_N, C^\omega_T \).

An important feature of the natural limit to foreign currency debt is that it is independent of the total amount of debt in the economy. Regardless of the magnitude

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13 Algebraically, the (degenerate) equilibrium would entail that \( p^\omega_N \to -\infty, C^\omega_T \to -\infty \).

14 The underlying economic dynamics are as follows: consumers increase consumption in an attempt to use up their tradable income \( Y^\omega_T - \bar{Y}_T - p_N R_L L \), but in doing so they appreciate the real exchange rate so much that their income from local currency investments \( p_N R_L L \) rises by as much as their additional expenditure. Tradable consumption can never catch up with local currency income, implying that the equilibrium is degenerate at \( \infty \).
or sign of the initial level of indebtedness $D$ of the emerging market economy, the total amount of foreign currency debt is always limited by the expected endowment of tradables $\bar{Y}_T$. In other words, the vulnerability of an economy to external debt crises depends more on its external foreign currency debt than its total external debt, since local currency provides insurance that mitigates any adverse aggregate shocks to the economy.

Note that our analysis also provides support for the common argument that an economy’s vulnerability to shocks such as sudden stops in capital inflows decreases the more open it is to trade (see e.g. Calvo et al. 2004): the larger the tradable endowment $\bar{Y}_T$ of the economy, the higher the natural limit on foreign currency debt. Similarly, equation (15) implies that for a given stock of foreign currency debt $F$, a larger tradable endowment $\bar{Y}_T$ entails a smaller percentage impact of output shocks on consumption.

### 3.3 Volatility and Risk Premia

A direct implication of proposition 1 on the mitigation/amplification of aggregate shocks is the following result:

**Proposition 3 (Consumption and exchange rate volatility)** Consumption volatility and exchange rate volatility fall in the amount of local currency debt $L$.

The same applies to exchange rate volatility, since the exchange rate is a linear transformation of tradable consumption. Formally, it follows from equation (11) that

$$\text{Var}(C_T) = \frac{\text{Var}(Y_T)}{(1 + \psi R_L)^2}$$
$$\text{Var}(p^\omega_N) = \frac{\psi^2 \text{Var}(Y_T)}{(1 + \psi R_L)^2}$$

(16)

It is interesting to note that the marginal insurance effect of local currency debt is declining, i.e. the insurance benefits of local currency debt have decreasing returns to scale: The more local currency debt in the economy, the less the exchange rate fluctuates and hence the lower the marginal insurance effect of an additional unit of local currency. In other words, the functional relationship between the variance of consumption and the amount of local currency debt is decreasing and convex.

By the same token, the more external foreign currency debt an economy is exposed to, the higher the marginal increase in risk brought on by an additional unit of foreign currency debt. (Recall that $F = D - E[p_N^\omega]L$, i.e. the two are negatively related.) Each additional unit of foreign currency debt raises exchange rate volatility and therefore increases the riskiness of the existing stock of foreign currency debt. When the natural foreign currency debt limit as discussed in proposition 2 is reached, consumption and exchange rate volatility diverge towards infinity.

**Proposition 4 (Risk premium on local currency debt)** The risk premium $\rho$ falls in the amount of local currency debt $L$. 


The risk premium $\rho$ on local currency debt is a result of lenders’ aversion to exchange rate volatility. More insurance in the form of local currency debt $L$ entails more stable exchange rates and therefore a smaller risk premium. To prove this, we take the following derivative:

$$\frac{d\rho}{dL} = -\rho(1 - \rho)\frac{\psi R_L R_F}{1 + \psi R_L L} < 0$$

This has an important implication: the supply of local currency debt is horizontal from the point of view of decentralized agents since they are small. However, for the aggregate small open emerging market economy, the supply of local currency debt is downward-sloping, as large lenders adjust the risk premium to the decreasing level of macroeconomic volatility when the aggregate amount of local currency debt to the economy increases.

**Proposition 5 (Current account)** The current account balance covaries positively with the output shock if $L > 0$ and negatively if $L < 0$.

We can write the economy’s current account balance as

$$CA^\omega = R_F F + p_N^\omega R_L L$$

In our model, the current account in period 1 simply reflects the repayments on external debt. Note that in the absence of local currency debt, the current account is deterministic since the payoffs of foreign currency debt are uncontingent. However, when local currency debt is held, the pro-cyclical fluctuations in the real exchange rate and in repayments are also reflected in the current account balance. The covariance of the current account with the output shock is therefore

$$\text{Cov}(CA^\omega, Y^\omega_T) = \psi R_L L \frac{\text{Cov}(C^\omega_T, Y^\omega_T)}{1 + \psi R_L L} \text{Var}(Y^\omega_T)$$

For positive values of local currency debt $L$, fluctuations in the current account mitigate the impact of output shocks on consumption: during negative shocks, repayments to foreigners fall and the current account balance is reduced, and conversely for positive shocks. For negative values of $L$, i.e. high levels of foreign currency debt, fluctuations in the current account amplify the impact of output shocks. The more foreign currency debt a country has contracted, the more severe the reaction of the current account to output shocks.

**Proposition 6 (Expected consumption)** Expected consumption declines as the amount of local currency debt $L$ rises.

Local currency debt carries a risk premium, i.e. insurance is costly. As indicated by the second part of equation (11) the expected level of tradable consumption equals
total expected tradable wealth divided by the term $1 + \rho \psi R L$, which captures the insurance premium paid to international lenders. If either $L = 0$ or $\rho = 0$ then expected consumption would equal expected total wealth. If both $\rho$ and $L$ are positive, then the payment of the insurance premium reduces the expected value of consumption.

$$\frac{dE[C_\omega^T]}{dL} = -\frac{\rho \psi R L E[C_\omega^T]}{1 + \psi R L} < 0$$

For negative $L$, domestic borrowers would earn a risk premium for providing insurance to international lenders.

### 4 Choice of Currency Denomination

Having characterized the equilibrium in the economy for period 1, given pre-determined levels $L$ and $F$ of local and foreign currency debt, we now turn to solving for the optimal choice of $L$ and $F$ in period 0. Before, however, let us take advantage of the market-clearing condition for the non-tradable sector and the fixed supply of $\bar{Y}_N$ so as to simplify our notation of the utility function $\hat{u}(\cdot)$ into a transformed utility function

$$u(C_T) = \hat{u}(C_T^{\sigma}C_N^{1-\sigma}) = \hat{u}(C_T^{\sigma}\bar{Y}_N^{1-\sigma})$$

Using this notation the first order condition FOC($L$) reads as

$$E\left\{u'(C_\omega^T)\left[p_\omega^N - (1 - \rho)E[p_\omega^N]\right]\right\} = 0$$

In equilibrium, a borrower’s valuation of one unit of local currency with payoff $p_\omega^N$ has to equal his valuation of the certainty equivalent, i.e. of the expected value $E[p_\omega^N]$ discounted by lenders’ risk premium $\rho$, where the borrower takes both the distribution of exchange rates $\{p_\omega^N\}$ and the risk premium $\rho$ as given. This can be rewritten as the standard insurance condition

$$-\text{Cov}\left\{\frac{u'(C_\omega^T)}{E[u'(C_\omega^T)]}, \frac{p_\omega^N}{E[p_\omega^N]}\right\} = \rho$$

Domestic agents hold local currency debt up to the point where the additional insurance effect per unit of local currency debt, as represented by the covariance term, equals the cost of obtaining the insurance, which is the risk premium on local currency. The higher this risk premium, the less insurance agents take on.

Substituting for the equilibrium exchange rate $\hat{p}_N^{\omega} = \psi C_T^{\omega}$ from (8) and approximating the utility function $u(C_T)$ to the second order yields the condition

$$- u''(E[C_\omega^T]) \cdot \text{Var}(C_\omega^T) = \rho E[C_\omega^T]u'(E[C_\omega^T])$$

Note that if the underlying output shock $Y_\omega^T$ is normally distributed, we could apply Stein’s lemma to obtain $\text{Cov}(u'(C_\omega^T), C_\omega^T) = E[u''(C_\omega^T)] \cdot \text{Var}(C_\omega^T)$; there would be no need for using an approximation for the covariance term.
Figure 3: The left panel of the figure depicts the supply locus $SS$ and demand locus $DD$ in a diagram of the risk premium on local currency-denominated debt against consumption volatility in the economy. The right panel shows the same two loci in a standard demand/supply diagram of local currency-denominated debt. The equilibrium $E$ is determined by the intersection of the two curves.

The optimal variance of tradable consumption is directly proportional to the risk premium on local currency debt, i.e. to the cost of insuring against volatility. For a positive risk premium $\rho > 0$ on local currency debt, domestic agents insure imperfectly. They opt for higher consumption volatility the higher the risk premium on local currency debt. In the case $\rho = 0$, the optimal variance of tradable consumption would be zero, i.e. domestic agents would insure perfectly against consumption fluctuations, but this would require an infinite amount of $L$, as we discussed before.

Equation (18) can also be expressed in terms of the coefficient of relative risk aversion $R$ of the agent’s utility function evaluated at $E[C_T^\omega]$:

$$\rho = R \cdot \frac{\text{Var}(C_T^\omega)}{E[C_T^\omega]^2}$$  \hspace{1cm} (19)

In this formulation, the risk premium in equilibrium has to equal the agent’s coefficient of relative risk times the variance per unit of consumption.

### 4.1 Solution to the Decentralized Equilibrium

The solution to the decentralized equilibrium is given by the 4 equations (6), (8), (10) and (19) in the 4 variables $(\rho, L, p^*_N, C_T^\omega)$, of which the last two are state-contingent. Condition (3) then yields the optimal $F$, and the market clearing condition for non-tradables implies $C_N^\omega = \bar{Y}_N$.

We can collapse the system of four into a system of two equations in two variables, $\rho$ and $\text{Var}(C_T^\omega)$. The first equation is derived from lenders’ supply condition (6), which states that the risk premium that international lenders charge on local currency debt is a function of the covariance of the exchange rate with lenders’ pricing kernel. Since
the exchange rate is linear in domestic tradable consumption $C^\omega_T$ as captured by (8), we obtain

$$\rho = -R_F \frac{\text{Cov}(C^\omega_T, M^\omega)}{E[C^\omega_T]} = \xi \cdot \frac{\text{Std}(C^\omega_T)}{E[C^\omega_T]} \quad (20)$$

where we define

$$\xi = -R_F \cdot \frac{\text{Cov}(Y^\omega_T, M^\omega)}{\text{Std}(Y^\omega_T)} \quad (21)$$

$\xi > 0$ is an exogenous constant that captures how much output fluctuations covary with lenders’ pricing kernel, i.e. lenders’ aversion to output risk in the emerging economy. The risk premium that lenders require is thus linear in the standard deviation of consumption: the more consumption and the real exchange rate fluctuate, the higher the premium that international lenders require to hold local currency debt. This is international lenders’ optimality condition or supply locus of local currency debt. An alternative way of interpreting the relationship in the equation is that the risk premium on local currency debt equals the risk inherent in local currency debt times the price of risk of international investors.

In the left panel of figure 3, the relationship is depicted as $SS$. The locus $SS$ is concave since the axis in the figure depicts the variance $\text{Var}(C^\omega_T)$, i.e. the square of the standard deviation.\footnote{The risk premium $\rho$ enters the term $E[C^\omega_T]$ on the left hand side of (20) indirectly, but for typical risk premia the effect on the shape of $SS$ can be neglected.}

The second equation is borrowers’ optimality condition (19). In the left panel of figure 3 this is depicted as the $DD$ locus, which represents all combinations of consumption volatility and risk premia for which agents’ portfolio problem is solved. This locus is approximately linear in $\rho$.\footnote{More precisely, the risk premium $\rho$ enters not only explicitly on the right-hand side of equation (18), but also implicitly in the term $E[C^\omega_T]$: the higher the risk premium, the more borrowers pay for the insurance services of local currency debt, reducing expected consumption. However, for realistic magnitudes, i.e. single digit or low double digit percentages, these indirect effects do not significantly impact the slope of the curve.}

In the left panel of figure 3 the amount of local currency debt (as represented by $L$) increases as we move from the top right of the figure to the bottom left along both the $SS$ and $DD$ loci, as local currency debt reduces volatility in the economy. In the limit, i.e. as $L \to \infty$, both loci end up at the origin. The linearity and concavity of the two schedules guarantee a unique non-degenerate equilibrium $E$ that pins down the optimal level of the risk premium and of volatility in the economy, and therefore the amount of local currency debt $L$.\footnote{An alternative way of establishing the uniqueness of equilibrium would be to focus on the formulation of the problem in terms of the share $\alpha$ of output risk that is insured, as discussed on page 12.}

Alternatively, in the right panel of the figure, we show the same two relationships in a traditional demand/supply diagram. Note that the level of consumption volatility depends inversely on the square of the amount of local currency debt in the economy, as given by equation (16); hence the right panel is an inverted version of the left panel.
The demand curve $DD$ is a quadratic hyperbola and the supply curve $SS$ a regular hyperbola. Supply is downward-sloping because macroeconomic volatility and exchange rate volatility fall as $L$ increases; this induces lenders to demand a smaller risk premium, as discussed in proposition 4.

While figure 3 allowed us to determine the optimum amount of local currency debt in the economy graphically, the solution to the two equations (18) and (20) can be expressed analytically as

$$\text{Var}(C_T) = \xi^2 \cdot \left( \frac{u'(E[C_T])}{u''(E[C_T])} \right)^2 \quad \rho = \xi^2 \cdot \frac{u'(E[C_T])}{u''(E[C_T])}E[C_T]$$

Both the desired level of consumption volatility and the risk premium that domestic agents are willing to pay are inversely related to their level of absolute risk aversion. Note that the resulting level of local currency debt $L$ can be either positive (debt) or negative (reserves) and depends on the risk aversion of domestic agents versus international lenders.

Empirically, Kucuk (2010) finds that local currency bonds in emerging market economies have exhibited average annual excess returns of 1.50% over foreign currency denominated bonds for the period of 2002 – 2009, suggesting that there are significant risk premia. Dodd and Spiegel (2005) report similar findings for 1993 – 2004. However, for reasons of data availability such studies cover only relatively short time horizons, which implies that they may be subject to a small sample bias. Burger et al. (2009) analyze what makes the local currency bond markets of emerging economies attractive for international investors and identify the following factors: high regulatory quality and creditor rights, an established domestic investor base, friendly tax regimes, low taxation of bonds, as well as liquidity and efficiency of bond markets. Higher participation by international investors is in turn likely to drive risk premia in local currency bond markets lower.

### 4.2 Change in International Risk-Aversion

External shocks, in particular those emanating from international capital markets, are viewed as having substantial effects on volatility in emerging markets. In this subsection we investigate the effects of a typical shock in this category, an exogenous increase in risk aversion in international capital markets.

Increases in lenders’ risk aversion raise the cost at which emerging markets can obtain insurance against domestic shocks, i.e. they raise the rate at which lenders supply funds for a given level of exchange rate volatility, as indicated by the $SS$ equilibrium locus (20). In addition, however, they also entail an important amplification effect: if lenders charge a higher risk premium for a given level of exchange rate volatility,

\[19\] Since the emerging market economy is small, the traditional effect that supply is upward-slopping because lenders become saturated with emerging market risk is not present. In a model of a ‘large’ emerging market economy, this traditional effect may counteract the decline in the risk premium that is discussed here.
borrowers demand less insurance, as indicated by the $DD$ equilibrium locus (18). This in turn raises consumption and exchange rate volatility in the economy, leading to further rounds of increases in the risk premium on local currency and consumption volatility.

We can model an increase in lenders’ risk aversion by performing a mean-preserving spread by a factor $\alpha$ on the pricing kernel $M^\omega$:

$$M^\omega_\alpha = M^\omega + \alpha \cdot (M^\omega - E[M^\omega])$$

We have depicted the case of an increase in risk aversion in figure 4. The direct effect, represented by the horizontal movement from $E_1$ to the right, is to shift lenders' optimality locus $SS_1$ by the factor $\alpha$ to the right to $SS_2$: for a given level of exchange rate volatility, lenders demand a risk premium that is by factor $\alpha$ higher than before. However, at lenders' new risk premium, borrowers reduce the amount of local currency that they demand, represented by the vertical upward movement. This results in less insurance and higher consumption and exchange rate volatility. This leads to further rounds of increases in lenders’ risk premium and reductions in borrowers’ demand for local currency. Taking into account these feedback effects, a small increase in international lenders’ risk aversion can lead to much larger increases in risk premia and volatility. When international lenders’ risk aversion falls, the opposite effect takes place: risk premia fall and the amount of insurance in the economy rises in a virtuous cycle.

Analytically, an increase in risk aversion by $\alpha$ scales the constant $\xi$ in lenders’ optimality condition (20) up by the factor $\alpha$, i.e. $\frac{d\xi}{d\alpha} = 1$. The direct effect on the risk premium from the shift of lenders’ $SS$ locus (the horizontal movement from $E_1$ to the right, i.e. holding the amount of insurance $L$ constant) is to increase the risk premium $\rho$ by the same factor:

$$\left.\frac{d\rho}{\rho}\right|_{dL=0} = 1$$
We can use the two equilibrium conditions in (22) to express the total effect on the risk premium and output volatility, taking into account general equilibrium effects, i.e. moving to the new equilibrium \( E_2 \):

\[
\frac{d\rho/\rho}{d\alpha} = 2 \quad \frac{d\text{Var}(C_T^\omega)/\text{Var}(C_T^\omega)}{d\alpha} = 2
\]

In other words, both the risk premium and the variance of consumption increase twice as much as the increase to lenders’ risk aversion.

Changes in international risk aversion often coincide with adverse shocks to the domestic economy in emerging markets. This is especially the case if the marginal investors in emerging economies are specialists who suffer losses when the value of local currencies goes down. In emerging economies that specialize in the production of commodities, global shocks may simultaneously affect the wealth of global investors and – through the effect on commodity prices – domestic output. As we will discuss in the ensuing section, a change in the riskiness of the domestic economy has only minor effects on consumption volatility in the emerging economy.

### 4.3 Changes in the Riskiness of the Domestic Economy

Let us next investigate the effects of an increase in volatility of the output shock in the domestic economy. We model this increase in riskiness by performing a mean-preserving spread by the factor \( \alpha \) on the output shock \( Y_T^\omega \), i.e.

\[
Y_{T,\alpha}^\omega = Y_T^\omega + \alpha(Y_T^\omega - \bar{Y}_T)
\]

However, as a first approximation, the two equilibrium loci \( DD \) and \( SS \) as given by (18) and (20) are unaffected: both domestic agents and international investors still have the same preferences over volatility in consumption and the real exchange rate. The equilibrium after the change in the volatility of the output shock thus exhibits the same risk premium and the same level of consumption volatility, as depicted in the left panel of figure 5. However, to reach this level of consumption volatility, the equilibrium amount of local currency debt \( L \) needs to increase. According to proposition 3

\[
\text{Std}(C_T^\omega) = \frac{\text{Std}(Y_T^\omega)}{1 + \psi R_L L}
\]

Naturally, if \( \text{Std}(Y_T^\omega) \) is scaled up by a factor of \( \alpha \), then the term \( 1 + \psi R_L L \) has to grow by the same factor in order to keep consumption volatility and the risk premium constant.

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\(^{20}\)As discussed in the preceding section, there is a small additional effect: the increase in insurance holdings \( L \) reduces the expected level of consumption \( E[C_T^\omega] \) a little. With decreasing absolute risk aversion, domestic agents would become slightly more risk averse as \( E[C_T^\omega] \) falls; hence they would increase the amount of insurance that they take on even further.
Figure 5: An increase in the volatility of the output shock in the emerging market economy does not change the equilibrium risk premium or consumption volatility as depicted in the left panel. However, the amount of local currency debt required to reach that equilibrium rises, as shown in the right panel.

Proposition 7 An increase in the volatility of the economy’s output shock raises the amount of local currency debt $L$ that agents incur so as to offset the impact of the higher volatility on consumption.

The right panel of figure 5 shows the effects of an increase in the volatility of the output shock in a standard diagram that plots demand and supply of local currency debt against its price: both demand and supply curves would shift right by the same factor. For a given price the amount of local currency debt demanded would rise because there is more need for insurance; the supply of local currency debt would rise because macroeconomic volatility increases. The new equilibrium is to the right of the old equilibrium, at an unchanged interest rate on local currency debt.

As we discussed in section 4.2, shocks to the riskiness of an emerging economy frequently occur in conjunction with shocks to global risk aversion. In such situations, the affected emerging economy cannot fully offset the increase in domestic risk through higher local currency debt issuance.

5 Conclusions

The goal of this paper was to develop a macroeconomic framework that jointly analyzes portfolio decisions and macroeconomic outcomes in emerging markets. One of the key elements of our analysis was aversion to emerging market risk among international investors, which leads to a risk premium on local currency debt.

We showed that foreign currency-denominated debts increase macroeconomic volatility and by extension risk premia on emerging markets. At the same time, individuals base the currency composition of their portfolios on a tradeoff between the risks of foreign currency debt and the higher interest rates on local currency debts.
There are a number of questions that are left for future research. Firstly, we performed our analysis in a simple two-period endowment economy so as to focus on a succinct analysis of the feedback channels between country portfolios and macroeconomic outcomes. In a model with multiple time periods, we would have to account (i) for the different maturities of debt flows and production flows and (ii) for individuals smoothing the wealth effects of changes in the valuation of their debts over time. However, in practice both factors lose much of their relevance precisely in those situations in which our analysis is of particular importance, i.e. when emerging markets experience large negative shocks: (i) in such episodes long-term debt markets typically dry up and the average duration of a country’s external debt portfolio declines sharply (De la Torre and Schmukler, 2004) and (ii) borrowing constraints tighten and prevent agents from smoothing the impact of valuation shocks over time (Calvo, 1998). The basic model presented here is extended in this direction in Korinek (2009a).

Secondly, we modeled international lenders’ aversion to emerging market risk by assuming an exogenous pricing kernel that negatively correlates with the aggregate shock in the emerging market economy. While this is sufficient for the insights presented in this paper, a closer examination of the determinants of lenders’ risk aversion may yield important insights for policy measures to reduce the risk premium on local currency debt, as discussed e.g. in Burger et al. (2009). Our analysis suggests that such measures would in turn incite emerging market borrowers to shift towards more local currency borrowing, which enhances macroeconomic stability and – in a virtuous circle – reduces risk premia further.

References


