B Extensions [Online Only]

B.1 Alternative State-Contingent Constraint Specifications

Let us consider two different specifications of the financial constraint that extend the constraint specification of our baseline model:

B.1.1 Constraints on Issuance of Individual Arrow Securities

First, consider the case that each individual Arrow security is subject to a separate financial constraint that allows domestic agents to issue a market value of state-contingent securities up to a fraction ϕ^{ω} of its income, where the fraction depends on the future state ω on which the security is contingent, as captured by the superscript ω,

$$m_{t+1}^{\omega} b_{t+1}^{\omega} + \phi^{\omega} \left[y_{T,t} + p_t y_{N,t} \right] \geq 0 \qquad \forall \omega$$

(Note that it would be equivalent to impose the constraint on the quantity of securities b_{t+1}^{ω} rather than the value $m_{t+1}^{\omega} b_{t+1}^{\omega}$ since the parameter ϕ^{ω} could simply be adjusted accordingly.)

In the optimization problem of domestic agents, this introduces a collection of constraints on security issuance with multiplier μ_t^{ω} each, giving rise to the Lagrangian

$$\begin{aligned} \mathscr{L} = E \sum_{t=0}^{\infty} \beta^t \Big\{ & u\left(c_{T,t}, c_{N,t}\right) - \lambda_t \left[c_{T,t} - y_{T,t} + \left(c_{N,t} - y_{N,t}\right) p_t + E\left[m_{t+1}^{\omega} b_{t+1}^{\omega} \right] - b_t \right] \\ & + E\left\{ \mu_t^{\omega} \left[m_{t+1}^{\omega} b_{t+1}^{\omega} + \phi^{\omega} \left(y_{T,t} + p_t y_{N,t} \right) \right] \right\} \Big\} \end{aligned}$$

and similarly for the social planner. Observe that the expectation in the second line is just an alternative way of writing the probability-weighted sum across all states of nature, $E\left[\cdot\right] = \int_{\omega} \left[\cdot\right] dP\left(\omega\right)$.

This leaves all the optimality conditions of private agents unchanged, except that it changes their state-contingent Euler equation (6) by making the shadow price on the financial constraint state-contingent,

$$FOC\left(b_{t+1}^{\omega}\right): \quad m_{t+1}^{\omega} \left(u_{T,t} - \mu_t^{\omega}\right) = \beta u_{T,t+1}^{\omega}$$

This reflects that domestic agents may be constrained in their security issuance contingent on some states of nature but not others. When domestic agents exhaust their security issuance against one state of nature, they may still be able to issue further securities against other states of nature. If domestic agents are subject to state-contingent taxes on their state-contingent security issuance, their Euler equations modify to $\left(m_{t+1}^{\omega} - \tau_{t+1}^{\omega}\right) u_{T,t} - m_{t+1}^{\omega} \mu_t^{\omega} = \beta u_{T,t+1}^{\omega}$.

In the social planner's problem, both the optimality condition on traded consumption and the generalized Euler equation are modified to

$$\begin{aligned} FOC\left(c_{T,t}\right) \quad &: \quad \tilde{\lambda}_t = u_{T,t} + E\left[\phi^{\omega} \tilde{\mu}_t^{\omega}\right] p'\left(c_{T,t}/y_{N,t}\right) \\ FOC\left(b_{t+1}^{\omega}\right) \quad &: \quad m_{t+1}^{\omega} \left(\tilde{\lambda}_t - \tilde{\mu}_t^{\omega}\right) = \beta \tilde{\lambda}_{t+1}^{\omega} \qquad \forall \omega \end{aligned}$$

In the first equation, the social value of liquidity $\tilde{\lambda}_t$ now depends on the weighted average of shadow prices $\tilde{\mu}_t^{\omega}$ on security issuance rather than the uncontingent shadow price $\tilde{\mu}_t$. The second equation is modified in the same manner as in the decentralized problem.

For the planner to implement the constrained efficient allocation, she needs to set the tax rates τ_{t+1}^{ω} such that the optimality conditions of private agents under taxation and of the planner coincide. We substitute for the λ_t's and $\tilde{\lambda}_t$'s in the two respective Euler equations and subtract the

Euler equation of domestic agents under taxation from the planner's generalized Euler equation to obtain

$$\tau^{\omega}_{t+1} u_{T,t} + m^{\omega}_{t+1} \left(\mu^{\omega}_t - \tilde{\mu}^{\omega}_t\right) = -m^{\omega}_{t+1} E\left[\phi^{\omega} \tilde{\mu}^{\omega}_t\right] p'\left(c_{T,t}/y_{N,t}\right) + \beta E\left[\phi^{\omega} \tilde{\mu}^{\omega}_{t+1}\right] p'\left(c_{T,t+1}/y_{N,t+1}\right) \quad \text{(A.5)}$$

Let us first consider the case that all financial constraints are slack at time t so $\mu_t = \tilde{\mu}_t = 0$ so that equation (A.5) simplifies to

$$\tau^{\omega}_{t+1} = \frac{\beta E\left[\phi^{\omega} \tilde{\mu}^{\omega}_{t+1}\right]}{u_{T,t}} \cdot p'\left(c^{\omega}_{T,t+1}/y^{\omega}_{N,t+1}\right) \quad \text{(A.6)}$$

In that case, the planner's optimal interventions are prudential, i.e. they focus solely on relaxing future constraints. Comparing this to expression (12) in our baseline model, the only difference is that the social cost of reducing the value of collateral by one dollar in state ω next period, captured by the shadow price $\phi \tilde{\mu}^{\omega}_{t+1}$, is replaced by the weighted average $E\left[\phi^{\omega} \tilde{\mu}^{\omega}_{t+1}\right]$, which captures the total social cost of reducing the value of collateral in all of the state-contingent constraints.[31] To the extent that the social costs of being constrained are close to each other in the two variants of the model, the resulting optimal tax rates are also close to each other. This is likely the case since we explicitly calibrated $\tilde{\mu}$ to capture the social cost of being constrained, i.e. the social value of being able to issue one more dollar of liabilities, without imposing assumptions on the specific functional form of the constraint [cf. equation (15)].

Let us next consider the case when some of the financial constraints are binding at time t. Assume that the planner continues to set the state-contingent tax rate (A.6) in constrained states of nature so that private agents correctly perceive the social cost of security issuance.[32] Then equation (A.5) implies that the shadow prices on the constraint of domestic agents and the planner for constrained states of nature satisfy

$$\tilde{\mu}^{\omega}_t = \mu^{\omega}_t + E\left[\phi^{\omega} \tilde{\mu}^{\omega}_t\right] p'\left(c_{T,t}/y_{N,t}\right)$$

This reflects that the planner internalizes that relaxing the constraint in state ω increases current traded goods consumption, which appreciates the current real exchange rate, and relaxes constraints on current security issuance in all other states of nature, with marginal welfare benefit $E\left[\phi^{\omega} \tilde{\mu}^{\omega}_t\right]$. If some of these constraints are binding ($\tilde{\mu}^{\omega}_t > 0$), then the planner will subsidize the issuance of unconstrained securities to increase consumption and relax these binding constraints, generating a motive for "ex-post" subsidies on security issuance.[33] Equation (A.5) implies that the resulting optimal tax rate on the issuance of securities in unconstrained states of nature (for which $\mu^{\omega}_t = \tilde{\mu}^{\omega}_t = 0$) is

$$\tau^{\omega *}_{t+1} = \tau^{\omega}_{t+1} - m^{\omega}_{t+1} \cdot \frac{E\left[\phi^{\omega} \tilde{\mu}^{\omega}_t\right]}{u_{T,t}} \cdot p'\left(c_{T,t}/y_{N,t}\right)$$

[31] The weighted average $E\left[\phi^{\omega} \tilde{\mu}^{\omega}_{t+1}\right]$ can also be decomposed into $E\left[\phi^{\omega} \tilde{\mu}^{\omega}_{t+1}\right] = E\left[\phi^{\omega}\right] E\left[\tilde{\mu}^{\omega}_{t+1}\right] + Cov\left[\phi^{\omega} \tilde{\mu}^{\omega}_{t+1}\right]$, highlighting how the measure is adversely affected by systematic correlation between the collateralizability parameter ϕ^{ω} and the social cost of binding constraints $\tilde{\mu}^{\omega}_{t+1}$.

[32] This is only one implementation out of a continuum of implementations – since domestic agents are constrained in their security issuance contingent on the state under consideration, a marginal change in the tax rate would simply induce a marginal change in the shadow price on the constraint but would not affect the real allocation.

[33] We use the term "ex-post" subsidies to reflect that the intervention occurs after a financial crisis state has materialized. In our baseline model, there is no motive for such intervention since all security issuance is limited by the same constraint – this constraint is either slack or binding, and if it is binding then subsidies on security issuance have no effects.

The optimal tax rate is the sum of the prudential tax τ_{t+1}^{ω} from equation (A.6), which is non-negative, and an additional ex-post intervention term, which is non-positive, indicating a potential subsidy. For example, if security issuance against low states of nature is constrained, the planner may subsidize issuance against high states of nature to increase current consumption, appreciate the real exchange rate, and relax the binding constraints.

B.1.2 Constraints on Composite Securities

Alternatively, consider the case that domestic agents have access to a set $\mathcal{S}$ of composite securities such as dollar debt, local currency debt, equity etc. As in our baseline model, let us denote the payoff vector of security $s \in \mathcal{S}$ by $X_{t+1}^{\omega}(s)$. For example, if $s = \mathcal{D}, \mathcal{L}$ represent dollar debt and local currency debt, then $X_{t+1}^{\omega}(\mathcal{D}) = 1$ and $X_{t+1}^{\omega}(\mathcal{L}) = p_{t+1}^{\omega} \forall \omega$. Furthermore, denote the position of security $s \in \mathcal{S}$ that domestic agents issue by $b_{t+1}(s)$. The budget constraint of domestic agents then is

$$c_{T,t} + p_t c_{N,t} + \sum_{s \in \mathcal{S}} E\left[m_{t+1}^{\omega} b_{t+1}(s) X_{t+1}^{\omega}(s)\right] = y_{T,t} + p_t y_{N,t} + \sum_{s \in \mathcal{S}} b_t(s) X_t(s)$$

The sum on the left-hand side represents the amount of finance raised from international investors with pricing kernel m_{t+1}^{ω} by issuing the different securities $s \in \mathcal{S}$. The sum on the right-hand side represents the realized payoffs of the N different securities issued in the previous period.

Assume that the market value of security $s \in \mathcal{S}$ that domestic agents are allowed to issue is constrained by

$$b_{t+1}(s) E\left[m_{t+1}^{\omega} X_{t+1}^{\omega}(s)\right] + \phi(s)\left[y_{T,t} + p_t y_{N,t}\right] \geq 0$$

where $\phi(s)$ is a security-specific collateralizability parameter. (Note that it would be equivalent to impose the constraint on the quantity of securities $b_{t+1}(s)$ rather than the market value, since the parameter $\phi(s)$ could simply be divided by the market value of one unit of security $E\left[m_{t+1}^{\omega} X_{t+1}^{\omega}(s)\right]$.)

In the Lagrangian of the optimization problem of domestic agents, this introduces a collection of constraints on security issuance with multipliers $\mu_t(s)$,

$$\begin{aligned}\mathscr{L} = E \sum_{t=0}^{\infty} \beta^t \Bigg\{ & u\left(c_{T,t}, c_{N,t}\right) - \\ & - \lambda_t \left[c_{T,t} - y_{T,t} + \left(c_{N,t} - y_{N,t}\right) p_t + \sum_{s \in \mathcal{S}} \left(b_{t+1}(s) E\left[m_{t+1}^{\omega} X_{t+1}^{\omega}(s)\right] - b_t(s) X_t(s)\right)\right] \\ & + \sum_{s \in \mathcal{S}} \left\{\mu_t(s)\left[b_{t+1}(s) E\left[m_{t+1}^{\omega} X_{t+1}^{\omega}(s)\right] + \phi(s)\left(y_{T,t} + p_t y_{N,t}\right)\right]\right\} \Bigg\}\end{aligned}$$

and similarly for the social planner.

The state-contingent Euler equation of domestic agents changes to

$$FOC\left(b_{t+1}(s)\right): \quad E\left[m_{t+1}^{\omega} X_{t+1}^{\omega}(s)\right]\left(u_{T,t} - \mu_t(s)\right) = \beta E\left[X_{t+1}^{\omega}(s) u_{T,t+1}^{\omega}\right] \qquad \forall s \in \mathcal{S}$$

If domestic agents are subject to specific taxes $t_{t+1}(s)$ on the issuance of security $s \in \mathcal{S}$, the left-hand side of this Euler equation modifies to $\left(E\left[m_{t+1}^{\omega} X_{t+1}^{\omega}(s)\right] - t_{t+1}(s)\right) u_{T,t} - E\left[m_{t+1}^{\omega} X_{t+1}^{\omega}(s)\right] \mu_t(s)$.

In the social planner's problem, both the optimality condition on traded consumption and the generalized Euler equation are modified to

$$\begin{aligned} FOC\left(c_{T,t}\right) &: \quad \tilde{\lambda}_t = u_{T,t} + \sum_{s \in \mathcal{S}} \left[\phi(s) \tilde{\mu}_t(s)\right] p'\left(c_{T,t}/y_{N,t}\right) \\ FOC\left(b_{t+1}(s)\right) &: \quad E\left[m_{t+1}^{\omega} X_{t+1}^{\omega}(s)\right]\left(\tilde{\lambda}_t - \tilde{\mu}_t(s)\right) = \beta E\left[X_{t+1}^{\omega}(s) \tilde{\lambda}_{t+1}^{\omega}\right] \qquad \forall s \in \mathcal{S} \end{aligned}$$

In the first equation, the social value of liquidity $\tilde{\lambda}_t$ now depends on the weighted sum of shadow prices $\tilde{\mu}_t(s)$ on the issuance of different securities. The second equation, the generalized Euler equation, is modified in the same manner as the decentralized problem. Subtracting the Euler equation of domestic agents under taxes from the social planner's generalized Euler equation and substituting for the λ_t's and $\tilde{\lambda}_t$'s, we obtain

$$t_{t+1}(s)\, u_{T,t} + E\left[m^{\omega}_{t+1} X^{\omega}_{t+1}(s)\right] \left[\mu_t(s) + \sum_{r\in\mathcal{S}} \left[\phi(r)\, \tilde{\mu}_t(r)\right] p'\left(c_{T,t}/y_{N,t}\right) - \tilde{\mu}_t(s)\right]$$
$$= \beta E\left[X^{\omega}_{t+1}(s) \sum_{r\in\mathcal{S}} \left[\phi(r)\, \tilde{\mu}^{\omega}_{t+1}(r)\right] p'\left(c^{\omega}_{T,t+1}/y^{\omega}_{N,t+1}\right)\right] \quad \text{(A.7)}$$

Let us define the externality pricing kernel

$$\tau^{\omega}_{t+1} = p'\left(c^{\omega}_{T,t+1}/y^{\omega}_{N,t+1}\right) \cdot \frac{\beta \sum_{r\in\mathcal{S}} \phi(r)\, \tilde{\mu}^{\omega}_{t+1}(r)}{u_{T,t}} \quad \text{(A.8)}$$

Compared to equation (12) in our baseline model, this expression replaces the social cost of reducing the value of collateral by one dollar in state ω next period, captured by the shadow price $\phi\tilde{\mu}^{\omega}_{t+1}$, by the sum of the social costs of reducing the value of collateral in each of the individual constraints on the different financial securities $\sum_{r\in\mathcal{S}} \phi(r)\, \tilde{\mu}^{\omega}_{t+1}(r)$. To the extent that the social costs of being constrained are close to each other in the two variants of the model, the resulting optimal tax rates are also close to each other. Again, this is arguably the case since we explicitly calibrated $\tilde{\mu}$ to capture the social cost of being constrained, i.e. the social value of being able to issue one more dollar of liabilities, without imposing assumptions on the specific functional form of the constraint [cf. equation (15)].[34]

Let us first consider the case when all financial constraints are slack at time t. Given the described adjustment to the externality pricing kernel, expression (A.7) implies that the optimal tax on a composite security $s \in \mathcal{S}$ with payoffs $X^{\omega}_{t+1}(s)$ under $\mu_t(s) = \tilde{\mu}_t(s) = 0$ is

$$t\left(X^{\omega}_{t+1}(s)\right) = E\left[\tau^{\omega}_{t+1} X^{\omega}_{t+1}(s)\right] \quad \text{(A.9)}$$

This is the same as the expression (14) reported in Corollary 1. In short, the prudential behavior of the planner is unchanged.

Conversely, if the issuance constraints on some of the securities are binding at time t and the planner imposes the tax (A.9) on those securities so that private agents correctly perceive the social cost of security issuance, then expression (A.7) implies that the private and social shadow prices on those binding constraints satisfy

$$\tilde{\mu}_t(s) = \mu_t(s) + \left(\sum_{r\in\mathcal{S}} \phi(r)\, \tilde{\mu}_t(r)\right) p'\left(c_{T,t}/y_{N,t}\right)$$

The sum on the right-hand side captures that the planner internalizes that allocating more wealth to the current time period increases current traded goods consumption, appreciates the current real

[34] To provide an example, consider an economy in which the issuance of dollar-denominated debt is subject to significantly tighter constraints than equity – $\phi(\mathcal{D}) \ll \phi(\mathcal{E})$. This implies that raising the value of collateral increases the issuance of equity significantly more than the issuance of dollar debt. The externality pricing kernel (A.8) reflects this by assigning much higher weight to the shadow price on the issuance of equity.

exchange rate, and relaxes the constraints on current security issuance. If some of these constraints are binding ($\tilde{\mu}_t(r) > 0$), then the planner will subsidize the issuance of unconstrained securities to increase consumption and relax these binding constraints, generating a similar motive for ex-post intervention as in section B.1.1. Expression (A.7) implies an optimal tax rate on the issuance of unconstrained securities $s \in \mathcal{S}$ with $\mu_t(s) = \tilde{\mu}_t(s) = 0$ of

$$t^*_{t+1}\left(X^{\omega}_{t+1}(s)\right) = t_{t+1}\left(X^{\omega}_{t+1}(s)\right) - E\left[m^{\omega}_{t+1}X^{\omega}_{t+1}(s)\right] \cdot \frac{\sum_{r\in\mathcal{S}}\left[\phi(r)\,\tilde{\mu}_t(r)\right]}{u_{T,t}} \cdot p'\left(c_{T,t}/y_{N,t}\right)$$

The first term in this expression, $t_{t+1}\left(X^{\omega}_{t+1}(s)\right)$, reflects the prudential motive captured by expression (A.9) and is non-negative, indicating a potential tax. The second term captures the additional ex-post intervention motive and is non-positive, indicating a potential subsidy. To provide an example, consider an economy in which all constraints on the issuance of debt are binding but the constraint on equity issuance is slack. The planner can relax the constraints on debt issuance by subsidizing the issuance of equity, which will appreciate the current real exchange rate and thus generate more collateral for all agents to issue more debt.

B.2 Capital Investment

Our baseline model extended by capital with depreciation rate δ is captured by the Lagrangian

$$\begin{aligned}\mathscr{L} = E\sum_{t=0}^{\infty}\beta^t \Big\{ & u\left(c_{T,t}, c_{N,t}\right) + \mu_t\left\{E\left[m^{\omega}_{t+1}b^{\omega}_{t+1}\right] + \phi\left[f\left(k_t\right) + p y_{N,t}\right]\right\} \\ & -\lambda_t\left[c_{T,t} - f\left(k_t\right) + \left(c_{N,t} - y_{N,t}\right)p_t + E\left[m^{\omega}_{t+1}b^{\omega}_{t+1}\right] - b_t + k_{t+1} - (1-\delta)\,k_t\right]\Big\}\end{aligned}$$

The additional optimality condition for capital investment k_{t+1} is

$$\lambda_t = \beta E\left[f'\left(k_{t+1}\right)\left(\lambda^{\omega}_{t+1} + \phi\mu^{\omega}_{t+1}\right) + (1-\delta)\,\lambda^{\omega}_{t+1}\right]$$

The social planner's problem is modified in the same way as in our baseline setup, and she arrives at an analogous condition for capital investment,

$$\tilde{\lambda}_t = \beta E\left[f'\left(k_{t+1}\right)\left(\tilde{\lambda}^{\omega}_{t+1} + \phi\tilde{\mu}^{\omega}_{t+1}\right) + (1-\delta)\,\tilde{\lambda}^{\omega}_{t+1}\right]$$

The only difference lies in the different shadow prices of private agents versus the planner. The planner can impose a subsidy s_t on new capital investment $i_t = k_{t+1} - (1-\delta)\,k_t$ to implement her preferred choice of investment. The subsidy ensures that the private optimality condition equals the social optimality condition,

$$1 - s_t = \frac{\tilde{\lambda}_t}{\lambda_t} \cdot \frac{E\left[f'\left(k_{t+1}\right)\cdot\left(\lambda^{\omega}_{t+1} + \phi\mu^{\omega}_{t+1}\right) + (1-\delta)\,\lambda^{\omega}_{t+1}\right]}{E\left[f'\left(k_{t+1}\right)\cdot\left(\tilde{\lambda}^{\omega}_{t+1} + \phi\tilde{\mu}^{\omega}_{t+1}\right) + (1-\delta)\,\tilde{\lambda}^{\omega}_{t+1}\right]} \tag{A.10}$$

Recall that $\tilde{\lambda}_t = \lambda_t + \phi\tilde{\mu}_t p' \geq \lambda_t$ and $\tilde{\mu}_t = \mu_t/\left(1-\phi p'\right) \geq \mu_t$, i.e. the planner's valuations of wealth and of relaxing the constraint are identical to that of private agents if the constraint is loose but are higher if the constraint is binding.

The first multiplicative term $\tilde{\lambda}_t/\lambda_t$ is greater than one if the constraint is binding in the current period t. In that case, the planner would like to encourage consumption expenditure, which falls on both traded and non-traded goods, and therefore appreciates the real exchange rate. She will therefore tax investment, which absorbs solely traded goods and has no contemporaneous real exchange rate effect.

The second multiplicative term in equation (A.10) is less than one if the constraint is expected to bind in period $t+1$. In that case, the planner subsidizes investment, which creates more traded goods in period $t+1$ and therefore appreciates the exchange rate and relaxes the constraint.

Capital Investment and Price of Future Tradable Goods Including capital investment in our model makes it, in principle, possible to obtain financial amplification in a setting in which the financial constraint depends on future rather than current prices. If (i) the financial constraint depends on the market value of future output and thus on future exchange rates, (ii) capital investment occurs only or largely in the traded sector and (iii) policymakers do not possess instruments to subsidize capital investment while discouraging consumption, then the following feedback loop could arise: a binding financial constraint reduces current investment in traded goods, which lowers future traded output and thus depreciates the future exchange rate, which in turn tightens the current constraint and leads to further reductions in current investment in traded goods, leading financial amplification.

B.3 Over-Optimism

Paternalistic Planner If the planner behaves paternalistically, she forms her own expectations $E_S[\cdot]$ of the discounted future flow of utility but recognizes that private investors price state-contingent securities using their own expectations operator $E_P\left[m^{\omega}_{t+1}b^{\omega}_{t+1}\right]$, which enters in the budget and borrowing constraint of private agents. This results in an optimization problem described by the Lagrangian

$$\mathscr{L} = E_S \sum_{t=0}^{\infty} \beta^t \left\{ u\left(c_{T,t}, y_{N,t}\right) - \tilde{\lambda}_t \left[c_{T,t} - y_{T,t} + E_P\left[m^{\omega}_{t+1}b^{\omega}_{t+1}\right] - b_t\right] \right.$$
$$\left. + \tilde{\mu}_t \left\{ E_P\left[m^{\omega}_{t+1}b^{\omega}_{t+1}\right] + \phi\left[y_{T,t} + p\left(c_{T,t}/y_{N,t}\right) y_{N,t}\right] \right\} \right\}$$

The resulting intertemporal optimality condition is

$$\pi^{P,\omega}_{t+1} m^{\omega}_{t+1} \left(\tilde{\lambda}_t - \tilde{\mu}_t\right) = \beta \pi^{S,\omega}_{t+1} \tilde{\lambda}^{\omega}_{t+1}$$

and the planner can implement the allocation by imposing a state-contingent tax/subsidy on security issuance of

$$1 + \tau^{\omega}_{t+1} = \frac{\pi^{S,\omega}_{t+1}}{\pi^{P,\omega}_{t+1}} \left(1 + \frac{\phi \tilde{\mu}_{t+1} p'}{u^{\omega}_{T,t+1}}\right)$$

as can be seen by simple comparison with the optimality condition of private agents under taxation (A.2).

Non-Paternalistic Planner A non-paternalistic planner respects the expectations of each individual agent and solves the optimization problem subject to these expectations but calculates the general equilibrium and the resulting externalities subject to her own expectations. Analytically, we set up our non-paternalistic planning problem using an $\varepsilon/1-\varepsilon$-approach. The planner maximizes the sum of welfare of all agents and asks how to regulate the behavior of a given mass ε of agents who employ the private expectations operator $E_P[\cdot]$ while internalizing that their behavior leads to general equilibrium effects and externalities that affect the welfare of the remaining mass

$1-\varepsilon$ of agents, which is evaluated using the planner's expectations operator $E_S[\cdot]$. The optimal level of regulation for the mass ε agents is imposed on all agents. In the limit as $\varepsilon \to 0$, this implements a symmetric planning allocation in which each agent maximizes her utility following her own subjective probability measure but the level of regulation corresponds to the externalities evaluated under the planner's probability measure.

We denote the allocations associated with the mass ε of domestic agents by lower-case letters and the variables of the remaining mass $1-\varepsilon$ agents by upper-case letters, which the planner takes as given (in equilibrium, they are identical to the lower-case letters). The real exchange rate in this setup is given by the expression

$$p_t = p\left(\left[\varepsilon c_{T,t} + (1-\varepsilon) C_{T,t}\right] / y_{N,t}\right)$$

The planner maximizes the sum of welfare of all agents, captured by the Lagrangian

$$\begin{aligned}\mathscr{L} = \varepsilon E_P \sum_{t=0}^{\infty} \beta^t \Big\{ & u\left(c_{T,t}, y_{N,t}\right) - \hat{\lambda}_t \left[c_{T,t} - y_{T,t} + E_P\left[m^{\omega}_{t+1} b^{\omega}_{t+1}\right] - b_t\right] \\ & + \hat{\mu}_t \left\{E_P\left[m^{\omega}_{t+1} b^{\omega}_{t+1}\right] + \phi\left[y_{T,t} + p\left(\left[\varepsilon c_{T,t} + (1-\varepsilon) C_{T,t}\right]/y_{N,t}\right) y_{N,t}\right]\right\}\Big\} \\ + (1-\varepsilon) E_S \sum_{t=0}^{\infty} \beta^t \Big\{ & u\left(C_{T,t}, y_{N,t}\right) - \tilde{\lambda}_t \left[C_{T,t} - y_{T,t} + E_P\left[m^{\omega}_{t+1} B^{\omega}_{t+1}\right] - B_t\right] \\ & + \tilde{\mu}_t \left\{E_P\left[m^{\omega}_{t+1} B^{\omega}_{t+1}\right] + \phi\left[y_{T,t} + p\left(\left[\varepsilon c_{T,t} + (1-\varepsilon) C_{T,t}\right]/y_{N,t}\right) y_{N,t}\right]\right\}\Big\}\end{aligned}$$

The optimality conditions for the variables of the mass ε agent are

$$\begin{aligned}\pi^{P,\omega}_t \varepsilon\left(u_{T,t} - \hat{\lambda}_t + \varepsilon \hat{\mu}_t \phi p'\right) + \pi^{S,\omega}_t (1-\varepsilon) \varepsilon \tilde{\mu}_t \phi p' &= 0 \\ m^{\omega}_{t+1}\left(\hat{\lambda}_t - \hat{\mu}_t\right) &= \beta \hat{\lambda}^{\omega}_{t+1}\end{aligned}$$

In the limit of $\varepsilon \to 0$, we can combine these to

$$\begin{aligned}\hat{\lambda}_t &= u_{T,t} + \frac{\pi^{S,\omega}_t}{\pi^{P,\omega}_t} \tilde{\mu}_t \phi p' \\ u_{T,t} + \frac{\pi^{S,\omega}_t}{\pi^{P,\omega}_t} \tilde{\mu}_t \phi p' - \hat{\mu}_t &= \frac{\beta\left(u^{\omega}_{T,t+1} + \pi^{S,\omega}_{t+1} / \pi^{P,\omega}_{t+1} \cdot \tilde{\mu}_{t+1} \phi p'\right)}{m^{\omega}_{t+1}}\end{aligned}$$

As described in Corollary 1, the latter equation, capturing the optimal intertemporal tradeoff, can be replicated by imposing a tax on individual agents that satisfies

$$\tau^{\omega}_{t+1} = \frac{\pi^{S,\omega}_{t+1}}{\pi^{P,\omega}_{t+1}} \cdot \frac{\phi \tilde{\mu}_{t+1} p'}{u^{\omega}_{T,t+1}}$$

B.4 Impulse Responses

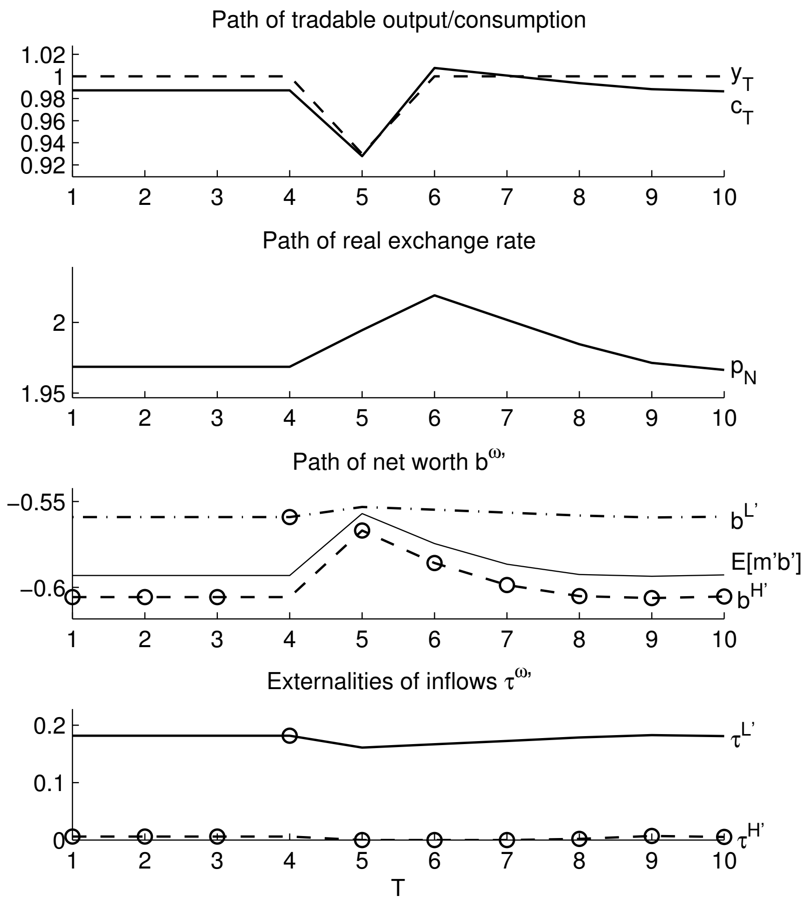

Figure 4: Impulse Response to Sudden Stop Shock in Planner's Allocation

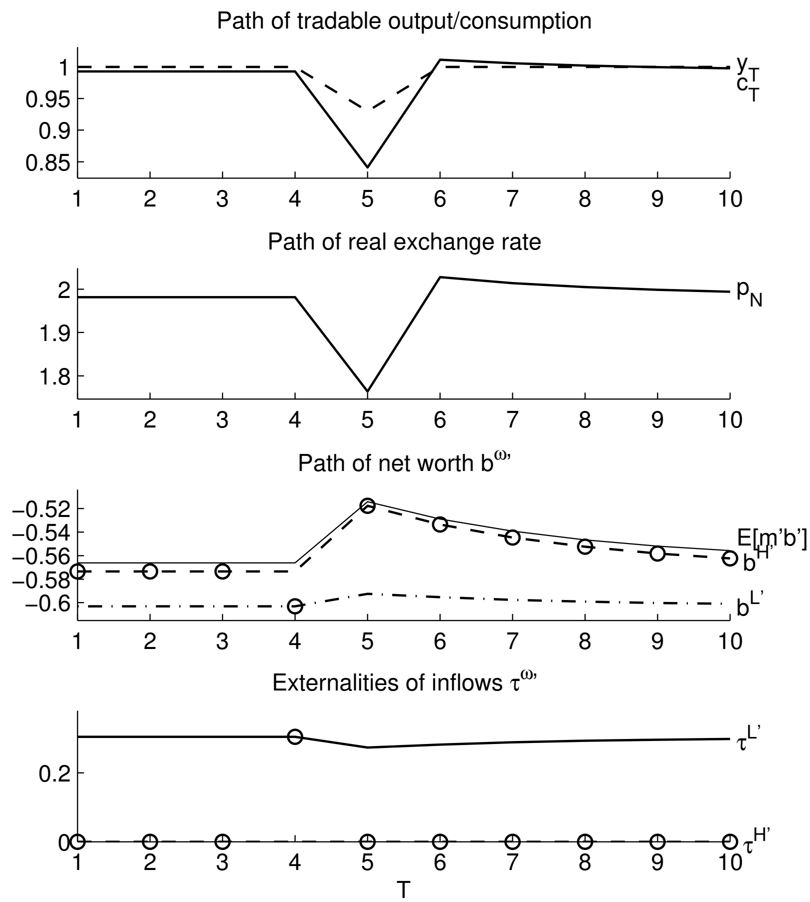

Figure 5: Impulse Response to Sudden Stop Shock in Restricted Planner's Equilibrium where $\tau_t^\omega = \bar{\tau}_t \forall \omega$

C Numerical solution method [Online only]

This appendix describes how the policy functions in the recursive equilibrium of our model can be solved numerically using the endogenous gridpoints bifurcation method as in Jeanne and Korinek (2010b). In a given period, the state of the economy can be summarized by the pair (b, ω), i.e. the security holdings carried into the period and the state of nature, which determines the realization of the output shocks. The dynamics of the model are captured by equilibrium policy functions $c_T(b, \omega)$, $p(b, \omega)$ and $b^{\omega\prime}(b, \omega)$ that satisfy

$$\begin{aligned} c_T(b,\omega) &= \min\left\{b + y + K(b,\omega), u_T^{-1}\left(\beta u_T\left(c_T\left(b^{\omega\prime},\omega'\right), y_N'\right)/m^{\omega\prime}, y_N\right)\right\} \quad \forall\omega' \\ p(b,\omega) &= \frac{u_N(c(b,\omega), y_N)}{u_T(c(b,\omega), y_N)} \end{aligned}$$

and the law-of-motion

$$c_T(b,\omega) + E\left[m^{\omega\prime} b^{\omega\prime}(b,\omega)\right] = y_T + b$$

where $K(b, \omega) = -\phi[y_T + p(b, \omega) y_N]$ reflects the maximum amount of security issuance allowed by the financial constraint and $y_T = y_T(\omega)$ as well as $y_N = y_N(\omega)$ are functions of the state of nature.

We introduce an auxiliary function $\bar{b}(b, \omega) = E[m^{\omega\prime}(\omega) b^{\omega\prime}(b, \omega)]$ that captures the total revenue from security issuance in the given period.

To implement our numerical solution method, we first define a discrete set $\Omega = \{\omega_1, ...\omega_N\}$ of N possible states of nature in each period and define by $\Pi = (\pi_{ij})$ the matrix of transition probabilities from state ω_i to state ω_j. We associate with each ω_i the output shocks $y_T(\omega)$ and $y_N(\omega)$ as well as a pricing kernel $m(\omega)$ of international investors and collect the three in the vectors $\mathbf{y}_T$, $\mathbf{y}_N$ and $\mathbf{m}$. We also define a grid $\mathbf{b}$ for bond holding. The lowest possible value $b_{\min}$ corresponds to the borrowing limit $\bar{b}$ when traded output is at its lowest possible value $\min\{y_T\}$ and the real exchange rate approaches its lowest possible value 0. The maximum value $b_{\max}$ can be an arbitrary positive number.

In each iteration step k, we start with a policy function $c_{T,k}(b, \omega)$ that is weakly increasing in b and $y_T(\omega)$. For simplicity we set the initial policy function to $c_{T,0}(b, \omega) = y_T(\omega) + (1-\beta) b$.

We iterate backward in time, i.e. we assume that we know the policy functions $c_{T,k}(b', \omega')$ etc next period and determine the equilibrium in the current period. For each possible realization of next period bond holdings $b' \in \mathbf{b}$ in the grid and for each possible state of nature $\omega' \in \Omega$, we determine the current period consumption c_T and bond holdings b under the assumption that the equilibrium is unconstrained from the Euler equation,

$$c_T^{unc}(b', \omega') = u_T^{-1}\left(\beta u_T\left(c_T\left(b^{\omega\prime}, \omega'\right), y_N'\right)/m^{\omega\prime}, y_N\right) \tag{A.11}$$

Given consumption $\bar{c}_T(b^{1\prime}) = c_T^{unc}(b^{1\prime}, \omega_1')$ in state ω_1', we search for the $b^{i\prime}$ for all other ω_i', $i > 1$ that correspond to the same level of consumption via interpolation,

$$c_T^{unc}\left(b^{i\prime}, \omega_i'\right) = \bar{c}_T\left(b^{1\prime}\right)$$

and calculate the total amount of security issuance that corresponds to this level of consumption, $E\left[m^{i\prime} b^{i\prime} | \omega_j\right]$ using the probability distribution of ω' for given $\omega_j \in \Omega$. From the period budget constraint, we can associate with each of the resulting pairs of $\left(\bar{c}_T\left(b^{1\prime}\right), E\left[m^{i\prime}b^{i\prime}|\omega_j\right]\right)$ an initial wealth level

$$b^{unc}\left(\omega_j\right) = \bar{c}_T\left(b^{1\prime}\right) + E\left[m^{i\prime}b^{i\prime}|\omega_j\right] - y_T\left(\omega_j\right)$$

This gives us a set of triplets $\left(\left\{b^{unc}\left(\omega_j\right)\right\}_{\omega_j\in\Omega}, \bar{c}_T, \left\{b^{\omega\prime}\right\}_{\omega'\in\Omega'}\right)$ in which different values of $\left(b,\omega_j\right)$ are associated with different values of $\bar{c}_T\left(b^{1\prime}\right)$ and $\left\{b^{\omega\prime}\right\}$ along the unconstrained branch of the system.

For each state $\omega_j \in \Omega$, we determine the threshold of $\hat{K}\left(\omega_j\right) = E\left[m^{i\prime}b^{i\prime}|\omega_j\right]$ at which the borrowing constraint is marginally binding in the unconstrained system, which satisfies

$$\hat{K}\left(\omega_j\right) = E\left[m^{i\prime}b^{i\prime}|\omega_j\right] = -\phi\left[y\left(\omega_j\right) + p\left(\bar{c}_T\left(b^{1\prime}\right), y_N\right)y_N\right]$$

In a given state ω_j, this is the lowest possible level of wealth carried into the next period in the unconstrained system, and it is also the lowest possible level of wealth in the constrained system of equations (since constrained consumption is always lower than unconstrained consumption and therefore the exchange rate is more depreciated). Any higher level of security issuance, i.e. any more negative $E\left[m^{i\prime}b^{i\prime}\right]$, cannot be supported in state ω_j of the economy since it would violate the collateral constraint. We denote the initial wealth level that leads to this level of security issuance by $\hat{b}\left(\omega_j\right)$. For any $\left(b,\omega_j\right)$ s.t. $b \geq \hat{b}\left(\omega_j\right)$, the unconstrained system is valid. On the other hand, if $b < \hat{b}\left(\omega_j\right)$, the system is constrained.

Next we solve for the constrained branch of the system: for each $\omega_j \in \Omega$, we consider the grid of values of $E\left[m^{i\prime}b^{i\prime}|\omega_j\right] \in \left\{\hat{K}\left(\omega_j\right), \phi y_T\left(\omega_j\right)\right\}$ that we obtained in the previous step and solve for the levels of $\bar{c}_T^{con}\left(\omega_j, E\left[m^{i\prime}b^{i\prime}|\omega_j\right]\right)$ that satisfy the financial constraint with equality for these levels of security issuance from the equation

$$E\left[m^{i\prime}b^{i\prime}|\omega_j\right] = -\phi\left[y\left(\omega_j\right) + p\left(\bar{c}_T^{con}, y_N\right)y_N\right]$$

We use the period budget constraint to obtain the initial wealth levels corresponding to the levels $\left(\bar{c}_T^{con}, E\left[m^{i\prime}b^{i\prime}|\omega_j\right]\right)$ as

$$b^{con} = \bar{c}_T^{con} + E\left[m^{i\prime}b^{i\prime}|\omega_j\right] - y_T\left(\omega_j\right)$$

For each ω_j, this gives us a set of triplets $\left(b^{con}, \bar{c}_T^{con}, \left\{b^{\omega\prime}\right\}_{\omega'\in\Omega'}\right)$ in which different values of $\left(b^{con},\omega_j\right)$ are associated with different values of $\bar{c}_T^{con}$ and $\left\{b^{\omega\prime}\right\}$ along the constrained branch of the system. The lowest possible value of b^{con} leads to zero consumption and total security issuance $E\left[m^{i\prime}b^{i\prime}|\omega_j\right] = \phi y_T\left(\omega_j\right)$. We denote this level by $b_{\min}\left(\omega_j\right)$. In summary, for a given ω_j, any $b \in \left(b_{\min}\left(\omega_j\right), \hat{b}\left(\omega_j\right)\right)$ leads to a constrained outcome and any $b \geq \hat{b}\left(\omega_j\right)$ leads to an unconstrained outcome. At $\hat{b}\left(\omega_j\right)$, the constrained and unconstrained policy functions coincide.

Next we construct the step $k+1$ policy function $c_{T,k+1}(b,\omega)$ by combining the constrained and unconstrained regimes.

For a given ω_j and $b \in [b_{\min}(\omega_j), \hat{b}(\omega_j)]$, we interpolate on the triplets $\left(b^{con}, \bar{c}_T^{con}, \left\{b^{\omega\prime}\right\}_{\omega'\in\Omega'}\right)$. For the same ω_j and $b \in [\hat{b}(\omega_j), b_{\max}]$, we interpolate on the triplets $\left(\left\{b^{unc}\left(\omega_j\right)\right\}_{\omega_j\in\Omega}, \bar{c}_T, \left\{b^{\omega\prime}\right\}_{\omega'\in\Omega'}\right)$. Concatenating the policy functions of these two regions, we get the step $k+1$ policy functions $c_{T,k+1}(b,\omega_j)$ and $b^{\omega\prime}\left(b,\omega_j\right)$. The resulting consumption function $c_{T,k+1}(b,\omega)$ is monotonically increasing in b and $y_T\left(\omega\right)$. We keep iterating this process until the distance between two successive functions $c_{T,k}(b,\omega)$ and $c_{T,k+1}(b,\omega)$ is sufficiently small.

A Matlab source code is available at `http://www.korinek.com/`