Capital Controls and Currency Wars

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Abstract

Capital account intervention generates international spillover effects that have recently raised concerns about global currency wars. This paper analyzes the spillover effects of capital controls imposed by one country on others and examines the desirability of global coordination of such policy measures. We find that if controls are designed to correct for domestic externalities, the resulting equilibrium is globally Pareto efficient, i.e. a global planner would impose the same measures and there is no role for global coordination. We illustrate this for a range of externalities that have recently been invoked as reasons for imposing capital controls: learning externalities, aggregate demand externalities in a liquidity trap, and pecuniary externalities arising from financial constraints. On the other hand, if controls are designed to manipulate a country's terms-of-trade or if policymakers face an imperfect set of instruments, such as targeting problems or costly enforcement, then multilateral coordination is desirable in order to mitigate the inefficiencies arising from such imperfections.

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1 Introduction

Capital controls lead to international spillover effects that have led to considerable controversy in international policy circles in recent years (see e.g. Ostry et al, 2012) and have raised concerns about global currency wars. This paper determines the welfare effects of such measures in a general equilibrium model of the world economy and analyzes under what conditions global coordination of capital account policies is desirable.

We describe the spillover effects from capital account intervention in an intertemporal benchmark model of a global economy in which individual countries engage in borrowing and lending. If one country imposes capital controls in the form of taxes on foreign borrowing, it reduces both borrowing and consumption and pushes down the world interest rate, leading to greater inflows to other countries. In an augmented model, it also depreciates its real exchange rate and appreciates the real exchange rate of other countries. Furthermore, we show an isomorphism between capital controls and reserve accumulation: any level of capital controls can be replicated by a corresponding level of reserve accumulation when the capital account is closed to private transactions.

Next we study several types of externalities that have recently been invoked as reasons why individual countries may want to impose capital controls: learning externalities, aggregate demand externalities in a liquidity trap, and pecuniary externalities arising from external financial constraints. For each of these domestic distortions, a national planner can improve domestic welfare by imposing capital controls that offset the externality, even though such controls create international spillover effects.

The resulting global equilibrium is Pareto efficient as long as national planners behave competitively and impose capital controls that offset domestic externalities while ignoring the general equilibrium effects of such controls. A global planner who internalizes all international spillover effects cannot improve on the described allocation. By contrast, if national planners refrain from imposing capital controls to correct for domestic externalities, global welfare is reduced. Conceptually, we can view the national planners that internalize domestic externalities in different countries but do not exert market power as competitive agents to which the welfare theorems apply. Changes in the world interest rate that stem from capital controls constitute pecuniary externalities that cancel out and do not impede Pareto efficiency. We also find that a seeming "arms race" of escalating capital controls does not necessarily indicate inefficiency but may be the tatonnement process through which multiple countries optimally adjust their capital controls.

On the other hand, capital controls to manipulate a country's intertemporal terms-of-trade constitute a beggar-thy-neighbor policy and are Pareto inefficient. A national planner in a large country may face incentives to exert market power over the country's intertemporal terms of trade, i.e. the world interest rate. For example, if a lending country restricts its lending, it benefits from an increase in the world interest rate.

Such monopolistic behavior reduces the global gains from intertemporal trade and is Pareto-inefficient. If countries engage in such behavior, it is desirable to come to a global agreement that capital controls aimed at manipulating the world interest rate will not be used.

The lesson for international policy coordination is that it is important to distinguish between 'corrective' capital controls that are imposed to offset domestic externalities and 'distortive' capital controls that are designed to manipulate a country's terms of trade. The former are generally desirable, whereas the latter are always undesirable.

An additional motive for coordinating capital controls arises when policymakers face restrictions on the set of available policy instruments. For example, if capital controls not only correct distorted incentives to borrow/lend but also impose an additional cost arising from costly implementation or corruption, then there is scope for global coordination of capital account policies: a global planner recognizes that adjusting all capital controls worldwide by the same factor may reduce the distortions created by capital controls but would leave the marginal incentives of all actors in the world economy unaffected.

Literature There is a growing recent literature that finds that capital controls may improve welfare from the perspective of a single country if they are designed to correct domestic externalities. An important example are prudential capital controls that reduce the risk of financial crises, as analyzed in the small open economy literature by Korinek (2007, 2010, 2011b), Ostry et al. (2010, 2011) and Bianchi (2011). This paper provides a normative analysis of the resulting general equilibrium effects and discusses whether global coordination of such policies is desirable.[1] We find that in a benchmark case in which national regulators can optimally control domestic externalities, coordination is not indicated. By contrast, Bengui (2013) studies the role for coordination between national regulators in a multi-country framework of banking regulation. He shows that liquidity in the global interbank market is a global public good. In the presence of such global externalities, there exists a case for global coordination of liquidity requirements.

Earlier work by MacDougall (1960), Kemp (1962), Hamada (1966), Jones (1967) and Obstfeld and Rogoff (1996) investigated how a national planner of a large country in the world economy may impose capital controls to exert monopoly/monopsony power over intertemporal prices. As in optimal tariff theory, such policies are beggar-thy-neighbor, i.e. they improve national welfare at the expense of reducing overall global welfare. In a recent contribution to this literature, Costinot et al. (2011) analyze the optimal time path of monopolistic capital controls under commitment and show how they can be used to distort relative prices in goods markets. Our paper contrasts the global welfare effects of distortive (monopolistic) capital controls

[1] Ostry et al. (2012) discusses the multilateral aspects of policies to manage the capital account from a policy perspective.

with corrective capital controls that are designed to offset domestic externalities, as was invoked by a rising number of countries that have imposed such controls in recent years. Jeanne et al. (2012), Gallagher et al. (2012) and Ostry et al. (2012) discuss the desirability and the multilateral implications of capital controls from a policy perspective.

Persson and Tabellini (1995) show that coordination of national fiscal and/or monetary policies is desirable if countries have incentives to employ such policies to exert monopoly power over international prices. Korinek (2011a) analyzes the positive implications of prudential capital controls in a multi-country setting.

The link between reserve accumulation and real exchange rate valuation is also investigated in Rodrik (2008) and Korinek and Serven (2010). Ghosh and Kim (2009) and Jeanne (2012) show how a combination of capital controls and tax measures can be used to undervalue a country's real exchange rate. These papers look at the exchange rate effects of various capital account policies in a small open economy, whereas we focus explicitly on global general equilibrium effects.

Magud et al. (2011) provide a survey of the empirical literature on the effects of capital controls on the country imposing the controls. Forbes et al. (2011) and Lambert et al. (2011) investigate the spillover effects of capital controls empirically. They find evidence that when Brazil imposed capital controls, there was diversion of capital flows to other countries that were expected to maintain free capital flows.[2] To the extent that the capital controls imposed by Brazil were imposed to correct a domestic distortion, our analysis suggests that this was a Pareto-efficient equilibrium response and did not introduce distortions in the global allocation of capital.

2 Benchmark Intertemporal Model

We describe a world economy with $N \geq 2$ countries indexed by $i = 1, ...N$ and a single homogenous tradable consumption good. Time is indexed by $t = 0, ...$ The mass of each country i in the world economy is $m^i \in [0, 1]$, where $\Sigma_{i=1}^N m^i = 1$. (A country with $m^i = 0$ corresponds to a small open economy.)

2.1 Country Setup

Country i is inhabited by a unit mass of identical consumers indexed by $z \in [0, 1]$ who value the consumption c_t^i of a tradable good according to the utility function

$$U^i = \sum_{t=0}^{\infty} \beta^t u\left(c_t^i\right) \tag{1}$$

[2]Forbes et al. (2011) also document negative spillover effects on countries that were likely to follow the example of Brazil to impose controls.

where $u(\cdot)$ is a standard neoclassical period utility function and $\beta < 1$ is a time discount factor, which we assume constant across countries.[3] For simplicity we drop the index z of individual consumers from our notation.

A representative consumer in country i starts period t with an endowment of y_t^i of tradable goods and financial net worth b_t^i, where the initial financial assets b_0^i in period 0 are given. He chooses how much to consume and how much to save by purchasing b_{t+1}^i zero coupon bonds at a price $1/R_{t+1}$ that pay off one unit of tradable goods in period $t+1$, where R_{t+1} represents the gross world interest rate between periods t and $t+1$. His budget constraint in period t is given by

$$c_t^i + \frac{\left(1-\tau_{t+1}^i\right) b_{t+1}^i}{R_{t+1}} = y_t^i + b_t^i + T_t^i \tag{2}$$

	$\tau_{t+1}^i > 0$	$\tau_{t+1}^i < 0$
$b_{t+1}^i > 0$ (saver)	outflow subsidy	outflow tax
$b_{t+1}^i < 0$ (borrower)	inflow tax	inflow subsidy

Table 1: Interpretation of capital control τ_{t+1}^i

The variable τ_{t+1}^i is a proportional subsidy to bond purchases b_{t+1}^i/R_{t+1}. We assume that the required revenue is raised as a lump-sum tax $T_t^i = -\tau_{t+1}^i b_{t+1}^i/R_{t+1}$. Depending on the signs of b_{t+1}^i and τ_{t+1}^i, we can interpret the policy measure τ_{t+1}^i in a number of different ways, as captured by Table 1: If the country is a net saver, $b_{t+1}^i > 0$, then τ_{t+1}^i constitues a subsidy to saving, i.e. a subsidy to capital outflows. Since capital outflows go hand in hand with positive net exports and since there is a single homogenous good in our economy, we can also think of it as a subsidy to net exports. If the country is a net borrower, $b_{t+1}^i < 0$, then the variable τ_{t+1}^i can be interpreted as a tax on foreign borrowing, or a tax on capital inflows. Since capital inflows imply positive net imports, the tax can be thought of as an import tariff. To ensure that bond demand is bounded, we impose the assumption that $\tau_{t+1}^i < 1$ $\forall i, t$. In the following, we will loosely refer to τ_{t+1}^i as the "capital control" imposed in period t.

Since there is a single representative consumer, his borrowing/saving decisions map into the current account statistics of the economy. The term b_t^i represents the gross return on savings that the consumer receives at the beginning of period t. The fraction b_t^i/R_t captures how much the economy saved in period $t-1$ in order to receive b_t^i units of goods in period t. Therefore the interest earnings in period t are $b_t^i(1-1/R_t)$. The trade balance tb_t^i in period t equals the difference between new

[3] In the following analysis, we will motivate international borrowing and saving by differences in endowments or output and intertemporal consumption smoothing considerations. Temporary differences in discount factors would offer an alternative route.

savings and the value of bond holdings at the beginning of the period, $tb_t^i = y_t^i - c_t^i = b_{t+1}^i/R_{t+1} - b_t^i$. The current account balance ca_t^i is the sum of the trade balance and interest earnings, $ca_t^i = tb_t^i + b_t^i(1 - 1/R_t) = b_{t+1}^i/R_{t+1} - b_t^i/R_t$, and corresponds to the change in the net asset position of the country between the end of periods $t-1$ and t.

2.2 Strategies

Representative Consumer We write the utility maximization problem of a representative consumer in recursive form as

$$V^i\left(b_t^i\right) = \max_{c_t^i, b_{t+1}^i} u\left(c_t^i\right) + \beta V^i\left(b_{t+1}^i\right) \tag{3}$$

The consumer takes T_t^i, R_{t+1} and τ_{t+1}^i as given and maximizes utility subject to the budget constraint (2). This leads to the Euler equation

$$\left(1 - \tau_{t+1}^i\right) u'\left(c_t^i\right) = \beta R_{t+1} u'\left(c_{t+1}^i\right) \tag{4}$$

The Euler equation implies a bond demand function $b_{t+1}^i\left(R_{t+1}; \tau_{t+1}^i\right)$ that is strictly increasing in the capital control τ_{t+1}^i. Strictly speaking, bond demand b_{t+1}^i is an equilibrium object that depends on the entire path of future interest rates and capital controls, but it is useful to focus in particular on its dependence on $\left(R_{t+1}, \tau_{t+1}^i\right)$. We impose the following assumption:

Assumption 1 (Elasticity of Intertemporal Substitution) *The elasticity of intertemporal substitution is greater than the borrowing/consumption ratio of country i,*

$$\sigma\left(c_t^i\right) > -\frac{b_{t+1}^i/R_{t+1}}{c_t^i}$$

This common assumption guarantees that bond demand $b_{t+1}^i\left(R_{t+1}; \tau_{t+1}^i\right)$ is strictly increasing in R_{t+1} and can be inverted into an indirect bond demand function $R_{t+1}^i\left(b_{t+1}^i; \tau_{t+1}^i\right)$. The assumption is satisfied for all countries that are net savers and for net borrowers as long as their borrowing is not too large in comparison to consumption. See Appendix A.1 for a detailed derivation.

The implications of increases in the world interest rate on saving b_{t+1}^i/R_{t+1} (as opposed to bond holdings b_{t+1}^i) depend on two effects:

$$\frac{\partial\left(b_{t+1}^i/R_{t+1}\right)}{\partial R_{t+1}} = \frac{\partial b_{t+1}^i/\partial R_{t+1}}{R_{t+1}} - \frac{b_{t+1}^i}{\left(R_{t+1}\right)^2} = \frac{b_{t+1}^i}{\left(R_{t+1}\right)^2}\left(\eta_{bR}^i - 1\right)$$

The first term in the expression in the middle captures the substitution effect – a higher interest rate makes it more desirable to save, as we assumed. The second term captures the income effect. For net borrowers, both terms are positive. For large savers, the income effect may offset the substitution effect and may lead to smaller net savings b^i_{t+1}/R_{t+1} in response to an increase in the world interest rate.

For net borrowers and modest net savers, a rise in the world interest rate is associated with a decline in consumption, which is necessary so net savings can rise, $\partial c^i_t / \partial R_{t+1} < 0$. For large savers, the inequality may be reversed.

2.3 Externalities

We will introduce several types of externalities into our setup that have been discussed as rationales for imposing capital controls in the public policy debate, including learning externalities, aggregate demand externalities at the zero-lower-bound, and financial stability externalities. For now, we capture their commonalities in reduced form. We assume a national planner who recognizes that aggregate capital flows impose an externality that is captured by an extra utility term in the social welfare function $x^i_t(tb^i_t)$ that operates through the effects of capital flows on the trade balance tb^i_t. We assume that $x^i_t(\cdot)$ is weakly convex but satisfies $x^{i\prime}_t(tb^i_t) < u'(c^i_t)$ for all allocations that we consider. We use this setup to derive a number of general results. In the ensuing three sections, we will analyze the three cited examples of externalities in more depth and show that our general results apply to each of these cases by mapping them into this reduced form framework.

National Planner The national planner picks a series of capital controls $\left\{\tau^i_{t+1}\right\}_t$ to maximize consumer welfare. She recognizes that welfare depends on the externalities created by the trade balance, which is an aggregate variable that each individual consumer in country i takes as given. In a symmetric equilibrium, the trade balance is $tb^i_t = b^i_{t+1}/R_{t+1} - b^i_t$ and we denote the optimization problem of the planner as

$$W^i\left(b^i_0\right) = \max_{\left\{b^i_{t+1}\right\}_t} \sum_{t=0}^{\infty} \beta^t \left[u\left(y^i_t + b^i_t - b^i_{t+1}/R_{t+1}\right) + x^i_t\left(b^i_{t+1}/R_{t+1} - b^i_t\right)\right]$$

We assume that the planner acts competitively and takes the path of world interest rates as given. One interpretation for this is that country i represents a small open economy within a region of atomistic economies of total mass m^i, and therefore the national planner in each of the atomistic economies cannot affect the world interest rate. We will analyze the behavior of monopolistic national planners in large countries who internalize their market power over the world interest rate in section 7 on distortive capital controls.

The Euler equation of a national planner who acts competitively in world markets is

$$u'\left(c^i_t\right) - x^{i\prime}_t\left(tb^i_t\right) = \beta R_{t+1}\left[u'\left(c^i_{t+1}\right) - x^{i\prime}_{t+1}\left(tb^i_{t+1}\right)\right] \tag{5}$$

Lemma 1 *A national planner who acts competitively corrects the domestic externalities $x_t^i(\cdot)$ by imposing a capital control*

$$\tau_{t+1}^{i*} = \frac{x_t^{i\prime}\left(tb_t^i\right) - \beta R_{t+1} x_{t+1}^{i\prime}\left(tb_{t+1}^i\right)}{u'\left(c_t^i\right)} \tag{6}$$

Proof. By substituting the described capital control into the Euler equation of private agents (4), it can be seen that the control τ_{t+1}^{i*} replicates the Euler equation of the planner (5). The resulting allocation therefore replicates the optimal allocation chosen by the planner. ■

2.4 Equilibrium

Definition 1 (Competitive Equilibrium) *For given initial bond holdings $\{b_0^i\}_i$ and capital controls $\left\{\tau_{t+1}^i\right\}_{i,t}$, a competitive equilibrium of the world economy is given by consumption allocations $\{c_t^i\}_{i,t}$ and bond holdings $\left\{b_{t+1}^i\right\}_{i,t}$ as well as interest rates $\{R_{t+1}\}_t$ such that private consumers in each country i solve their optimization problem (3) subject to their budget constraint (2) and the global bond market clears,*

$$B_{t+1}\left(R_{t+1};\tau_{t+1}\right) := \sum_{i=1}^{N} m^i b_{t+1}^i\left(R_{t+1};\tau_{t+1}^i\right) = 0 \quad \forall t \tag{7}$$

In this expression, we define $\tau_{t+1} = \left\{\tau_{t+1}^i\right\}_i$ as the vector of capital controls across countries and B_{t+1} as the global excess demand for bonds in period t, which is by Assumption 1 strictly increasing in R_{t+1}.[4] For future use, we define rest-of-the-world bond holdings $B_{t+1}^{-i} = \sum_{j\neq i} m^j b_{t+1}^j$ and we define the upper-case variables C_t and Y_t as the weighted global sums of consumption $\{c_t^i\}_t$ and output $\{y_t^i\}_t$ and similarly for rest-of-the-world C_t^{-i} and Y_t^{-i}.

We define the laissez faire equilibrium (LF) as the competitive equilibrium that prevails if all capital controls are set to zero $\tau_{t+1}^i = 0\forall i, t$.

Furthermore, we define the Nash equilibrium among national planners (NP) as the competitive equilibrium that prevails if the national planner in each country i imposes the capital control $\tau_{t+1}^{i*}\forall i, t$ given by Lemma 1 to correct for domestic externalities while taking the capital controls of all other national planners and the world interest rate as given.

Let us now focus on the effects of changes in capital controls on the equilibrium of the world economy. We perform a comparative static exercise in which we assume that the national planner in country i increases her capital control by $d\tau_{t+1}^i > 0$.

[4] This is our analogon of the Marshall-Lerner condition that an increase in the world interest rate increases the global excess demand for bonds.

Lemma 2 (Effects of Capital Controls) *An increase in the capital control* $d\tau^i_{t+1} > 0$ *in country* i

1. *increases bond holdings* b^i_{t+1} *and saving* b^i_{t+1}/R_{t+1} *and reduces consumption* c^i_t *in country* i *for a given world interest rate* R_{t+1},

2. *if* $m^i > 0$, *it also reduces the world interest rate* R_{t+1} *and reduces bond holdings* B^{-i}_{t+1} *and saving* B^{-i}_{t+1}/R_{t+1} *while increasing consumption in the rest of the world.*

3. *The decline in the world interest rate benefits all borrowing countries and hurts all saving countries.*

Proof. Point 1 follows from implicitly differentiating the Euler equation of the consumer to express $\partial b^i_{t+1}/\partial\tau^i_{t+1} > 0$. We divide by R_{t+1} and apply the period t budget constraint to obtain the statements about saving and consumption.

For point 2, we apply the implicit function theorem to the global market clearing condition (7) to obtain

$$\frac{dR_{t+1}}{d\tau^i_{t+1}} = -\frac{m^i b^i_\tau}{B_R} < 0 \tag{8}$$

where the partial derivatives satisfy $B_R = \sum_i m^i b^i_R > 0$ and $B_\tau = m^i b^i_\tau > 0$. The decline in rest-of-the world bond holdings B^{-i}_{t+1} and saving B^{-i}_{t+1}/R_{t+1} and the increase in rest-of-the-world consumption C^{-i}_t follow from market clearing.

Point 3 is obtained by taking the derivative of the welfare function of individual countries

$$\frac{dW^j}{dR_{t+1}} = \beta^t \left[u'\left(c^j_t\right) - x^{j\prime}\left(tb^j_t\right)\right] \frac{b^j_{t+1}}{\left(R_{t+1}\right)^2} \gtreqless 0 \quad \text{depending on} \quad b^j_{t+1} \gtreqless 0$$

■

Intuitively, capital controls introduce a wedge into the Euler equation of consumers that raises desired bond holdings while reducing consumption today. This shifts the global excess demand for bonds $B_{t+1}\left(R_{t+1};\tau_{t+1}\right)$ upwards. For the global bond market to clear, a decline in the world interest rate is required, which makes the rest of the world supply fewer bonds (i.e. save less) and consume more. The decline in the interest rate benefits borrowers because they obtain credit at lower rates and hurts lenders because they earn less in interest.

Figure 1 illustrates our findings graphically for a world with two countries i, j of equal mass. $R^j\left(b^j\right)$ depicts the inverse bond supply of country j, $R^i\left(-b^i\right)$ represents the inverse bond demand in country i in the absence of capital controls. The intersection of the two, marked by R^{LF} and b^{LF}, indicates the laissez faire equilibrium of the economy. However, suppose that there is a negative externality associated with borrowing by country i. Then a competitive national planner would demand less borrowing, as indicated by $R^{i*}\left(-b^i\right)$, and impose a capital control τ^{i*} on borrowing

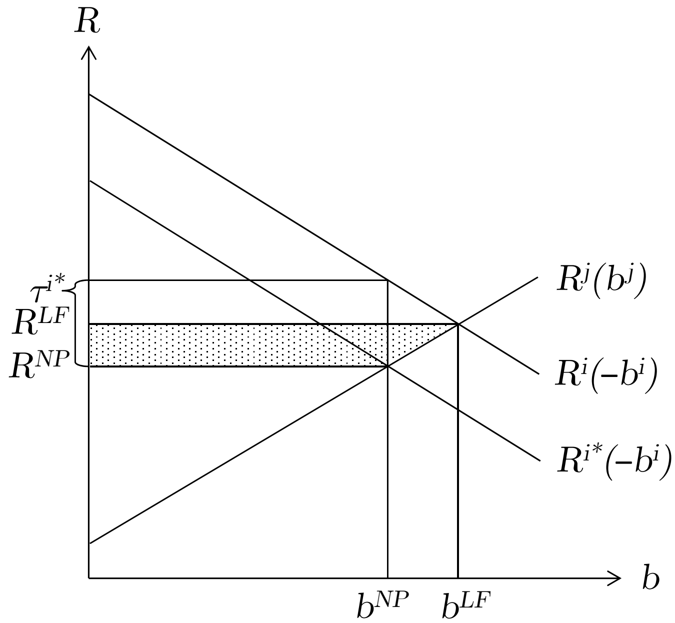

Figure 1: Optimal capital control to internalize domestic externalities

to make private agents internalize the externality. The resulting equilibrium exhibits less borrowing/lending b^{NP} and a lower world interest rate R^{NP}. Country j looses the surplus that is marked by the shaded area in the figure.

Numerical Illustration To illustrate the effects of changes in capital controls numerically, we consider an economy i of mass $m^i > 0$ with CES utility $u(c) = c^{1-1/\sigma}/(1-1/\sigma)$. Assume a steady state with $\beta R = 1$, $b^i = 0$, $\tau^i = 0$ and $c^i = y^i = C = Y$ constant. The two partial derivatives of the saving/output ratio b^i/y^i with respect to the capital control and the interest rate are

$$\frac{\partial b^i/y^i}{\partial \tau^i} = b^i_\tau/y^i = \frac{\sigma}{1+\beta}$$
$$\frac{\partial b^i/y^i}{\partial R} = b^i_R/y^i = \frac{\beta\sigma}{1+\beta}$$

An increase in the capital control or an increase in the world interest rate both increase the net savings of the country by approximately half of the intertemporal elasticity of substitution. (The second expression is pre-multiplied by β because interest is compounded in period $t+1$ whereas the capital control is imposed in period t.) For the standard value of the elasticity of substitution $\sigma = 1/2$, both an increase in the capital control or the interest rate result in an increase in domestic savings by approximately a quarter percent of GDP.[5]

[5] We note that there is considerable disagreement among economists about the value of the intertemporal elasticity of substitution. See e.g. Bansal and Yaron (2004) for a discussion. The

Country	GDP^i	$\$\Delta b^i/R$	$\Delta R/R$
World	$69,899bn	···	–1%
United States	$15,076bn	$30.2bn	–0.216%
China	$7,298bn	$16.7bn	–0.104%
Japan	$5,867bn	$13.7bn	–0.084%
Brazil	$2,493bn	$6.1bn	–0.036%
India	$1,827bn	$4.5bn	–0.026%
South Korea	$1,116bn	$2.8bn	–0.016%

Table 2: Effects of 1% capital control on saving and the world interest rate (Source: IMF 2011 IFS data and author's calculations for $\sigma = \frac{1}{2}$)

Global bond demand as a fraction of world output B/Y satisfies

$$\frac{\partial B/Y}{\partial R} = B_R/Y = \frac{\beta\sigma}{1+\beta}$$

We combine this with the expression $b^i_\tau/y^i = \frac{\sigma}{1+\beta}$ in equation (8) to find that the effect of capital controls in country i on the world interest rate is

$$\frac{dR_{t+1}/R}{d\tau^i_{t+1}} = -\frac{b^i_\tau/R}{B_R} = -m^i$$

In short, if a country that has a relative share m^i of world GDP imposes a 1% capital control, the world interest rate will decline by m^i %. Observe that this expression is independent of the intertemporal elasticity of substitution (as long as it is constant across countries).

Accounting for the adjustment in the world interest rate, the general equilibrium effect of a capital control in country i is mitigated to a fraction $(1 - m^i)$ of the partial equilibrium effect,

$$\frac{db^i/y^i}{d\tau^i} = b^i_\tau/y^i + b^i_R/y^i \cdot \frac{dR_{t+1}}{d\tau^i_{t+1}} = \left(1 - m^i\right)\frac{\sigma}{1+\beta}.$$

In Table 2, we illustrate the effects of capital controls on bond holdings and the world interest rate for a number of countries that were important players in global capital markets and/or currency wars in recent years. For example, Brazil represents 3.6% of the world economy. If the country increases a 1% capital control, it will reduce capital inflows by $6.1bn, which in turn lowers the world interest rate by 0.036% according to our calibration.

formulas we derived deliver transparent results for any value of the intertemporal elasticity of substitution preferred by the reader.

2.5 Welfare Analysis

Let us now analyze the global efficiency of capital controls that are imposed to correct domestic externalities. Even though the previous subsection illustrated that capital controls may lead to significant spillover effects, we show here that unilaterally imposed optimal controls nonetheless lead to a Pareto efficient global outcome. Then we proceed to investigating the scope for 'Pareto-improving' capital controls that make all countries better off, either by providing compensatory transfers or by regulating flows in both inflow and outflow countries.

We start our welfare analysis with the following result:

Proposition 1 (Efficiency of Unilaterally Correcting Externalities) *The Nash equilibrium among national planners (NP) in which each national planner imposes the capital control $\tau^{i*}_{t+1} \forall i, t$ given by lemma 1 to correct for domestic externalities is Pareto efficient.*

Proof. An allocation is Pareto efficient if it maximizes the weighted sum of welfare of all countries for some vector of welfare weights $\left\{\phi^i > 0\right\}_{i=1}^N$ subject to the global resource constraint $\sum_i m^i \left(y^i_t - c^i_t\right) = \sum_i m^i tb^i_t = 0 \; \forall t$, to which we assign the Lagrange multiplier $\beta^t \nu_t$,

$$\max_{\left\{tb^i_t\right\}_{i,t}} \sum_t \beta^t \left\{ \sum_i \phi^i m^i \left[u\left(y^i_t - tb^i_t\right) + x^i\left(tb^i_t\right)\right] + \nu_t \sum_i m^i tb^i_t \right\}$$

The optimality condition to this problem is

$$\phi^i \left[u'\left(c^i_t\right) - x^{i\prime}\left(tb^i_t\right)\right] = \nu_t \quad \forall i \tag{9}$$

The Nash equilibrium among national planners satisfies this optimality condition for all i, t if we normalize ν_0 to an arbitrary positive value and set $\phi^i = \nu_0 / \left[u'\left(c^i_0\right) - x^{i\prime}\left(tb^i_0\right)\right] \forall i$ and $\nu_{t+1} = \nu_t / \left(\beta R_{t+1}\right) \forall t > 0$. ■

The proposition is a version of the first welfare theorem. If each national planner determines her country's optimal bond demand and acts competitively, the general conditions of the first welfare theorem apply and the resulting competitive equilibrium is Pareto efficient. After each national planner has internalized her domestic externalities, her bond demand correctly reflects the country's social marginal rate of substitution between consumption today and tomorrow. In the global market for bonds, the marginal rates of substitution are all equated to the world interest rate and the resulting equilibrium is Pareto efficient.

The spillover effects on the world interest rate from imposing capital controls therefore constitute efficient pecuniary externalities. They reflect the response of the market to the balance of demand and supply for bonds. As we emphasized in Lemma 2, such pecuniary externalities entail redistributions between borrowers and lenders, but Proposition 1 shows that they do not lead to Pareto inefficiencies – Pareto optimality is independent of redistributive concerns.

Efficiency of Laissez-Faire Equilibrium A straightforward corollary to Proposition 1 is that the laissez faire equilibrium is generally not Pareto efficient if some countries are subject to externalities from international capital flows.

Arms Race of Capital Controls Optimally imposed capital controls may lead to dynamics that look like an arms race, but this does not necessarily indicate inefficiency. Suppose that there is one lending country l as well as two borrowing countries i and j that each have capital controls in place in order to correct a domestic externality $x_t^i\left(b_{t+1}^i\right)$, which is increasing and convex in b_{t+1}^i and similarly for country j. Assume that country i experiences an exogenous increase in $x_t^{i\prime}\left(b_{t+1}^i\right)$ that raises the optimal capital control τ_{t+1}^{i*}. As a result, the supply of capital to country j increases, i.e. b_{t+1}^j rises, and it is optimal for country j to raise its capital control as well. However, based on the response of country j, country i may find it optimal to increase its capital control τ_{t+1}^i yet further.

The resulting dynamics may give the appearance of an arms race but are nonetheless efficient. As long as the conditions of Proposition 1 are satisfied, this "arms race" is the mechanism of tatonnement through which the efficient equilibrium is restored. In the described example, each successive round of increases in capital controls will be smaller and the capital controls in the two countries τ_{t+1}^i and τ_{t+1}^j will converge towards the efficient levels, which involves higher capital controls in both borrowing countries and reduced lending by country l.

2.5.1 Implementation

Proposition 2 (Indeterminacy in Implementation) *Consider a competitive equilibrium allocation with initial bond holdings $\{b_0\}$ and capital controls $\left\{\tau_{t+1}^i\right\}_{i,t}$ that implement an allocation of consumption $\{c_t^i\}_{i,t}$ and bond holdings $\left\{b_{t+1}^i\right\}_{i,t}$. A global planner can implement the described consumption allocation via a continuum of alternative competitive equilibria, in which interest rates, capital controls and bond holdings differ.*

Proof. There is a one-to-one mapping between a given allocation of consumption $\{c_t^i\}_{i,t}$ and of trade balances $\{tb_t^i\}_{i,t}$ because $tb_t^i = y_t^i - c_t^i \ \forall i,t$. An competitive equilibrium implements a given allocation of trade balances if and only if

$$tb_t^i = \frac{b_{t+1}^i}{R_{t+1}} - b_t^i \ \forall i,t \tag{10}$$

In the following, we will demonstrate that a global planner can pick alternative combinations of bond holdings and interest rates to implement the same allocation of trade balances. We limit our attention to picking bond holdings and interest rates and refer to equation (4) for the implied level of capital controls.

We start from a given competitive equilibrium allocation in a world economy with N countries. Consider a global planner who perturbs bond holdings $\left\{b^i_{t+1}\right\}$ for T periods (for b^i_{t+1} indexed from $t+1$ to $t+T$) and perturbs interest rates $\{R_{t+1}\}$ for $T+1$ periods (from $t+1$ to $t+T+1$). This provides the planner with $(N-1)\,T$ degrees of freedom to determine bond holdings, since he needs to observe market clearing every period, and $T+1$ degrees of freedom to set interest rates for $T+1$ periods, for a total of $NT+1$ degrees of freedom. (The planner can always adjust capital controls in each country so as to make the chosen level of bond holdings optimal for consumers in each country, given the chosen interest rate.)

If the planner wants to make these perturbations while keeping consumption $\{c^i_t\}_{i,t}$ fixed at all times, he has to keep the trade balance given by equation (10) unchanged for all $i \in \{1, ...N\}$ and, in particular, for the $T+1$ periods from t to $t+T$ in which the perturbed variables enter the expression for the trade balance. Market clearing $\sum_i m^i b^i_{t+1} = 0$ implies that if equation (10) is satisfied for $N-1$ countries in a given period, it automatically holds for country N. Therefore the requirement to keep consumption fixed imposes $(N-1)\,(T+1)$ constraints on the planner. If the planner perturbs bond holdings and interest rates for sufficiently many periods $T \geq N-2$, then he has more degrees of freedom than constraints and there is a continuum of bond holdings and interest rates that implement the described consumption allocation. ■

The proposition reflects that there is an indeterminacy in implementation because the global planner has $NT+1$ independent instruments ($(N-1)\,T$ capital controls plus $T+1$ world interest rates) at his disposal to satisfy $(N-1)\,(T+1)$ independent equilibrium conditions. As we will see later, this indeterminacy hands the planner a powerful mechanism to circumvent binding financial constraints. To provide further intuition we consider the simplest possible example in which the planner perturbs bond holdings for $T=1$ period in a world economy with two countries $N=2$, satisfying the condition for indeterminacy, $T > N-2$:

Example 1 (Two Countries) *Assume a world economy with two countries i, j of equal mass and focus on the Nash equilibrium among national planners (NP). By market clearing, observe that $b^j_{t+1} = -b^i_{t+1}\ \forall t$. W.l.o.g. assume that country i is a lender $b^i_{s+1}, b^i_{s+2} > 0$ in periods s and $s+1$. A global planner can set $\tilde{b}^i_{s+1}$ to an arbitrary level $\tilde{b}^i_{s+1} > \max\left\{-tb^i_{s+1}, 0\right\}$ while leaving all consumption allocations $\left\{c^i_t, c^j_t\right\}_t$ as well as $\left\{b^i_{t+1}\right\}_{t\neq s}$ unchanged.*

Proof. For all consumption allocations to remain unchanged, the global planner has to observe two constraints

$$tb^i_s = \frac{\tilde{b}^i_{s+1}}{\tilde{R}_{s+1}} - b^i_s \qquad \text{and} \qquad tb^i_{s+1} = \frac{b^i_{s+2}}{\tilde{R}_{s+2}} - \tilde{b}^i_{s+1}$$

By market clearing the analogous conditions for country j are linearly dependent, reflecting that the planner is subject to $(N-1)\,(T+1) = 2$ constraints. The planner can therefore pick the three variables $\left\{\tilde{b}^i_{s+1}, \tilde{R}_{s+1}, \tilde{R}_{s+2}\right\}$ subject to only two constraints. If the planner picks bond holdings $\tilde{b}^i_{s+1} \neq b^i_{s+1}$, then he has to rescale $\tilde{R}_{s+1}$ by the same factor as bond holdings so that the fraction $\tilde{b}^i_{s+1}/\tilde{R}_{s+1} = b^i_{s+1}/R_{s+1}$ remains constant and the trade balance in period s is unchanged. Since the gross interest rate $\tilde{R}_{s+1}$ is restricted to be positive, this is only possible for $\tilde{b}^i_{s+1}$ that is of the same sign as b^i_{s+1}.[6] Furthermore, the planner has to change $\tilde{R}_{s+2}$ in the opposite direction so that the trade balance in period $s+1$ is unaffected,

$$\tilde{R}_{s+2} = \frac{b^i_{s+2}}{tb^i_{s+1} + \tilde{b}^i_{s+1}}$$

where the terms b^i_{s+2} and $tb^i_{s+1} + \tilde{b}^i_{s+1}$ need to be of equal sign for the gross interest rate to be positive. The capital controls required to implement these bond positions need to be scaled in proportion to the world interest rate to guarantee that the Euler equations of consumers hold in period s,

$$\frac{1 - \tilde{\tau}^i_{s+1}}{\tilde{R}_{s+1}} = \frac{1 - \tau^i_{s+1}}{R_{s+1}}$$

and similarly in period $s+1$. ■

In the two country example, the global planner has 3 instruments $\left\{\tilde{b}^i_{s+1}, \tilde{R}_{s+1}, \tilde{R}_{s+2}\right\}$ to meet 2 independent equilibrium conditions. In a given period s, a parallel increase in all capital controls τ^i_{s+1} and τ^j_{s+1} (i.e. a subsidy to outflows in country i and a tax on inflows in country j) reduces global demand for bonds and lowers the world interest rate proportionately to the change in capital controls. This redistributes from the saving country to the borrowing country. In the following period, capital controls τ^i_{s+2} and τ^j_{s+2} are reduced (i.e. a tax on outflows in country i and a subsidy to inflows in country j) to raise the world interest rate and redistribute from borrowers back to lenders. We will show below that this mechanism provides a global planner with a powerful tool to relax temporarily binding financial constraints in the world economy.

The example also highlights why the global planner generally needs to perform this perturbation over multiple periods if the number of countries is greater than two (except in knife-edge cases). If $N = 3$ but $T = 1$, we would allow the planner to choose bond holdings $\left\{b^i_{t+1}\right\}_i$ for only one period and interest rates $\{R_{t+1}, R_{t+2}\}$ for two periods. If the planner wants to perturb these variables without affecting consumption allocations in the economy, he has $NT + 1 = 4$ independent choice varibles but needs to satisfy the constraint (10) for periods t and $t+1$, implying

[6] If we impose the restriction that the gross interest rate is bounded above some positive value, for example because of the zero-lower-bound on the nominal interest rate, then the set of permissible perturbations is further reduced. Further implications of the zero-lower-bound on nominal interest rates are discussed in section 5.

$(N-1)(T+1)=4$ constraints. It is impossible for the planner to deviate from the original allocation while satisfying all four constraints, except in knife-edge cases when these constraints are linearly dependent.[7] One such knife-edge case is if we focus on a steady state equilibrium in which the bond positions of countries are unchanged over three consecutive time periods so that the constraints (10) for periods t and $t+1$ are linearly dependent. Another important example of such a knife edge case is the following:

Example 2 (Identical Countries) *Assume a world economy with N identical countries with zero bond holdings that experience domestic externalities and impose domestically optimal capital controls $\tau_{t+1}^{i*}>0\ \forall i$ in a given time period t. The consumption allocations of the Nash equilibrium among national planners (NP) can be replicated by setting the capital controls in all countries to zero, $\tau_{t+1}^{i}=0\ \forall i$.*

The intuition is that capital controls have no wealth effects from changes in the interest rate if bond positions $\left\{b_{t+1}^{i}\right\}_{i}$ are zero, as is the case for identical symmetric countries. Technically, the term b_{t+1}^{i}/R_{t+1} in constraint (10) is unaffected by a change in R_{t+1}. Lowering capital controls across all countries just implies a higher world interest rate, all consumption allocations are unchanged. We will take advantage of this result in section 8 to show how coordination between countries can be used to reduce implementation costs when capital controls are costly to impose.

2.5.2 Pareto-Improving Capital Controls

If the objective of a global planner is not to achieve Pareto efficiency but the more stringent standard of achieving a Pareto improvement, then capital controls generally need to be accompanied by transfers. Even if capital controls are Pareto efficient, they still lead to changes in the world interest rate (i.e. pecuniary externalities) that redistribute between borrowing and lending countries. In the following proposition, we show that access to lump-sum transfers enables a global planner to always implement a Pareto improvement when correcting for the domestic externalities in any number of economies.

Proposition 3 (Pareto-Improving Capital Controls, With Transfers) *Starting from the laissez faire equilibrium, a global planner who identifies domestic externalities $\left\{x_t^i\left(b_{t+1}^i/R_{t+1}\right)\right\}_{i=1}^N$ with at least one $x_t^{i\prime}(\cdot)\neq 0$ can achieve a Pareto improvement by setting capital controls in all countries such that $\tau_{t+1}^{i}=\tau_{t+1}^{i*}$ and engaging in compensatory international transfers $\hat{T}_t^i \lesseqgtr 0$ that satisfy $\sum_i \hat{T}_t^i=0$.*

[7] This is always the case in a steady state equilibrium in which the bond positions of all countries are unchanged over two consecutive periods.

Proof. Denote the saving/consumption allocations and the associated world interest rate in the laissez faire equilibrium by $\left\{c_t^{i,LF}, b_{t+1}^{i,LF}\right\}_{i,t}$ and $\left\{R_{t+1}^{LF}\right\}_t$ and in the global planner's equilibrium that results from imposing optimal capital controls τ_{t+1}^{i*} and transfers by $\left\{c_t^{i,GP}, b_{t+1}^{i,GP}\right\}_{i,t}$ and $\left\{R_{t+1}^{GP}\right\}_t$. Assuming the planner provides transfers $\hat{T}_t^i = c_t^{LF,i} - c_t^{GP,i} + \frac{b_{t+1}^{LF,i} - b_{t+1}^{GP,i}}{R_{t+1}^{GP}}$, then $\sum \hat{T}_t^i = 0$ since both sets of allocations (LF and GP) satisfy market clearing. Furthermore, given these transfers, consumers in each country i can still afford the allocation that prevailed in the laissez faire equilibrium. For non-zero capital controls, the allocation differs from the laissez faire equilibrium since the Euler equations (4) and (5) differ. Given that the old allocation is still feasible, revealed preference implies that every country is better off under the new allocation. ■

In an international context, compensatory transfers may be difficult to implement. As an alternative, we show that a planner who can coordinate the capital control policies of both inflow and outflow countries can correct the domestic externalities of individual economies while holding the world interest rate constant so that no wealth effects arise. As a result, the global planner's capital control policies constitutes a global Pareto improvement at a first-order approximation.

The following lemma demonstrates how a global planner can manipulate the world interest rate by simultaneously adjusting the capital controls in all countries worldwide; then we show how this mechanism can be used to undo changes in the world interest rate and the associated wealth effects if individual countries unilaterally impose capital controls.

Lemma 3 *Consider a competitive equilibrium with an allocation of bond holdings* $\left\{b_{t+1}^j\right\}_{j,t}$ *and a series of capital controls* $\left\{\tau_{t+1}^j\right\}_{j,t}$ *and world interest rates* $\left\{R_{t+1}\right\}_t$. *A global planner can increase the world interest rate in a given period by* dR_{t+1} *while keeping the bond allocations for all countries constant by moving the capital control in each country* $j = 1...N$ *by*

$$\frac{d\tau_{t+1}^j}{dR_{t+1}} = -\frac{b_R^j}{b_\tau^j} \tag{11}$$

Proof. We set the total differential of the bond demand function of a given country j to zero,

$$db_{t+1}^j = b_R^j dR_{t+1} + b_\tau^j d\tau_{t+1}^j = 0$$

and rearrange to obtain equation (11). Since $db_{t+1}^j = 0$, all future allocations are unaffected by the described perturbation. ■

Corollary 1 (Pareto-Improving Capital Controls, No Transfers) *Assume an exogenous marginal increase in the domestic externality in country* i *that raises the* *optimal unilateral capital control by $d\tau^{i*}_{t+1} > 0$ in period t. A global planner can correct for this while keeping the world interest rate constant $dR_{t+1} = 0$ to avoid income and wealth effects by adjusting*

$$\frac{d\tau^j_{t+1}}{d\tau^{i*}_{t+1}} = -\frac{m^i b^i_\tau}{B_R} \cdot \frac{b^j_R}{b^j_\tau} \qquad \text{and} \qquad \frac{d\tau^i_{t+1}}{d\tau^{i*}_{t+1}} = 1 - \frac{m^i b^i_R}{B_R}$$

In the resulting equilibrium, bond holdings $\left\{b^j_{t+1}\right\}_j$ are altered but, by the envelope theorem, welfare is unchanged at a first-order approximation.

Proof. If the planner implemented the unilaterally optimal increase $d\tau^{i*}_{t+1} > 0$ in the capital control of country i, then the world interest rate would move by $dR_{t+1}/d\tau^{i*}_{t+1} = -m^i b^i_\tau / B_R$. According to Lemma 3, the move in the interest rate can be undone if the capital controls of all countries $j = 1...N$ are simultaneously adjusted by $-d\tau^j_{t+1}/dR_{t+1} \cdot dR_{t+1}/d\tau^{i*}_{t+1}$, which delivers the first equation of the proposition. The second equation is obtained by adding the initial unilateral response of the capital control plus the adjustment given in the first equation with $j = i$. In the resulting equilibrium, the increase in the externality $d\tau^{i*}_{t+1}$ is corrected but the world interest rate is unchanged. Furthermore, for a constant world interest rate, the change in welfare that results from a marginal change in bond holdings is

$$\left.\frac{dW^j}{db^j_{t+1}}\right|_{dR_{t+1}=0} = -\frac{\beta^t \left(1 - \tau^j_{t+1}\right) u'\left(c^j_t\right)}{R_{t+1}} + \beta^{t+1} u'\left(c^j_{t+1}\right) = 0$$

■

For non-infinitesimal changes in capital controls, changes in bond holdings Δb^j_{t+1} have second-order effects on welfare (i.e. effects that are negligible for infinitesimal changes but growing in the square of the deviation) even if the world interest rate is held constant. Under certain conditions, e.g. if there are only two types of countries in the world economy, a global planner can undo these second-order effects via further adjustments in the world interest rate R_{t+1}.

In Figure 1 on page 10, a national planner corrects for a negative externality to borrowing τ^{i*} in country i. A global planner could achieve a Pareto improvement by splitting the burden of regulating capital flows between the two countries. He would tax outflows in country j such that $1 - \tau^j = R^{NP}/R^{LF}$ and tax inflows for the remaining part of the externality such that $1 - \tau^i = \frac{1-\tau^{i*}}{1-\tau^j}$ in country i. As a result, the interest rate would be unchanged at R^{LF} and the welfare loss by country i, indicated by the shaded area in the figure, would be limited to the Harberger triangle between b^{NP}, b^{LF} and R^{LF}.[8]

[8] Since there are only two countries in this example, country i could be compensated for this second order loss by raising the interest rate on the remainder of its bond holdings, as described in Lemma 3, achieving an unambiguous Pareto improvement.

Such a policy response shares certain characteristics with voluntary export restraints (VERs) in trade policy: If a borrowing country imposes controls on capital inflows, the world interest rate will decline and all lending countries experience negative wealth effects. However, if lenders restrict outflows by imposing controls of their own, they can keep the surplus. A global planner would share the burden of adjustment between borrower and lender in proportion to their elasticities of demand and supply so as to keep the world interest rate constant.

Numerical Illustration Assuming the economy starts in a steady state (as described in the illustration in section 2.4), an increase in the externality in country i that would call for an optimal unilateral response $d\tau_{t+1}^{i*}$ in the country's level of capital controls can also be corrected by setting

$$\begin{aligned} \frac{d\tau_{t+1}^{i}}{d\tau_{t+1}^{i*}} &= 1 - m^i \\ \frac{d\tau_{t+1}^{j}}{d\tau_{t+1}^{i*}} &= -m^i \end{aligned}$$

In short, the country that experiences the externality corrects only a fraction $(1 - m^i)$ of it and the rest of the world imposes a capital control to correct the remaining fraction m^i corresponding to the country's weight in the world economy. For example, small open economies would meet the burden of adjustment by themselves since $m^i = 0$ and they do not affect the world interest rate. For large economies, we refer to the country weights implied by Table 2 on page 11. For example, if China experienced a positive externality from current account surpluses that calls for a 1% unilateral subsidy to capital outflows, then a global planner who follows the described scheme would impose a 0.90% subsidy on outflows in China and a 0.10% subsidy to inflows in the rest of the world to keep the world interest rate unchanged. Similarly, if Brazil experienced a –1% externality from capital inflows, the global planner would impose a 0.97% tax on inflows to Brazil and a 0.03% tax on outflows in the rest of the world in order to keep the world interest rate stable.

3 Generalizations

This section generalizes our benchmark setup from the previous section along three dimensions. First, we extend the model to include a real exchange rate and delineate conditions under which there is an isomorphism between capital controls and real exchange rate intervention. Second, we show that our results are robust to introducing uncertainty either if there is only a single real bond traded in global capital markets or if there is a complete market and policymakers have a complete set of instruments. Third, we introduce investment and capital into our model to show that the equilibrium in a world economy in which each national planner unilaterally imposes her optimal capital control is still Pareto efficient.

3.1 Real Exchange Rates and Reserve Accumulation

Policymakers are often concerned about the effects of capital flows on the real exchange rate. To capture such effects, we extend our benchmark model to include a non-traded good in each country, which allows us to introduce a real exchange rate.

We distinguish variables that refer to traded versus non-traded goods by the subindices T and N. We maintain our assumption that there is a single homogenous traded good which is the numeraire good, and we denote the relative price of non-traded goods in country i by p_N^i, which constitutes a measure of the real exchange rate.[9] Observe that we index p_N^i by country i since the prices of non-traded goods in different countries are different.

A representative consumer in country i values tradable and non-tradable consumption according to the function

$$V^i\left(b_t^i\right) = \sum \beta^t u\left(c_{T,t}^i, c_{N,t}^i\right)$$

We denote the partial derivatives of the utility function as $u_T = \partial u / \partial c_{T,t}^i$ and similar for u_N, u_{NT} etc. We impose the assumptions $u_T > 0 > u_{TT}$, $u_N > 0 > u_{NN}$ and $u_{NT} u_T - u_N u_{TT} > 0$, i.e. the two goods are complements or at most mild substitutes in the utility function of domestic agents.[10]

The period 0 consumer budget constraint augmented by non-traded goods is

$$c_{T,t}^i + p_{N,t}^i c_{N,t}^i + \left(1 - \tau_{t+1}^i\right) b_{t+1}^i / R_{t+1} = y_{T,t}^i + p_{N,t}^i y_{N,t}^i + b_t^i + T_t^i \tag{12}$$

The consumer's optimality conditions are

$$\left(1 - \tau_{t+1}^i\right) u_T\left(c_{T,t}^i, c_{N,t}^i\right) = \beta R_{t+1} u_T\left(c_{T,t+1}^i, c_{N,t+1}^i\right) \tag{13}$$

$$p_{N,t}^i = \frac{u_N\left(c_{T,t}^i, c_{N,t}^i\right)}{u_T\left(c_{T,t}^i, c_{N,t}^i\right)} \tag{14}$$

The first optimality condition is analogous to the Euler equation (4) and defines bond demand b_{t+1}^i as an increasing function of the world interest rate R_{t+1} under Assumption 1. The second optimality condition states that the real exchange rate

[9] The official definition of the real exchange rate is the price of a consumption basket of domestic goods expressed in terms of a consumption basked of foreign goods. Ceteris paribus, a rise in the relative price of non-tradables increases the price of a consumption basket of domestic goods, implying a strictly monotonic relationship between the official real exchange rate and our measure p_N^i.

[10] Empirically, Mendoza (1995) and Stockman and Tesar (1995) find that traded and non-traded goods are clear complements.

is the marginal rate of substitution between traded and non-traded goods. Market clearing for non-traded goods requires that $c^i_{N,t} = y^i_{N,t}$.

The structure of the real exchange rate model is such that the results from our benchmark model carry over. To put it formally:

Corollary 2 (Real Exchange Rate Model) *For given levels of non-traded output $\left\{y^i_{N,t}\right\}_{i,t}$, the real exchange rate model is isomorphic to our benchmark model, with the real exchange rate being a strictly increasing function of tradable consumption $p^i_{N,t} = p^i_N(c^i_{T,t})$.*

Proof. To show the isomorphism, we define the utility function $u\left(c^i_{T,t}\right) = u\left(c^i_{T,t}, y^i_{N,t}\right)$ for each country by substituting the market-clearing condition for non-traded goods $c^i_{N,t} = y^i_{N,t}$. This utility function satisfies the restrictions required by the benchmark model. Non-traded consumption and endowment cancel from the budget constraint (12). The remaining problem is identical to our benchmark setup and leads to identical optimality conditions.

After substituting the market-clearing condition in (14), we observe that tradable consumption is the only endogenous variable driving the real exchange rate. The derivative

$$\frac{\partial p^i_{N,t}}{\partial c^i_{T,t}} = \frac{u_{NT}u_T - u_N u_{TT}}{(u_T)^2} > 0$$

is positive by our earlier assumptions on the utility function. ■

The intuition is that higher availability of traded goods implies that non-traded goods, which are in fixed supply in the economy, become relatively more valuable. Capital controls shift tradable consumption from one country to another, and the real exchange rate moves in line with tradable consumption. For example, an increase in the capital control in country i depreciates the exchange rate $\partial p^i_{N,t}/\partial \tau^i_{t+1} < 0$ there and tends to appreciate the real exchange rate in the rest of the world.

Remark Introducing an additional tax instrument $\tau_{N,t+1}$ on non-tradable consumption that is rebated lump-sum does not affect real allocations. Such a tax scales down the real exchange rate $p_{N,t}$ by a factor $1+\tau_{N,t+1}$, but market clearing implies that non-tradable consumption is unchanged. Therefore the tax does not affect the marginal utility of tradable consumption and the intertemporal Euler equation of consumers.[11]

[11] This observation would change if we endogenize the supply of non-tradable goods: a tax on non-tradable goods reduces their relative supply, which decreases the marginal utility of tradable goods u_T and induces consumers to save more in international capital markets.

3.1.1 Reserve Accumulation

We extend our framework to study reserve accumulation.[12] Assume a planner in country i accumulates bond holdings a_t on behalf of domestic consumers, where any accumulation/decumulation $a_{t+1}/R_{t+1} - a_t$ is financed/rebated via lump-sum transfers. We may think of these bond holdings as reserves. This changes the period t budget constraint of consumers to

$$c^i_{T,t} + p^i_{N,t} c^i_{N,t} + \frac{a^i_{t+1} + \left(1 - \tau^i_{t+1}\right) b^i_{t+1}}{R_{t+1}} = y^i_{T,t} + p^i_{N,t} y^i_{N,t} + a^i_t + b^i_t + T^i_t \tag{15}$$

In the following, we distinguish two diametrically opposed cases. We describe the capital account in an economy i as open when domestic consumers can trade international bonds b^i_{t+1}, as we have assumed so far. By contrast, we call the capital account closed when domestic consumers are forbidden from borrowing or saving abroad. This imposes the constraint $b^i_{t+1} = 0 \forall t$.

Proposition 4 (Reserve Accumulation) *(i) Under open capital accounts, domestic consumers undo any reserve holdings a^i_{t+1} by adjusting their private bond holdings such that $b^i_{t+1} = \tilde{b}^i_{t+1} - a^i_{t+1}$, where $\tilde{b}^i_{t+1}$ corresponds to the optimal choice of consumers in the absence of reserves.*

(ii) Under closed capital accounts, reserve accumulation cannot be undone. It reduces domestic consumption $\partial c^i_{T,t}/\partial a^i_{t+1} < 0$ and depreciates the real exchange rate $\partial p^i_{N,t}/\partial a^i_{t+1} < 0$ of country i. If the mass of the country is positive, it also reduces the world interest rate $\partial R_{t+1}/\partial a^i_{t+1} < 0$.

(iii) There is a one-to-one correspondence between a given level of capital controls τ^i_{t+1} under open capital accounts and a given amount of reserve accumulation a^i_{t+1} under closed capital accounts.

Proof. For part (i), assume an equilibrium with zero reserves $a^i_{t+1} = 0 \ \forall t$ and denote the associated level of private bond holdings by $\tilde{b}^i_{t+1}$. If a planner accumulates a non-zero level of reserves $a^i_{t+1} \neq 0$ in some periods, then an allocation in which private bond holdings satisfy $b^i_{t+1} = \tilde{b}^i_{t+1} - a^i_{t+1}$ will leave all other variables unchanged and will therefore satisfy the optimality conditions of the consumer.

If consumers have unconstrained access to capital markets, then reserve accumulation is ineffective, even if the planner has imposed price controls τ^i_{t+1} on international capital flows. What matters for the real allocations of the consumer is solely the level of capital controls τ^i_{t+1}, not the level of reserves a^i_{t+1}. This is a form of Ricardian

[12] This extension can be introduced either in our benchmark model or in the model with a real exchange rate. We opt for the latter since this naturally allows us to discuss the implications of reserve accumulation for the real exchange rate.

equivalence – a representative consumer internalizes that government bond holdings are equivalent to private bond holdings.[13]

Under closed capital accounts in part (ii), private agents are restricted to a zero international bond position $b^i_{t+1} = 0$ and international capital flows are solely determined by reserve accumulation. Reserve accumulation/decumulation constitutes forced saving/dissaving. The effects of reserve accumulation therefore mirror the effects of private capital flows in Proposition 2.

To show point (iii), we observe that a capital control τ^i_{t+1} under open capital accounts leads private consumers to accumulate $b^i_{t+1}\left(R_{t+1}; \tau^i_{t+1}\right)$ bonds and is therefore equivalent to reserve accumulation $a^i_{t+1} = b^i_{t+1}\left(R_{t+1}; \tau^i_{t+1}\right)$ under closed capital accounts. Since bond holdings $b^i_{t+1}\left(R_{t+1}; \tau^i_{t+1}\right)$ are strictly decreasing in τ^i_{t+1} and their range is $\Re$, any level of reserve accumulation can be replicated by a commensurate capital control τ^i_{t+1}. ■

Numerical Illustration We continue our numerical illustration to investigate the isomorphism between reserve accumulation and capital controls. Consider a small economy that is in steady state. An increase in reserve accumulation as a fraction of GDP a^i/y^i if the economy's capital account is closed is equivalent to an increase in capital controls if the economy's capital account is open of

$$\frac{\partial \tau^i}{\partial a^i/y^i} = \frac{1+\beta}{\sigma}$$

For the standard value of the intertemporal elasticity of substitution $\sigma = 1/2$, this term is approximately $\partial \tau^i/\partial(a^i/y^i) \approx 4$. In short, accumulating an extra percent of GDP in reserves under closed capital accounts is equivalent to imposing a 4% capital control under open capital accounts or, vice versa, a 1% capital control is equivalent to accumulating a quarter percent of GDP in reserves.

For more detailed numerical results, we refer back to Table 2 on page 11. In the Table, we illustrated that a 1% capital control improves the current account by $\Delta b^i/R$. But, given the isomorphism, we can read the table in both directions. If China, for example, accumulates an extra \$13bn in foreign reserves and its capital account is closed, this is equivalent to a 1% capital control under an open capital account. Similarly, if Brazil accumulates an extra \$5bn in foreign reserves under closed capital accounts, it is equivalent to a 1% capital control under fully open capital accounts. In practice, many developing countries that have liberalized their capital accounts exhibit intermediate values of capital account openness so that only part of their reserve accumulation is undone by private agents.

[13] The result is therefore subject to the same limitations as Ricardian equivalence. In particular, it critically relies on the assumption that consumers can access bond markets at the same conditions as governments.

3.2 Uncertainty

Our benchmark model can also be extended to incorporate uncertainty. This will be useful later on when we analyze targeting problems in imposing capital controls. Assume that at the beginning of each period t, a state of nature $\omega_t \in \Omega_t(\omega_{t-1})$ is realized with probability $\pi(\omega_t)$ where where $\sum_{\omega_t \in \Omega_t(\omega_{t-1})} \pi(\omega_t) = 1$ and where the set of possible outcomes $\Omega_t(\omega_{t-1})$ depends on the state of nature ω_{t-1} in the prior period. Through this dependence, the state ω_t captures the entire history of states of nature in the economy. We denote random variables as functions of the state of nature, for example, $y_t^i(\omega_t)$ denotes the stochastic endowment shock of the economy.

Assume a complete set of securities contingent on the next-period state of nature ω_{t+1} and denote a representative consumer i's holdings of securities contingent on state ω_{t+1} by $b_{t+1}^i(\omega_{t+1})$. The required return of a security that pays off one unit in state ω_{t+1} is $R_{t+1}(\omega_{t+1})$. We denote the inverse of this required return as the state price $q_{t+1}(\omega_{t+1}) = 1/R_{t+1}(\omega_{t+1})$, i.e the price of a security that pays off one unit in state ω_{t+1} of period $t+1$. Furthermore, we assume that a planner imposes a capital control $\tau_{t+1}^i(\omega_{t+1})$ on the issuance of such securities and rebates the net revenue as a lump sum $T_t^i = -\sum_{\omega_{t+1} \in \Omega_{t+1}} \tau_{t+1}^i(\omega_{t+1}) b_{t+1}^i(\omega_{t+1})$.

A representative consumer maximizes the expectation of his utility (1) subject to the period budget constraint

$$c_t^i(\omega_t) + \sum_{\omega_{t+1} \in \Omega_{t+1}(\omega_t)} \left(1 - \tau_{t+1}^i(\omega_{t+1})\right) q_{t+1}(\omega_{t+1}) b_{t+1}^i(\omega_{t+1}) = y_t^i(\omega_t) + b_t^i(\omega_t) + T_t^i(\omega_t)$$

We define a global excess demand function for state-contingent securities $B(q_{t+1}(\omega_{t+1}), \tau_{t+1}(\omega_{t+1}); \omega$ $\sum_{i=1}^N b^i\left(q_{t+1}(\omega_{t+1}), \tau_{t+1}^i(\omega_{t+1}); \omega_{t+1}\right)$ and impose market clearing $B_{t+1}(\omega_{t+1}) = 0$ for each state ω_{t+1}. The logic of proposition 2 implies that a capital control $\tau_{t+1}^i(\omega_{t+1})$ in country i and state ω_{t+1} pushes up the price $q_{t+1}(\omega_{t+1})$ of payoffs in that state, i.e. $\partial q_{t+1}(\omega_{t+1}) / \partial \tau_{t+1}^i(\omega_{t+1}) > 0$ and induces other countries to save less contingent on that state, i.e. reduce $b_{t+1}^j(\omega_{t+1})$.

3.3 Capital Investment

Our benchmark model can also easily be extended to include capital investment. Assume that output is a function of the capital stock $y_t^i = f(k_t^i)$ and that capital is augmented by investment i_t^i according to the law of motion $k_{t+1}^i = (1-\delta) k_t^i + i_t^i$. The budget constraint of a representative consumer in country i is modified to

$$c_t^i + \frac{\left(1 - \tau_{t+1}^i\right) b_{t+1}^i}{R_{t+1}} + i_t^i = f\left(k_t^i\right) + b_t^i - T_t^i$$

The consumption Euler equation of both domestic consumers and the national planner is unchanged. Optimal capital investment is pinned down by the condition

$$\begin{aligned} u'\left(c_t^i\right) &= \beta u'\left(c_{t+1}^i\right) \left[f'\left(k_{t+1}^i\right) + (1-\delta)\right] \\ \text{or} \quad R_{t+1} &= \left(1 - \tau_{t+1}^i\right) \left[f'\left(k_{t+1}^i\right) + (1-\delta)\right] \end{aligned}$$

For a given world interest rate, a positive capital control τ_{t+1}^i induces consumers to accumulate less capital. If a national planner has also access to an instrument targeting investment, such as an investment tax credit or subsidy to investment s_{t+1}^i, then the condition is modified to

$$R_{t+1} = \frac{1-\tau_{t+1}^i}{1-s_{t+1}^i} \cdot \left[f'\left(k_{t+1}^i\right) + (1-\delta) \right]$$

and the planner can insulate capital investment from the effects of capital controls by setting $s_{t+1}^i = \tau_{t+1}^i$.

However, for a given set of available policy instruments, a national planner and a global planner agree on the social benefits and costs of imposing capital controls, and Proposition 1 on the efficiency of the Nash equilibrium among national planners continues to hold. Adverse effects on capital investment are an important factor to consider for a national planner who considers what level of capital controls to impose, but they do not entail any differences for the multilateral implications of capital controls.[14]

In the following three sections, we will analyze the microfoundations of three different motivations for imposing capital controls or related capital flow management measures. We begin our analysis with learning externalities and show that such effects may justify the use of capital controls. We continue by studying aggregate demand externalities at the zero lower bound. Then we focus on capital controls to mitigate financial constraints in world capital markets.

4 Learning Externalities

This section applies our analysis of capital controls and international spillover effects to the case of production externalities that arise from learning effects. There is a considerable theoretical literature that postulates that such effects are important for developing countries in the phase of industrialization. See for example Rodrik (2008) and Korinek and Servén (2010). In the empirical literature there have been some studies that document the existence of learning externalities, whereas others are more skeptical. For a survey see e.g. Giles and Williams (2000).

Even if one is skeptical of the existence of learning externalities, this is an important question to analyze since policymakers have explicitly invoked negative externalities on industrial development when they imposed capital controls, exemplified by numerous comments of the Brazilian finance minister Guido Mantega (see Wheatley and Garnham, 2010). We leave the debate on the validity of learning effects to other

[14]The global planning equilibrium even continues to be subject to the same indeterminacy that we identified in proposition 2, since what is relevant for domestic investment decisions is the domestic interest rate (post capital controls), which differs from the world interest rate.

works and focus our analysis here on the international dimensions of capital account policies that are designed to internalize such effects.

We investigate the regulation of such externalities and the resulting spillover and efficiency effects in two separate examples in which capital controls represent first-best and second-best instruments respectively to address learning-by-exporting externalities and learning-by-doing externalities.

4.1 Learning-by-Exporting Externalities

Assume that output growth Δy_{t+1}^i in economy i is a continuous increasing function of the economy's trade balance $tb_t^i = \bar{b}_{t+1}^i / R_{t+1} - \bar{b}_t^i$ in the previous period,

$$y_{t+1}^i = y_t^i + \Delta y_{t+1}^i \left(\bar{b}_{t+1}^i / R_{t+1} - \bar{b}_t^i \right) \tag{16}$$

where $\bar{b}_t^i$ denotes the aggregate bond holdings of the economy. The representative consumer in economy i optimizes utility (1) subject to the budget constraint (2), but does not internalize his contribution to aggregate bond holdings.

Learning-by-exporting effects are most relevant for developing economies that are on a convergence path, not for economies that have already reached the world technology frontier (see e.g. Rodrik, 2008). We capture the temporary nature of such externalities here by making the simplifying assumption that $\Delta y_t^i \left(tb_t^i \right) \equiv 0 \; \forall t \geq 2$, i.e. the economy under consideration experiences learning-by-exporting effects only between periods 0 and 1 but not beyond. A characterization of the general problem and detailed steps of the derivation are provided in appendix A.2.

National Planner The Euler equation of a national planner who internalizes the externalities in period $t = 0$ is

$$u' \left(c_t^i \right) = \beta R_{t+1} u' \left(c_{t+1}^i \right) + \beta v_{t+1} \Delta y_{t+1}^{i\prime} \left(tb_t^i \right)$$

where we denote by $v_{t+1} = \sum_{s=0}^{\infty} \beta^s u' \left(c_{t+s+1}^i \right)$ the cumulative value of an additional marginal unit of consumption in every future period. In short, the planner internalizes that an increase in the trade balance in period 0 raises output by $\Delta y_{t+1}^{i\prime}$ in every future period, which has utility value v_{t+1}. She finds it optimal to impose a capital control to induce private consumers to internalize the externality,

$$\tilde{\tau}_{t+1}^i = \frac{\beta v_{t+1} \Delta y_{t+1}^{i\prime} \left(tb_t^i \right)}{u' \left(c_t^i \right)} \tag{17}$$

Observe that this capital control is a first-best policy tool to internalize learning-by-exporting externalities in our framework, since it directly targets net saving and hence the trade balance of the economy.

Global Planner A global planner who internalizes the learning-by-exporting externalities would arrive at an optimality condition for each country i

$$u'\left(c_t^i\right) = \frac{\lambda_t}{\lambda_{t+1}} u'\left(c_{t+1}^i\right) + \beta v_{t+1} \Delta y_{t+1}^{i\prime}\left(tb_t^i\right)$$

where λ_t represents the shadow price on the resource constraint of the global economy in period t. Since the relative marginal valuation of resources between periods t and $t+1$ is given by βR_{t+1}, we find that the optimality condition coincides with those of the national planners in countries $i = 1, ...N$.

Proposition 5 (Efficiency of Unilaterally Correcting LBE-Externalities) *The global equilibrium in which each national planner i corrects domestic learning-by-exporting externalities by imposing the unilaterally optimal capital control $\tilde{\tau}_{t+1}^i$ is Pareto efficient.*

Proof. See appendix A.2. ■

The intuition behind the result is that a national planner who does not internalize market power fully internalizes the social benefits and costs of her policies. Since there are no frictions in the global capital market itself in our model, the market interest rate βR_{t+1} correctly reflects the social marginal rate of substitution between different time periods, as can be seen from comparing the Euler equations of a national planner and the global planner. As long as national planners take the market interest rate as given, it guides their allocations in a socially efficient way. In a sense, we can view each national planner as a competitive agent and apply the welfare theorems to the global economy.

4.2 Learning-by-Doing Externalities

Capital controls may also serve as a second-best instrument to internalize learning-by-doing externalities in a production economy in which productivity is an increasing function of employment, but no direct (first-best) policy instrument to subsidize employment is available. Assume that the output of a representative worker in economy i is given by $y_t^i = A_t^i \ell_t^i$, where labor ℓ_t^i imposes a convex disutility $d\left(\ell_t^i\right)$ on workers and where the growth in productivity A_t^i is an increasing function of aggregate employment $\bar{\ell}_t^i$,

$$A_{t+1}^i = A_t^i + \Delta A_{t+1}^i\left(\bar{\ell}_t^i\right)$$

In the described economy, the first-best policy instrument to internalize such learning effects would be a subsidy s_t^i to wage earnings in the amount of

$$s_t^i = \frac{\beta \Delta A_{t+1}^{i\prime}\left(\ell_t^i\right) V_{A,t+1}}{u'\left(c_t^i\right) A_t^i}$$

where $V_{A,t} = \sum_{s=t}^{\infty} \beta^{s-t} u'\left(c_s^i\right) \ell_s^i$ is the present value of a permanent marginal increase in productivity.

If a first-best instrument is not available (for example because of a large informal sector or because of the risk of corruption), the national planner may be able to resort to capital controls to improve the equilibrium. In the constrained second-best policy problem, a planner maximizes

$$\max V\left(b_t^i, A_t^i\right) = u\left(c_t^i\right) - d\left(\ell_t^i\right) + \beta V\left(b_{t+1}^i, A_t^i + \Delta A_{t+1}^i\left(\ell_t^i\right)\right)$$

subject to the standard budget constraint and to an implementability constraint that reflects the restriction on the set of instruments,

$$\begin{aligned} c_t^i &= A_t^i \ell_t^i + b_t^i - b_{t+1}^i / R_{t+1} \\ A_t^i u'\left(c_t^i\right) &= d'\left(\ell_t^i\right) \end{aligned} \tag{18}$$

As in our earlier example, we assume for simplicity that learning-by-doing externalities are only present in the first period $t = 0$. (Appendix A.3 provides a more general treatment.) Then the Euler equation of a national planner is

$$\begin{aligned} u'\left(c_t^i\right) + \mu_t^i A_t^i u''\left(c_t^i\right) &= \beta R_{t+1} u'\left(c_{t+1}^i\right) \\ &\quad \text{where} \quad \mu_t^i = \frac{\beta W_{A,t+1} \Delta A_{t+1}^{i\prime}\left(\ell_t^i\right)}{d''\left(\ell_t^i\right) - \left(A_t^i\right)^2 u''\left(c_t^i\right)} > 0 \end{aligned} \tag{19}$$

Intuitively, the shadow price μ_t^i captures the welfare benefits of learning-by-doing externalities that can be reaped from relaxing the implementability constraint (18), i.e. from inducing consumers to work harder. The Euler equation reflects that each unit of consumption not only benefits consumers by providing utility directly $u'\left(c_t^i\right) > 0$ but also has the effect of reducing the incentive to work since it lowers the marginal utility $\mu_t^i A_t^i u''\left(c_t^i\right) < 0$. The planner can implement this second-best solution by imposing a capital control

$$\tilde{\tau}_{t+1}^i = -\frac{\mu_t^i A_t^i u''\left(c_t^i\right)}{u'\left(c_t^i\right)} \tag{20}$$

This control encourages consumers to save more, which reduces their consumption and makes them work harder. Put differently, the capital control reduces capital inflows and stimulates demand for domestic production, which in turn triggers learning-by-doing externalities.

Again, we compare the Nash equilibrium between national planners to the constrained optimum that would be chosen by a global planner.

Proposition 6 (Efficiency of Unilaterally Correcting LBD-Externalities) *The global Nash equilibrium in which each domestic planner i imposes the unilaterally second-best capital control $\tilde{\tau}_{t+1}^i$ to correct domestic learning-by-doing externalities is constrained Pareto efficient.*

Proof. See appendix A.3. ■

Intuitively, even though the capital control (20) is just a second-best instrument, it is chosen so as to equate the marginal social benefit from indirectly triggering the LBD-externality to its marginal social cost. Reducing domestic consumption by running a trade surplus is the only way that a planner can induce domestic agents to work harder. Since the global planner does not have any superior instruments, he cannot do better than this and chooses an identical allocation.

Remark There has been a lively debate on the multilateral desirability of capital controls (see e.g. IMF, 2012; Forbes et al., 2012; Ostry et al., 2012). It has sometimes been suggested that capital controls that are imposed as a second-best measure, such as those to internalize LBD externalities in our example above, should be viewed with particular skepticism. This section establishes that the global efficiency implications of second-best capital controls to internalize LBD-externalities are no different from other reasons to implement capital controls, given the restrictions on the set of policy instruments. A global planner who faces the same constraints on his policy instruments would implement an identical allocation. The second-best nature of capital controls is only relevant in the debate on global coordination if a global planner has access to a superior set of policy instruments than national planners.

5 Aggregate Demand Externalities at the ZLB

In this section we study the multilateral implications of capital controls that are imposed to counter aggregate demand externalities at the zero lower bound (ZLB) on nominal interest rates. We develop a stylized framework that captures the essential nature of such externalities in the spirit of Krugman (1998) and Eggertsson and Woodford (2003), adapted to an open economy framework as in Jeanne (2009).

Assume that a representative consumer in country i derives utility from consuming c_t^i units of a composite final good and experiences disutility from providing ℓ_t^i units of labor, expressed in recursive form as

$$V^i\left(b_t^i\right) = u\left(c_t^i\right) - d\left(\ell_t^i\right) + \beta V\left(b_{t+1}^i\right) \quad \text{where} \quad c_t^i = y_t^i + b_t^i - \left(1 - \tau_{t+1}^i\right) b_{t+1}^i / R_{t+1} - T_t^i$$

where y_t^i represents the sum of his wage income and profits. As is common in the New Keynesian literature, we assume that there is a continuum $z \in [0,1]$ of monopolistic intermediate goods producers who are collectively owned by consumers and who each hire labor to produce an intermediate good of variety z according to the linear function $y_t^{iz} = \ell_t^{iz}$, where labor market clearing requires $\int \ell_t^{iz} dz = \ell_t^i$. All the varieties are combined in a CES production function to produce final output

$$y_t^i = \left(\int_0^1 \left(y_t^{iz}\right)^{\frac{\varepsilon-1}{\varepsilon}} dz\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where the elasticity of substitution is $\varepsilon > 1$. We assume that the monopoly wedge arising from monopolistic competition is corrected by a proportional subsidy $\frac{1}{\varepsilon-1}$ that is financed by a lump-sum tax on producers. This implies that the wage income and profits of consumers equal final output, which in turn equals labor supply $y_t^i = \ell_t^i$.

Let us denote the constraints of consumers and monopolistic producers in nominal terms and introduce price stickiness so as to clarify how the zero lower bound on interest rates affects aggregate demand and ouput. We assume that prices are set one period in advance and that intermediate goods producers are committed to satisfying the demand that they experience at the given price every period. We normalize the price of the final good in period t to $p_t^i = 1$ and assume that the monetary authority is expected to implement an exogenous inflation rate π_{t+1}^i between periods t and $t+1$.[15] Then intermediate goods producers find it optimal to set prices $p_{t+1}^{iz} = \left(1 + \pi_{t+1}^i\right) p_t^i$, and given the symmetric production function this implies an aggregate price level $p_{t+1}^i = \left(1 + \pi_{t+1}^i\right) p_t^i$. For simplicity, we assume that after period $t+1$ prices are perfectly flexible and price stickiness is irrelevant.

The nominal period t budget constraint of the representative consumer is

$$p_t^i c_t^i = p_t^i y_t^i + p_t^i b_t^i - p_{t+1}^i b_{t+1}^i / \left(1 + i_{t+1}^i\right) - T_t^i$$

For a given inflation rate, the zero-lower bound on the domestic nominal interest rate $i_{t+1}^i \geq 0$, in combination with the Euler equation of consumers, can be expressed as

$$u'\left(y_t^i + b_t^i - b_{t+1}^i / R_{t+1}\right) \geq \frac{\beta}{1 + \pi_{t+1}^i} u'\left(y_{t+1}^i + b_{t+1}^i - b_{t+2}^i / R_{t+2}\right) \tag{21}$$

The higher inflation, the lower the minimum permissible consumption growth rate of consumers.

No arbitrage implies that the domestic nominal interest rate has to satisfy

$$1 + i_{t+1}^i = \frac{1 + \pi_{t+1}^i}{1 - \tau_{t+1}^i} R_{t+1}$$

For given inflation and capital controls, avoiding the zero-lower bound on the domestic nominal interest rate $i_{t+1}^i \geq 0$ requires that the world real interest rate satisfies

$$R_{t+1} > \bar{R}_{t+1}^i := \frac{1 - \tau_{t+1}^i}{1 + \pi_{t+1}^i} \tag{22}$$

The higher the domestic inflation rate π_{t+1}^i or the tax on inflows (subsidy on outflows) τ_{t+1}^i, the lower the world interest rate can be without plunging country i into a liquidity trap.

[15] It is well known in the New Keynesian literature that the problems associated with the zero lower bound can be avoided if the monetary authority is able to commit to a higher inflation rate. See e.g. Eggertsson and Woodford (2003).

In the laissez-faire equilibrium, this constraint is satisfied if world demand for bonds and by extension the world interest rate is sufficiently high, i.e. if $R_{t+1} \geq \frac{1}{1+\pi^i_{t+1}}$, because optimal consumption smoothing by domestic consumers implies sufficiently high consumption growth. If the ZLB constraint is slack, then output y^i_t is determined by the optimality condition

$$u'\left(c^i_t\right) = d'\left(\ell^i_t\right)$$

We call the output level implied by this condition potential output y^{i*}_t.

If the world real interest rate hits the threshold $R_{t+1} = \frac{1}{1+\pi^i_{t+1}}$, then the zero lower-bound on the nominal interest rate is reached in the laissez faire equilibrium and the domestic interest rate cannot fall any further. Instead, any increase in the world supply of bonds will flow to economy i, which offers a real interest rate of $\frac{1}{1+\pi^i_{t+1}}$. In economy i, today's consumption goods are too expensive compared to tomorrow's consumption goods and consumers reduce their aggregate demand for today's consumption goods. Since output is demand-determined, y^i_t falls below potential output y^{i*}_t in order to satisfy equation (21).

This situation captures the essential characteristic of a liquidity trap: at the prevailing nominal interest rate of zero, consumers do not have sufficient demand to absorb both the output from their domestic economy and the capital inflows b^i_{t+1}. Intermediate producers cannot reduce their prices but let domestic output adjust so that demand equals supply. If the domestic real interest rate could fall, consumers would have incentive to consume more in period t and the problem would be solved.

National Planner A planner in the economy recognizes that output capacity is wasted when the economy hits the zero lower bound since $u'\left(c^i_t\right) > d'\left(y^i_t\right)$. She can insulate her domestic economy from low world interest rates and aggregate demand by erecting barriers against capital inflows. As captured by equation (22), the world real interest rate at which a country's nominal interest rate hits the zero lower bound is a declining function of the capital control. A national planner internalizes that $y^i_t = \ell^i_t$ and sets the capital controls so as to implement the level of borrowing that solves

$$\max_{y^i_t, b^i_{t+1}} u\left(y^i_t + b^i_t - b^i_{t+1}/R^i_{t+1}\right) - d\left(y^i_t\right) + \beta V\left(b^i_{t+1}\right) \quad \text{s.t.} \quad (21)$$

Assigning the multiplier μ^i_t to the ZLB constraint, the associated optimality conditions are

$$FOC\left(y^i_t\right) \quad : \quad u'\left(c^i_t\right) = d'\left(y^i_t\right) - \mu^i_t u''\left(c^i_t\right) \tag{23}$$

$$FOC\left(b^i_{t+1}\right) \quad : \quad u'\left(c^i_t\right) = \beta R_{t+1} u'\left(c^i_{t+1}\right) - \mu^i_t \left[u''\left(c^i_t\right) + \frac{\beta R_{t+1}}{1+\pi^i_{t+1}} u''\left(c^i_{t+1}\right)\right]$$

If the ZLB constraint is slack ($\mu^i_t = 0$), these reduce to the standard optimality conditions and imply that the national planner finds it optimal to impose zero capital controls.

If the ZLB constraint is binding, on the other hand, then the first optimality condition of the planner implies that output is reduced below potential. The planner perceives the utility cost of the ZLB constraint as

$$\mu_t^i = \frac{u'\left(c_t^i\right) - d'\left(y_t^i\right)}{-u''\left(c_t^i\right)}$$

She recognizes that borrowing less from abroad (saving more) increases domestic demand and relaxes the ZLB constraint on domestic output, as captured by the term in square brackets in the second optimality condition. She implements her optimal allocation by setting the capital control τ_{t+1}^i such that the Euler equation of private agents replicates her optimality condition (23),

$$\begin{aligned}\tau_{t+1}^i &= \frac{-\mu_t^i \left[u''\left(c_t^i\right) + \frac{\beta R_{t+1}}{1+\pi_{t+1}} u''\left(c_{t+1}^i\right)\right]}{u'\left(c_t^i\right)} = \\ &= \frac{u'\left(c_t^i\right) - d'\left(y_t^i\right)}{u'\left(c_t^i\right)} \left[1 + \frac{\beta R_{t+1}}{1+\pi_{t+1}^i} \cdot \frac{u''\left(c_{t+1}^i\right)}{u''\left(c_t^i\right)}\right] > 0\end{aligned}$$

Intuitively, the tax rate reflects the loss from giving up socially profitable production opportunities $u'\left(c_t^i\right) - d'\left(y_t^i\right)$ in relation to the cost of relaxing the ZLB by deviating from optimal borrowing/lending with foreigners.

6 Financial Constraints and Pecuniary Externalities

This section analyzes how capital controls can be employed to deal with financial constraints in international capital markets. We delineate circumstances under which a global planner can fully circumvent financial constraints. Even though the conditions necessary for this may not always be met in practice, they are instructive for how globally coordinated capital controls may contribute to mitigating financial constraints. Next we study prudential capital controls that are imposed to alleviate domestic pecuniary externalities as in Korinek (2010).

6.1 Financial Constraints and Welfare Effects

Assume that consumers in country i are subject to a commitment problem that limits how much they can borrow from international lenders.[16] For now, we assume that consumers may threaten to abscond and renegotiate their debts after obtaining loans. If they do so, international lenders can take them to court and seize at most $-\phi^i > 0$

[16] Since all agents within a given economy are identical, there is no domestic bond market.

from them, which is a country-specific constant that reflects the quality of creditor protections in country i. To avoid absconding, lenders impose a constraint on new borrowing of[17]

$$\frac{b_{t+1}^{i}}{R_{t+1}} \geq \phi^{i} \tag{24}$$

When this constraint (24) is binding, equilibrium borrowing is determined by $b_{t+1}^{i}/R_{t+1} = \phi^{i}$, and there is a wedge in the Euler equation of constrained consumers that corresponds to the shadow price of the constraint λ_{t+1}^{i},

$$\left(1-\tau_{t+1}^{i}\right) u^{i\prime}\left(c_{t}^{i}\right) = \beta R_{t+1} u^{i\prime}\left(c_{t+1}^{i}\right) + \lambda_{t+1}^{i} \tag{25}$$

The welfare effects of marginal changes in ϕ^{i} depend on both the constraint itself and on the resulting general equilibrium effect on the world interest rate. The marginal welfare cost of tightening the constraint $d\phi^{i} > 0$ for a given interest rate is is λ_{t+1}/R_{t+1}. The tighter borrowing limit reduces the effective global demand for bonds and reduces the world interest rate by

$$\frac{dR_{t+1}}{d\phi^{i}} = -m^{i}\frac{\partial R_{t+1}}{\partial B_{t+1}^{-i}} < 0$$

which is always beneficial for country i since a constrained country is by definition a borrower. The total welfare effect is the sum of the two,

$$\frac{dW_{t}^{i}}{d\phi^{i}} = u_{T,t}^{i} m^{i} \eta_{RB^{-i}} - \frac{\lambda_{t+1}^{i}}{R_{t+1}} \gtrless 0 \tag{26}$$

For relatively lax borrowing constraints in large economies with $m^{i} > 0$, the interest effect is larger and the constrained country benefits from a tightening of the constraint. This may seem counter-intuitive, but recall that a tighter constraint moves the country closer to the level of borrowing that would be chosen by a monopolistic planner who internalizes the country's market power. For relatively tight constraints, the welfare cost of the constraint outweighs any positive terms of trade effects on the world interest rate. The cutoff at which $dW_{t}^{i}/d\phi^{i} = 0$ corresponds to the monopolistic level of borrowing that is described in more detail in section 7.

The interest rate effects of one country's tightening borrowing limit on other countries are identical to those described in equation (XXX) – they improve the welfare of other borrowing countries (who compete for funds) and reduce the welfare of lending countries (who experience a decline in the effective demand for their lending).

[17] Our main findings are unaffected if we impose the constraint on repayments $b_{t+1}^{i} \geq \phi^{i}$ instead of new borrowing.

6.2 Restoring the First-Best Allocation

Binding financial constraints impede optimal consumption smoothing and therefore pose a challenge to a planner who wants to equate the marginal rates of substitution of different agents. Here we delineate circumstances under which a global planner can in fact employ capital controls to fully undo the effects of financial constraints $\phi^i < 0$ in a two-country framework.

The planner can do so by taking advantage of the indeterminacy in the setting of capital controls and the world interest rate that we identified in proposition 1. What matters for the decisions of decentralized agents is the fraction $\frac{R_{t+1}}{1-\tau^i_{t+1}}$ not the levels of the interest rate and the capital controls.

Although the conditions necessary for restoring the first-best are unlikely to be met in practice, they are instructive for how globally coordinated capital controls may contribute to mitigating financial constraints. nces, a global planner can restore the first-best equilibrium in our setup.

Proposition 7 (Restoring the First-Best) *In a world with two countries i, j that are subject to the financial constraint (24), a global planner who can determine the capital controls τ^i_t, τ^j_t of both countries can implement the first-best equilibrium.*

Proof. Denote variables in the first-best allocation by $\{c^{i*}_t\}$, $\left\{b^{i*}_{t+1}\right\}$ and $\left\{R^*_{t+1}\right\}$ and focus on a period t in which the first-best level of new borrowing is below what the financial constraint permits, i.e. $b^{i*}_{t+1}/R^*_{t+1} < \phi^i$. The global planner implements the first-best allocation by reducing both the repayment and the new borrowing in period t by the excess over the borrowing limit $\Delta = \phi^i - b^{i*}_{t+1}/R^*_{t+1}$, i.e. by setting $b^i_t = b^{*i}_t + \Delta$ and $b^i_{t+1}/R_{t+1} = b^{*i}_{t+1}/R^*_{t+1} + \Delta = \phi^i$. This leaves period t consumption unchanged. At the same time, the planner uses his control over the interest rates R_t and R_{t+1} to keep borrowing in the previous period $b^i_t/R_t = b^{*i}_t/R^*_t$ and the repayment next period $b^i_{t+1} = b^{*i}_{t+1}$ constant at the first-best levels, which guarantees that consumption in all time periods is unchanged. Substituting the latter two equations into the former two, we find

$$R_t = R^*_t \cdot \underbrace{\frac{b^{i*}_t + \Delta}{b^{i*}_t}}_{<1} \qquad \text{and} \qquad R_{t+1} = R^*_{t+1} \cdot \underbrace{\frac{b^{i*}_{t+1}/R^*_{t+1}}{b^{i*}_{t+1}/R^*_{t+1} + \Delta}}_{>1}$$

In other words, the planner reduces the world interest rate for repayments and increases it proportionately for new borrowing in period t. To achieve this, he imposes capital controls

$$\tau^i_t = -\frac{\Delta}{b^{i*}_t} > 0 \qquad \text{and} \qquad \tau^i_{t+1} = \frac{\Delta}{b^{i*}_{t+1}/R^*_{t+1} + \Delta} < 0$$

By engaging in this manipulation in a given period t, the planner can circumvent any level of the borrowing constraint that satisfies $\bar{b}^i_{t+1} < 0$. The intervention can be repeated for arbitrarily many periods. ■

Intuitively, the global planner circumvents the financial constraint by reducing both the repayment and the (constrained) level of new borrowing in period t by identical amounts while manipulating interest rates such that nothing changes in adjacent time periods. In period $t-1$, both countries agree to impose capital controls $\tau_t > 0$ (i.e. controls on inflows in the borrowing country and subsidies to outflows in the lending country) to push down the world interest rate and "help" country i, which would otherwise be constrained in the following period, to reduce its repayment b_t^i for a given level of borrowing b_t^i/R_t. In period t, both countries agree to impose capital controls in the opposite direction (i.e. subsidies on inflows in the borrowing country and taxes on outflows in the lending country) to push up the world interest rate. This implies that the borrowing country obtains less b_{t+1}^i/R_{t+1} for a given face value of debt b_{t+1}^i, which makes up for the loss in interest payments that the lending country would otherwise have suffered.

Remark 1: The results of proposition 7 are robust to alternative specifications of the financial constraint. For example, the same argument could be applied to period $t+1$ if the interest rate was omitted in the denominator of constraint (24). What matters is the the planner can change the amount borrowed b_t/R_t and repaid b_t independently in two consecutive periods because he can determine the level of the interest rate R_t.

Remark 2: Our results can easily be generalized to a world with multiple states of nature in which two countries trade contingent securities b_{t+1}^{ω} in a complete market. Following the recipe of proposition 7, a global planner would reduce the payoffs of contingent liabilities of the borrowing country i that pay out in states of nature when the constraint is binding by imposing capital controls $\tau_t^{\omega} > 0$ and reduce new borrowing once such a state is reached. In practice, securities that pay out in constrained states of nature can be interpreted as "hard claims" such as dollar debt. The planner would impose inflow controls in the recipient country and subsidies to outflows in the source country. This reduces the need for new financing in country i if one of those states of nature materializes. In that event, the planner would subsequently impose capital controls in the opposite direction on *all* securities (i.e. subsidies on inflows in the recipient country and on outflows in source country) to push up the world interest rate and compensate the source country for the lower returns in the prior period. If a different state of nature materializes in which there is no risk of binding constraints, the planner would take no further action in period t. Again, the resulting real allocations replicate the first-best.

Limitations There are also a number of limitations to restoring the first best.

Multiple Countries The result relies on the planner's ability to proportionately scale down the repayment and new borrowing of all countries. However, in the case of multiple countries, this only implements the first best if the ratio of repayment and new borrowing is the same for all countries, i.e. if the following condition is met:

$$b_t^i / b_{t+1}^i = b_t^j / b_{t+1}^j \forall i, j \tag{27}$$

For two countries, this condition is naturally fulfilled since $mb_t^i = -mb_t^j \; \forall \; t$. However, if there are multiple countries subject to idiosyncratic shocks, this condition is no longer likely be satisfied.

Commitment Furthermore, the implementation of proposition 7 requires that unconstrained countries set capital controls in favor of the the constrained country in period $t-1$ and constrained countries return the favor in period t.

For these reasons, the first-best may be difficult to implement through capital controls in practice. If a global planner has a superior enforcement technology, similar mechanisms such as crisis lending to provide a constrained country with additional borrowing capacity may restore the first best.

7 Distortive Capital Controls

7.1 Setup

Suppose that there is a monopolistic planner in each country i with positive mass $m^i > 0$ that maximizes the utility of the representative consumer U^i and internalizes that she has market power over the world interest rate R_{t+1}, which affects consumer welfare as we observed in lemma 2.

Global market clearing requires that world-wide savings add up to zero,

$$m^i b_{t+1}^i + B_{t+1}^{-i} = 0 \qquad \text{where} \qquad B_{t+1}^{-i} = \sum_{j \neq i} m^j b_{t+1}^j$$

B_{t+1}^{-i} denotes the rest-of-the-world bond holdings excluding country i. We can express these as a function $B_{t+1}^{-i}\left(R_{t+1}; \tau_{t+1}^{-i}\right)$ that is strictly increasing in R_{t+1} and increasing in each element of the rest-of-the world's capital controls $\tau_{t+1}^{-i} = \left\{\tau_{t+1}^j\right\}_{j \neq i}$. We invert this function to obtain the inverse rest-of-the-world bond demand $R^{-i}\left(B_{t+1}^{-i}; \tau_{t+1}^{-i}\right)$, which is strictly increasing in B_{t+1}^{-i} and declining in each element of τ_{t+1}^{-i}.

A monopolistic planner in country i recognizes that market clearing requires $B_{t+1}^{-i} = -m^i b_{t+1}^i$ and that the world interest rate therefore satisfies $R_{t+1} = R^{-i}\left(-m^i b_{t+1}^i; \tau_{t+1}^{-i}\right)$. We formulate the recursive optimization problem of planner who takes the vector of policies imposed by other countries as given, by[18]

$$V^i\left(b_t^i\right) = \max_{b_{t+1}^i} u\left(y_t^i + b_t^i - \frac{b_{t+1}^i}{R^{-i}(-m^i b_{t+1}^i; \tau_{t+1}^{-i})}\right) + \beta V^i\left(b_{t+1}^i\right)$$

[18] The described setup solves for the optimal level of distortive capital controls in a time-consistent setting. For an analysis of optimal distortive capital controls under commitment see **?**.

leading to the generalized Euler equation

$$\left(1 - m^i \eta_{RB^{-i}}\right) u'\left(c_t^i\right) = \beta R_{t+1} V^{i\prime}\left(b_{t+1}^i\right) \tag{28}$$

where we denote by $\eta_{RB^{-i}} = -\partial R_{t+1}/\partial B_{t+1}^{-i} \cdot b_{t+1}^i/R_{t+1}$ the inverse elasticity of global savings. The term $m^i \eta_{RB^{-i}}$ in the Euler equation of the monopolistic planner reflects that increasing domestic saving b_{t+1}^i pushes down the world interest rate $\partial R_{t+1}/\partial b_{t+1}^i = -m^i \partial R_{t+1}/\partial B_{t+1}^{-i} < 0$. If the country is a net saver, then $\eta_{RB^{-i}} < 0$ and the planner finds it desirable to push up the world interest rate by taxing capital outflows and reducing saving; if the country is a net borrower, then $\eta_{RB^{-i}} > 0$ and the planner finds it optimal to tax capital inflows. Naturally the effect is scaled by the weight m^i, which reflects the country's market power in the global bond market.

A monopolistic planner distorts domestic saving decision to the point where the marginal benefit of manipulating the world interest rate – the interest rate impact times the amount saved valued at the country's marginal utility $m^i \partial R_{t+1}^{-i}/\partial B_{t+1}^{-i} \cdot b_{t+1}^i/R_{t+1} \cdot u'\left(c_t^i\right)$ – equals the marginal cost of having an unsmooth consumption profile $\beta R_{t+1} u'\left(c_{t+1}^i\right) - u'\left(c_t^i\right)$.

Proposition 8 (Market Power and Capital Controls) *A monopolistic planner who internalizes her country's market power over the world interest rate imposes a monopolistic capital control*

$$\tau_{t+1}^{i,M} = m^i \eta_{RB^{-i}} = -\frac{\partial R_{t+1}}{\partial B_{t+1}^{-i}} \frac{m^i b_{t+1}^i}{R_{t+1}} \tag{29}$$

Proof. The tax $\tau_{t+1}^{i,M}$ ensures that the private optimality condition of consumers (4) replicates the planner's Euler equation (28). ■

This leads us to the following observations about the optimal monopolistic capital control:

1. The optimal tax carries the opposite sign as the country's bond position b_{t+1}^i. If the country is a net saver, the planner taxes saving $\tau_{t+1}^i < 0$. If the country is a net borrower, the planner taxes inflows $\tau_{t+1}^i > 0$.

2. Ceteris paribus, the absolute value of the optimal tax $|\tau_{t+1}^i|$ is linear in the mass of the country m^i and in the absolute size of the country's bond position $|b_{t+1}^i|$. If the country is small compared to the world economy $m^i = 0$ or is a zero saver $b_{t+1}^i = 0$, then the optimal tax rate is $\tau_{t+1}^i = 0$.

3. The absolute magnitude of the optimal tax is higher the greater the elasticity of the world interest rate with respect to global savings $\eta_{RB^{-i}}$.

4. For a given elasticity $\eta_{RB^{-i}}$, the optimal tax rate is a decreasing function of initial output, $\partial \tau_{t+1}^i/\partial y_t^i > 0$ because output increases saving. Countries that are comparatively rich in period 0 tax saving; countries that are comparatively poor tax borrowing.

5. For a given country, the optimal tax rate reduces the magnitude of capital flows but does not change their direction.

6. The optimal policy can equivalently be implemented via a quantity restriction $\bar{b}^{i,M}_{t+1} = b^i\left(R_{t+1}; \tau^{i,M}_{t+1}\right)$. If a country is a net saver, the tax is equivalent to a quota or ceiling on capital outflows $\bar{b}^{i,M}_{t+1}$ that restricts $b^i_{t+1} \leq \bar{b}_{t+1}$. If the country is a net borrower, the tax is equivalent to a quota or ceiling on capital inflows $b^i_{t+1} \geq \bar{b}^{i,M}_{t+1}$.

7. Finally, under closed capital accounts, optimal monopolistic capital controls are isomorphic to reduced reserve accumulation/decumulation.[19]

Numerical Illustration In the following we determine the monopolistically optimal level of capital controls for a variety of countries numerically based on equation (29). From our earlier analysis, we observe that the steady-state response of the world interest rate to additional saving is $\partial R/\partial B^{-i} = \frac{1+\beta}{\sigma\beta\bar{Y}^{-i}}$ in a steady state. We identify $m^i b^i/R$ in the data as the net external wealth NW^i_t of different countries and express the monopolistic capital controls of country i as

$$\tau^{i,M}_{t+1} = \frac{1+\beta}{\sigma\beta} \cdot \frac{NW^i}{\bar{Y}^{-i}}$$

For our earlier value of the intertemporal elasticity of substitution $\sigma = 2$, the first term of this expression is approximately 4. In short, the monopolistically optimal capital control of a country is roughly four times its current account relative to the GDP of the rest of the world.

We report the resulting calculations for a number of countries in Table 3. Countries for which the external wealth represents a significant fraction of rest-of-the-world GDP have a strong motive for imposing monopolistic capital controls. The United States, for example, would optimally impose a 4% tax on capital inflows so as to exert monopoly power over the availability of global savings instruments and benefit from a lower world interest rate. By contrast, China would optimally impose a 2% tax on capital outflows (or subsidy on capital inflows) so as exert monopoly power over its supply of worldwide savings and raise the interest rate. Countries that make up a smaller share of the world capital market have less market power and choose accordingly smaller capital controls.

The table highlights that it is difficult to reconcile the capital controls observed in the real world with the monopolistic motive for imposing capital controls. This suggests that some of the other motives for imposing controls that we studied in earlier sections were more relevant for most countries that imposed capital controls in recent years.

[19] For example, when a country that accumulates reserves is concerned that it is not earning "sufficient" interest on its reserves because its accumulation is pushing down the world interest rate, this is non-competitive behavior and is equivalent to distortive capital controls.

Country	GDP^i	NW^i	NW^i/Y^{-i}	$\tau^{i,M}$
World	$62,634bn	···	···	···
United States	$14,447bn	$-474bn	-0.98%	4.02%
China	$5,739bn	$281bn	0.49%	-2.02%
Japan	$5,459bn	$123bn	0.22%	-0.88%
Brazil	$2,089bn	$-63bn	-0.1%	0.42%
India	$1,722bn	$-63bn	-0.1%	0.42%
South Korea	$1,014bn	$30bn	0.05%	-0.2%

Table 3: Monopolistically optimal capital controls (Source: IMF IFS and author's calculations) [**needs updating**]

7.2 Welfare Analysis of Exerting Market Power

In this subsection we analyze the welfare effects if one country imposes capital controls to exert market power. We first discuss the spillover effects on other countries; then we investigate the Pareto efficiency of the resulting global equilibrium.

Corollary 3 (Spillover Effects of Exerting Market Power) *An increase in the capital control τ^i_{t+1} has positive welfare effects for borrowing countries $b^j_{t+1} < 0$ and negative welfare effects for lending countries $b^j_{t+1} > 0$.*

Proof. We observed in proposition 2 that the effects of raising the capital control τ^i on the world interest rate is $dR/d\tau^i = -m^i b^i_\tau / B_R < 0$. The interest rate in turn affects the welfare of another country j as follows

$$\frac{dV^j_t}{dR_{t+1}} = \beta^t u'\left(c^j_t\right) \cdot \frac{b^j_{t+1}}{R^2_{t+1}} \gtrless 0$$

If the country is a net saver ($b^j_{t+1} > 0$), it is hurt by a low interest rate and by capital controls in country i. If the country is a net borrower ($b^j_{t+1} < 0$), it benefits from a low interest rate and from capital controls in country i. In short, it is in the interest of all lending countries to improve their intertemporal terms-of-trade and push up the world interest rate by reducing the supply of bonds on world capital markets. ■

Figure 2 illustrates our results in a framework of two countries i and j of equal mass for a given time period. The line $R^i(b^i)$ represents the (inverse) supply of bonds, the two lines $R^j(-b^j)$ and $R^j(-b^j) - b^j R_b$ represent the demand for bonds as well as the 'marginal revenue' curve for country i that takes into account the decline in the interest rate from supplying additional bonds. The laissez faire equilibrium is characterized by an interest rate R^{LF} and bond positions $b^i = b^{LF} = -b^j$. A monopolistic planner in country i would reduce the quantity of bonds supplied to j

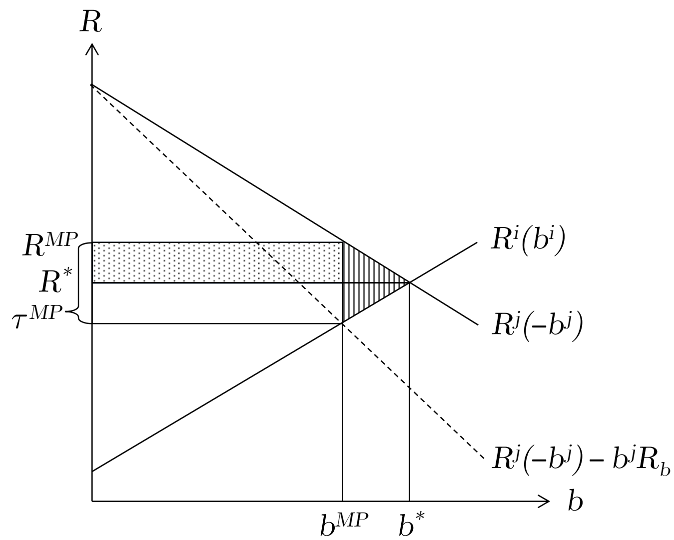

Figure 2: Optimal capital control imposed by a domestic planner to exert market power

such that her marginal valuation $R^i(b^i)$ equals the marginal revenue derived from country j. This monopolistic equilibrium is indicated by the quantity of bonds sold b^{MP} and interest rate R^{MP}. The described policy shifts the surplus between R^{MP} and R^*, marked by the dotted area in the figure, from country j to country i. It also introduces a deadweight loss indicated by the triangular vertically-shaded area. Monopolistic capital controls constitute a classic beggar-thy-neighbor policy and are always inefficient: they introduce a distortion into the Euler equation of domestic agents, which reduces global welfare, in order to shift welfare from foreigners to domestic agents – the policy represents a "negative-sum" game overall.

Proposition 9 (Inefficiency of Exerting Market Power) *An equilibrium in which domestic planners impose capital controls to exert market power is Pareto-inefficient.*

Proof. The result is a straightforward application of the first welfare theorem that we captured in proposition 1. ■

If one or more countries impose capital controls to exert market power, then capital controls are not equal across countries (lenders impose outflow controls; borrowers impose inflow controls). The intertemporal marginal rates of substitution of different agents differ, and the necessary conditions for Pareto efficiency of proposition 1 are violated.

Market Power and the Real Exchange Rate The real exchange rate model allows us to study the effect of monopolistic capital controls on the real exchange rate as well as the scope for monopolistic real exchange rate intervention.

If a country experiences capital outflows and is a net saver in a given period ($b^i_{t+1} > 0$), then its exchange rate is depreciated compared to the autarky level. A monopolistic planner would tax saving abroad ($\tau^i_{t+1} < 0$), which would reduce capital outflows and push up the world interest rate. In the domestic economy, this policy appreciates the real exchange rate. Alternatively, in a country with closed capital accounts, the planner would reduce a^i_{t+1}, i.e. reduce reserve accumulation to keep the world interest rate elevated.

If a country is a net borrower ($b^i_{t+1} < 0$), the opposite lessons apply. The country's real exchange rate is appreciated compared to the autarky level. A monopolistic planner would tax capital inflows to push down the world interest rate. In doing so, she would also put downward pressure on the domestic real exchange rate. Alternatively, with closed capital accounts, the planner would increase a^i_{t+1}, i.e. increase reserves or borrow less from abroad, to push down the world interest rate.

In short, a monopolistic planner would reduce deviations of the real exchange rate from its steady state.

Market Power and Uncertainty Our findings on market power carry over to the model with uncertainty that we outlined in section 3.2. In particular, a monopolistic domestic planner finds it optimal to impose state-contingent capital controls of

$$\tau^{i,\omega}_{t+1} = m^i \eta_{RB^{\omega,-i}}$$

In a world economy with idiosyncratic country shocks, optimal risk-sharing requires that each country purchases insurance contingent on states in which it is relatively worse off: a country sells more securities contingent on states in which it is relatively better off than on states in which it is worse off compared to the rest of the world.

For example, if the country is a net lender and is relatively better off in state ω than in state ψ, then optimal risk-sharing implies $0 < b^{i,\omega} < b^{i,\psi}$ which insures consumers against state ψ. Under the usual regularity conditions for $\partial R^\omega / \partial B^{\omega,-i}$, a monopolistic domestic planner sets $0 > \tau^{i,\omega} > \tau^{i,\psi}$, i.e. the planner taxes carrying resources (insurance) into state ψ more than carrying resources into state ω. This diminishes international risk-sharing. Practically speaking, lending countries will lend too much in hard claims and too little in contingent forms of finance such as FDI.

By the same token, countries that are net borrowers and want to exert monopoly power will borrow too much in foreign currency and too little in terms of FDI, again diminishing international risk-sharing.

8 Imperfect Capital Controls

This section analyzes capital controls that are imperfect policy tools and investigates under what circumstances such imperfections lead to a case for global coordination of capital control policies. In the previous section, we emphasized that the international spillover effects of perfectly targeted capital controls constitute pecuniary externalities that are mediated through a well-functioning market and therefore lead to Pareto-efficient outcomes, as long as domestic policymakers act competitively and impose such controls to internalize domestic externalities. This result follows from the first welfare theorem if we view the domestic policymakers in each country as competitive agents who optimize domestic welfare. By implication, we found that there is no need for global coordination to achieve Pareto-efficient outcomes. Our result relies on the assumption that domestic policymakers have the instruments to perfectly and costlessly control the amount of capital flows to the country.

In practice capital controls sometimes differ from the perfect policy instruments that we have depicted in our earlier analysis in that they create ancillary distortions (see e.g. Carvalho and Marcio, 2006). In the following two subsections, we analyze two types of such distortions: implementation costs of capital controls and imperfect targeting of capital controls. We formalize both examples and analyze whether a global planner could achieve a Pareto improvement by coordinating the capital control policies of different countries in the presence of such ancillary distortions.

8.1 Costly Capital Controls

The simplest specification of such a setup is to assume that capital controls impose a resource cost $C^i(\tau)$ on the economy that represents enforcement costs or distortions arising from attempts at circumvention. Assume that the function $C^i(\cdot)$ is twice continuously differentiable and satisfies $C(0) = C'(0) = 0$ and $C''(\tau) > 0 \forall \tau$, i.e. it is convex.[20]

The optimization problem of a national policymaker, where we use the summary notation $W^i(b^i) = V^i(b^i; b^i)$, is then

$$\max_{b^i, c^i, \tau^i} u\left(c^i\right) + \beta W^i\left(b^i\right) - \lambda^i \left[c^i - y^i + \frac{b^i}{R} + C^i\left(\tau^i\right)\right] - \mu^i \left[\left(1-\tau^i\right) u'\left(c^i\right) - \beta R V^{i\prime}\left(b^i\right)\right]$$

The first-order conditions are

$$\begin{aligned} FOC\left(b^i\right) &: \beta W^{i\prime}\left(b^i\right) = \lambda^i / R - \mu^i \beta R V^{i\prime\prime}\left(b^i\right) \\ FOC\left(c^i\right) &: u'\left(c^i\right) = \lambda^i + \mu^i \left(1-\tau^i\right) u''\left(c^i\right) \\ FOC\left(\tau^i\right) &: \lambda^i C^{i\prime}\left(\tau^i\right) = \mu^i u'\left(c^i\right) \end{aligned}$$

and can be combined to the optimality condition

$$u'\left(c^i\right) = \beta R W'(b) \frac{1 + \frac{\beta R V' u''}{(u')^2} C^{i\prime}}{1 - \frac{\beta R V''}{u'} C^{i\prime}} \tag{30}$$

We find:

[20] Analogous results can be derived if the cost of capital controls is proportional to the amount of bond holdings, e.g. $c(\tau, b) = C(\tau b)$, which may specifically capture the costs associated with attempts at circumvention.

Proposition 10 (Costly Capital Controls) *If capital controls impose a resource cost $C^i(\tau^i)$ as defined above and if $\xi^i \neq 0$, then a national planner imposes an optimal level of capital controls of the same sign as ξ^i but of smaller absolute magnitude, i.e. τ^i satisfies $0 < |\tau^i| < |\xi^i|$.*

Proof. The planner implements the optimality condition (30) by setting the capital control in the decentralized optimality condition (4) to

$$\tau^i = \frac{\beta R \xi^i}{u'(c^i)} + \beta R C^{i\prime} \cdot \frac{V'u'' + u'V''}{(u')^2 + \beta R V' u'' C'}$$

The first additive term corresponds to the optimal costless capital controls $\tilde{\tau}^i$. If this term is positive because the country experiences a negative externality $\xi^i > 0$ from capital inflows, then $C^{i\prime} > 0$ and the second additive term is negative, which mitigates the optimal magnitude of the capital control to $\tau^i < \tilde{\tau}^i$. (This holds as long as the denominator is positive, i.e. $(u')^2 + \beta R V' u'' C' > 0$, which is satisfied as long as the marginal cost of the capital control C' is not too large.) For $\xi^i > 0$, the second term never flips the sign of the control τ^i to make it negative. If it did, then $C^{i\prime}$ would switch sign as well and the second term would become positive, leading to a contradiction. The argument for $\xi^i < \tau^i < 0$ follows along the same lines. ■

8.2 Global Coordination of Costly Capital Controls

We next determine under what conditions the equilibrium in which each national planner imposes capital controls according to equation (30) is globally Pareto efficient. In other words, if national planners follow the described rule, can a global planner achieve a Pareto improvement on the resulting equilibrium? It turns out that the answer depends critically on the set of instruments available to the planner.

8.2.1 Global Coordination with Transfers

First, we analyze a global planner who maximizes global welfare in the described environment who has access to lump-sum transfers between countries. This implies that he is not bound by the period 1 budget constraints of individual countries and can undo the redistributions that stem from changes in the world interest rate.

Formally, a global planner maximizes the sum of the surplus of all nations for some set of welfare weights $\{\phi^i\}$. He internalizes that the world interest rate R is a choice variable and that the optimality conditions of individual agents (with shadow price μ^i) as well as global market clearing must hold, i.e. $\Sigma_i m^i b^i = 0$ (with shadow price ν). In addition, we include a transfer T^i in our optimization problem, which needs to satisfy global market clearing $\Sigma_i m^i T^i = 0$ (with shadow price γ). The associated

Lagrangian is

$$\mathcal{L} = \sum_i \phi^i \left\{ u\left(c^i\right) + \beta W^i\left(b^i\right) - \lambda^i\left[c^i - y^i + b^i/R + C^i\left(\tau^i\right) - T^i\right] - \right.$$
$$\left. -\mu^i\left[\left(1-\tau^i\right)u'\left(c^i\right) - \beta R V^{i\prime}\left(b^i\right)\right]\right\} - \nu \sum_i m^i b^i - \gamma \sum_i m^i T^i$$

The first-order conditions of the global planner are

$$\begin{aligned}
FOC\left(b^i\right) &: \beta W^{i\prime}\left(b^i\right) = \lambda^i/R - \mu^i \beta R V^{i\prime\prime}\left(b^i\right) + m^i \nu/\phi^i \\
FOC\left(c^i\right) &: u'\left(c^i\right) = \lambda^i + \mu^i\left(1-\tau^i\right)u''\left(c^i\right) \\
FOC\left(\tau^i\right) &: \lambda^i C^{i\prime}\left(\tau^i\right) = \mu^i u'\left(c^i\right) \\
FOC\left(R\right) &: \sum_i \phi^i \left\{\frac{\lambda^i b^i}{R} + \mu^i\left(1-\tau^i\right)u'\left(c^i\right)\right\} = 0 \\
FOC\left(T^i\right) &: \phi^i \lambda^i = \gamma m^i
\end{aligned}$$

The uncoordinated Nash equilibrium among national planners is constrained Pareto efficient under the given set of instruments if and only if we can find a set of welfare weights $\left\{\phi^i\right\}$ such that the allocations of national planners satisfy the maximization problem of the global planner. If we substitute the allocations from the Nash equilibrium, we find that the second and third optimality conditions are unchanged compared to the national planner's equilibrium and can be solved for λ^i and μ^i that are identical to the shadow prices in the Nash equilibrium between national planners. Substituting these in the optimality condition $FOC\left(b^i\right)$, we find that this condition is satisfied for all countries if we set $\nu = 0$. The fifth optimality condition is satisfied if we set $\phi^i = \gamma m^i/\lambda^i \forall i$. The Nash equilibrium among planners is therefore efficient if the described variables also satisfy the fourth optimality condition $FOC\left(R\right)$.

Proposition 11 (Coordination of Costly Controls with Transfers) *If capital controls to correct national externalities are costly, then the uncoordinated Nash equilibrium between national planners is Pareto efficient with respect to a global planner who can engage in transfers if and only if the resulting allocation satisfies*

$$\sum_i m^i\left(1-\tau^i\right)C'\left(\tau^i\right) = 0 \tag{31}$$

Proof. The optimality condition (31) can be obtained by substituting $FOC\left(T^i\right)$ into the condition $FOC\left(R\right)$ and accounting for market clearing $\Sigma_i m^i b^i = 0$ as well as for $FOC\left(\tau^i\right)$. ■

In a Pareto-optimal allocation, the weighted average marginal distortion imposed by capital controls must be zero. If there are no externalities, this can be achieved by having zero controls in all countries. Otherwise, the planner combines controls in capital inflow and outflow countries in a way that their weighted average marginal distortion is zero.

The planner's country weights ϕ^i do not show up in condition (31) since the condition is purely about efficiency, i.e. about minimizing the overall resource cost of imposing capital controls. Since the planner has access to lump-sum transfers, she can undo any redistributions created by movements in the interest rate according to her welfare weights.

We illustrate our findings in the following examples:

Example 1: Single country/symmetric countries Assume a world economy that consists of $k \geq 1$ identical countries that impose costly capital controls $0 < \tau^i < \xi$ to offset domestic externalities. In doing so they incur a resource cost $C^i\left(\tau^i_{t+1}\right) > 0$. However, since they are identical, their net bond positions is $b^i_{t+1} = 0$ in equilibrium. There is a clear scope for reducing capital controls to zero and avoiding the resource cost, making all countries better off. Observe that the reduction in capital controls leads to a parallel increase in the world interest rate R.

Analytically, since all countries are symmetric, the only non-degenerate solution to equation (31) is $C^{i\prime}\left(\tau^i_{t+1}\right) = 0 \forall i$. A global planner would reduce the controls in all countries to zero.

Example 2: Two countries with asymmetric externality Assume two countries that are identical, except that one of them experiences a negative externality from selling bonds $\xi > 0$. In the Nash equilibrium of national planners, country i imposes a capital control $0 < \tau^i < \xi$ (the inequality holds because capital controls are costly) and country j doesn't. Country i therefore experiences capital outflows and incurs a resource cost $C\left(\tau^i\right) > 0$, whereas country j receives capital inflows. The decentralized equilibrium is inefficient and the optimality condition (31) is not satisfied.

The global planner would lower the capital control $\tau^i > 0$ on inflows in country i and impose a control on outflows $\tau^j < 0$ in country j to minimize the total resource cost $C(\cdot)$ of controls. This would increase the world interest rate, but the planner can undo the resulting redistribution to make sure that a Pareto-improvement takes place.

Example 3: Two countries, restricted instruments Let us add to the previous example a restriction that country j cannot use its capital control instrument so $\tau^j \equiv 0$. The Nash equilibrium of national planners is unaffected since country j already found it optimal to impose a zero capital control.

For the global planner, we may capture the restriction $\tau^j \equiv 0$ by assuming that it is arbitrarily costly for country j to deviate from zero capital controls, e.g. $C^j\left(\tau^j\right) = \alpha\left(\tau^j\right)^2$ with $\alpha \to \infty$. In the limit, the optimality condition (31) is satisfied for the allocation in the Nash equilibrium among national planners. The global planner balances the marginal cost of changing the capital control in countries i and j which requires $m^i\left(1-\tau^i\right)C^{i\prime}\left(\tau^i\right)=-m^j\left(1-\tau^j\right)C^{j\prime}\left(\tau^j\right)$ and which holds for $\tau^j\to 0$. The intuition for the result is that there is no scope for sharing the burden of regulation with country j if it is infinitely expensive for country j to impose even minimal capital controls.

Example 4: More countries Observe that all our examples on coordination continue to hold for the case of more than two countries. In particular, equation (31) weighs each country by its mass m^i in the world economy and adds up the marginal distortion it experiences. If we replace one country of mass m^i by k identical countries of mass m^i/k, then the national planner in each of these countries will find it optimal to choose precisely the same allocation as the one in the large country of mass m^i. Moreover, the global planner treats the sum of the k small countries in the same fashion as the one large country. This is because we assumed that the planners in the current section do not exert market power. (The case of $k>1$ in example 1 is an application of this finding.)

In summary, our examples illustrate that there may be a rationale for global coordination of capital controls if such controls impose deadweight costs that can be reduced by sharing the burden of controlling capital flows and if a global planner can engage in compensatory transfers. The goal of coordination is to minimize the aggregate deadweight loss from capital controls by distributing the burden of imposing controls between borrowing and lending countries.

8.2.2 Global Coordination without Transfers

If the global planner cannot engage in transfers between the countries involved, then the conditions under which a global Pareto improvement can be achieved are highly restrictive.

A global planner maximizes the sum of the surplus of all nations for some set of welfare weights $\left\{\phi^i\right\}$. He internalizes that the world interest rate R is a choice variable and that global market clearing must hold, i.e. $\Sigma_i m^i b^i=0$ (with shadow price ν):

$$\mathcal{L}=\sum_i \phi^i\left\{u\left(c^i\right)+\beta W^i\left(b^i\right)-\lambda^i\left[c^i-y^i+b^i/R+C^i\left(\tau^i\right)\right]-\right.$$
$$\left.-\mu^i\left[\left(1-\tau^i\right)u'\left(c^i\right)-\beta R V^{i\prime}\left(b^i\right)\right]\right\}-\nu\sum_i m^i b^i$$

The first-order conditions of the global planner are the same as the first four conditions above. The uncoordinated Nash equilibrium among national planners is constrained Pareto efficient under the given set of instruments if and only if we can find a set of welfare weights $\{\phi^i\}$ such that the allocations of national planners satisfy the maximization problem of the global planner.

If we substitute the allocations from the Nash equilibrium, we find as before that the first three optimality condition are satisfied for all i if we set $\nu = 0$. The fourth optimality condition captures the effects of varying the world interest rate, which has both a redistributive effect that depends on the sign of b^i/R and an effect on the tightness of the implementability constraint μ^i of each country. We can reformulate this optimality condition as

$$\sum_i \phi^i \lambda^i \left\{ \frac{b^i}{R} + \left(1-\tau^i\right) C^{i\prime}\left(\tau^i\right) \right\} = 0 \tag{32}$$

Proposition 12 (Coordination of Costly Controls, No Transfers) *If capital controls to correct national externalities are costly, then the uncoordinated Nash equilibrium between national planners is Pareto efficient with respect to a constrained global planner who cannot engage in transfers as long as either (i) there is no trade or (ii) there is at least one borrower and one lender for whom the marginal distortions imposed by costly capital controls are smaller than the country's bond positions, i.e.* $(1-\tau^i) C^{i\prime}(\tau^i) < -b^i/R$ *for borrowers and* $-(1-\tau^i) C^{i\prime}(\tau^i) < b^i/R$ *for lenders.*

Proof. The optimality condition (32) has a non-trivial solution with non-degenerate welfare weights $\phi^i > 0 \forall i$ if and only if the term $\left\{ \frac{b^i}{R} + (1-\tau^i) C^{i\prime}(\tau^i) \right\}$ is either zero for all countries or is positive for some and negative for other countries. ■

These conditions are typically satisfied since some countries are lenders $b^i > 0$, others are borrowers $b^i < 0$, and since the capital control τ^i and the marginal distortion $C^{i\prime}$ are small in absolute value.

Inefficiency arises if the capital controls of national planners impose significant marginal costs $C^{i\prime}$ compared to the amount of borrowing/lending b^i/R that agents engage in. We provide two examples in which this may be the case:

Example 5: Single country/symmetric countries Returning to our earlier example of $k \geq 1$ identical countries with costly capital controls $0 < \tau^i < \xi$, we observe that their net bond positions are $b^i = 0$ in equilibrium. Therefore a coordinated reduction in capital controls and the associated increase in the world interest rate do not have redistributive effects. A planner can achieve a Pareto improvement without engaging in transfers by reducing capital controls to zero.

Analytically, we observe that in the Nash equilibrium between national planners, $b^i = 0$ and $C^{i\prime}(\tau^i) > 0 \forall i$. Therefore the only solution to the optimality condition (32) is the degenerate solution $\phi^i = 0 \forall i$. The allocation therefore cannot be the outcome of the constrained planner's optimization and is constrained inefficient.

Example 6: Highly distortive capital controls Assume a world economy that consists of a borrowing country i with $b^i < 0$ and a lending country j of the same size with $b^j = -b^i > 0$. The national planner in the borrowing country is subject to an externality ξ^i and corrects it using a capital control τ^i that is so highly distortive that $(1 - \tau^i) C^{i\prime}(\tau^i) > -b^i/R$. By contrast, the lender does not suffer from externalities and sets $\tau^j = 0$. In the described situation, a global planner recognizes that both countries would be better off if she reduces the capital controls in both countries in parallel. ($\tau^j < 0$ for the lending country j amounts to an export tax on capital.) This policy reduces the marginal distortion $C^{i\prime}(\tau^i)$ in country i while introducing a small distortion in country j. (Recall that C^i is convex.) However, in general equilibrium the parallel reduction in capital controls pushes up the world interest rate R, which benefits country j. In country i, the cost of the increase in the interest rate is offset by the reduced distortion since $(1 - \tau^i) C^{i\prime}(\tau^i) > -b^i/R$. Therefore both countries are better off.

Analytically, all the terms in the curly brackets of condition (32) are positive so that the only solution is degenerate, $\phi^i = 0 \forall i$. The allocation therefore cannot be the outcome of the constrained planner's optimization and is constrained inefficient.

Example 7: Modestly distortive capital controls We continue to assume that country j is a lender with zero capital controls and country i is a borrower that imposes a capital control $\tau^i > 0$, but that the distortion arising from the capital control is more modest, i.e. $(1 - \tau^i) C^{i\prime}(\tau^i) < -b^i/R$. This is plausible if we believe that the marginal cost of capital controls is less than the stock of foreign capital that a country is borrowing. Then the term in curly brackets in optimality condition (32) is negative for the borrower and positive for the lender. It is clear that we can find welfare weights ϕ^i and ϕ^j such that the optimality condition is satisfied and we can conclude that the capital control imposed by the national planner in country i is constrained Pareto efficient.

Observe that a critical element of proposition 12 is that it is sufficient to achieve constrained Pareto efficiency if there is a single borrowing and a single lending country in the world economy without large externalities or without large distortions from capital controls. As long as this is the case, there will be a loser for any policy that shifts the world interest rate, and it is impossible for a global planner to achieve a Pareto improvement.

8.3 Imperfectly Targeted Capital Controls

We now introduce the possibility that a planner cannot perfectly target different forms of capital flows and study the implications for the desirability of international coordination in the setting of capital controls.

For simplicity, we use our earlier state-contingent setup and assume that there are two states of nature $\omega = L, H$ at $t = 1$ with probabilities π^ω and two securities b^ω that are contingent on these two states, but the planner in each country i has only one capital control instrument τ^i that equally applies to both. One possible interpretation of the two securities is that security L represents a payoff in a low state of nature in which additional insurance mandated by the planner is desirable and H represents a payoff in a high state in which no insurance is necessary.

8.3.1 Single Country Problem

$$\max_{b^{i,\omega},c^i,\tau^i} u\left(c^i\right)+\beta E\left[W^\omega\left(b^{i,\omega}\right)\right]-\lambda^i\left[c^i+\Sigma_\omega q^\omega b^{i,\omega}-y^i\right]-\sum_\omega \mu^{i,\omega}\left[1-\tau^i-\frac{\beta\pi^\omega V^{\omega\prime}\left(b^{i,\omega}\right)}{q^\omega u'\left(c^i\right)}\right] \tag{33}$$

The first-order conditions are

$$\begin{aligned}
FOC\left(c^i\right) &: \lambda^i = u'\left(c^i\right)-\left(1-\tau^i\right)\frac{u''\left(c^i\right)}{u'\left(c^i\right)}\sum_\omega \mu^{i,\omega} \\
FOC\left(b^{i,\omega}\right) &: q^\omega\lambda^i = \pi^\omega\beta W^{\omega\prime}\left(b^{i,\omega}\right)+\mu^{i,\omega}\left(1-\tau^i\right)\frac{V^{\omega\prime\prime}\left(b^{i,\omega}\right)}{V^{\omega\prime}\left(b^{i,\omega}\right)} \\
FOC\left(\tau^i\right) &: \sum_\omega \mu^{i,\omega}=0
\end{aligned}$$

The first condition captures that the marginal utility of wealth is equal to the marginal utility of consumption plus the benefit of relaxing the planner's implementability constraints that stems from consumption. Combining this condition with the third condition yields $\lambda^i = u'\left(c^i\right)$. The second condition is the Euler equation for state ω, by which the planner equates the marginal cost of saving in state ω (lhs) to the marginal social benefit (first term on rhs) plus the effects on the implementability constraint in state ω. It can be reformulated to express the shadow value

$$\mu^{i,\omega}=\frac{q^\omega u'\left(c^i\right)-\pi^\omega\beta W^{\omega\prime}\left(b^{i,\omega}\right)}{\left(1-\tau^i\right)V^{\omega\prime\prime}\left(b^{i,\omega}\right)/V^{\omega\prime}\left(b^{i,\omega}\right)}$$

If private agents save too much in that state compared to what is optimal in the first-best, $q^\omega u'\left(c^i\right)>\pi^\omega\beta W^{\omega\prime}$, then the shadow price $\mu^{i,\omega}$ is negative, indicating that it is desirable to increase the capital control from the perspective of this state; if they save less than optimal, then the shadow price in that state is positive. The third optimality condition states the planner sets the capital control τ^i such that saving is on average at the right level, as indicated by these $\mu^{i,\omega}$'s.

As in the case of costly capital controls, there are two variants of the global planning problem that we can solve. The first variant corresponds to the traditional test for Pareto efficiency, in which a planner is only concerned about the efficiency implications of her actions not the redistributive effects. In this setup, we allow the planner to have access to lump-sum transfers and ask if we can find weights ϕ^i such that the global planner's solution replicates the allocations in the decentralized equilibrium. If such weights can be found, then we call the decentralized equilibrium Pareto efficient.

8.3.2 Global Coordination with Transfers

We set up a global planning problem with compensatory transfers to determine if there is scope for international cooperation in the setting of imperfectly targeted capital controls. We assume that the global planner places weight ϕ^i on the objective of each country i as described in problem (33). The planner is subject to the constraint on global market clearing in each state-contingent security $\sum_i m^i b^{i,\omega} = 0$, to which we assign the shadow prices ν^ω, and a resource constraint on international transfers $\sum_i m^i T^i = 0$ to which we assign the shadow prices ζ. Furthermore, the planner internalizes the endogeneity of the state-contingent prices q^ω. We summarize her optimization problem as

$$\max_{b^{i,\omega},c^i,\tau^i,q^\omega,T^i} \sum_i \phi^i \left\{ u\left(c^i\right) + \beta E\left[W^\omega\left(b^{i,\omega}\right)\right] - \lambda^i\left[c^i + \Sigma_\omega q^\omega b^{i,\omega} - y^i - T^i\right] - \right.$$

$$\left. - \sum_\omega \mu^{i,\omega}\left[1 - \tau^i - \frac{\beta\pi^\omega V^{\omega\prime}\left(b^{i,\omega}\right)}{q^\omega u'\left(c^i\right)}\right]\right\} - \zeta \sum_i m^i T^i + \sum_i m^i \sum_\omega \nu^\omega b^{i,\omega}$$

The global planner's optimality conditions are

$$\begin{aligned}
FOC\left(c^i\right) &: \lambda^i = u'\left(c^i\right) - \left(1-\tau^i\right)\frac{u''\left(c^i\right)}{u'\left(c^i\right)}\sum_\omega \mu^{i,\omega} \\
FOC\left(b^{i,\omega}\right) &: q^\omega \lambda^i = \pi^\omega \beta W^{\omega\prime}\left(b^{i,\omega}\right) + \mu^{i,\omega}\left(1-\tau^i\right)\frac{V^{\omega\prime\prime}\left(b^{i,\omega}\right)}{V^{\omega\prime}\left(b^{i,\omega}\right)} + m^i \nu^\omega/\phi^i \quad \forall \omega, i \\
FOC\left(\tau^i\right) &: \sum_\omega \mu^{i,\omega} = 0 \\
FOC\left(q^\omega\right) &: \sum_i \phi^i\left[\lambda^i b^{i,\omega} + \mu^{i,\omega}\frac{1-\tau^i}{q^\omega}\right] = 0 \quad \forall \omega \\
FOC\left(T^i\right) &: \phi^i \lambda^i = \zeta m^i \quad \forall i
\end{aligned}$$

We combine the fourth and fifth optimality conditions and use bond market clearing to obtain

$$\sum_i \phi^i\left(1-\tau^i\right)\mu^{i,\omega} = 0 \quad \forall \omega$$

The planner chooses the wedges $\mu^{i,\omega}$ such that their weighted average is zero in each state ω.

Observe from the second condition cominbed with the fifth that

$$\mu^{i,\omega} = \frac{\lambda^i\left[q^\omega - \nu^\omega/\zeta\right] - \pi^\omega \beta W^{\omega\prime}\left(b^{i,\omega}\right)}{\left(1-\tau^i\right)V^{\omega\prime\prime}\left(b^{i,\omega}\right)/V^{\omega\prime}\left(b^{i,\omega}\right)}$$

Since ν^ω doesn't show up anywhere else, this tells us that the planner can choose an arbitrary combination of $[q^\omega - \nu^\omega/\zeta]$ such that the difference between the two is constant. In short, the market prices q^ω are indeterminate.

$$\text{OBSERVE IN LF:} \quad \mu^{i,\omega} = \frac{q^\omega u'\left(c^i\right) - \pi^\omega \beta W^{\omega\prime}\left(b^{i,\omega}\right)}{\left(1-\tau^i\right) V^{\omega\prime\prime}\left(b^{i,\omega}\right) / V^{\omega\prime}\left(b^{i,\omega}\right)}$$

Example: assume 2 countries, 2 states, and country j in state L is a borrower and has an externality

- in Nash equilibrium: country j imposes inflow tax corresponding to average externality $\mu^{j,L} > 0 > \mu^{j,H}$
- in global equilibrium: country i also imposes outflow tax such that the average of the two countries is zero
- is this actually possible? under laissez faire, $\mu^{i,\omega} = 0$ for both states. if we impose an outflow tax then both $\mu^{i,\omega}$ for country i turn in the same direction, violating $FOC\left(\tau^i\right)$!!
- what is the intuition?

Proposition 13 *The decentralized equilibrium in our problem with imperfect targeting is Pareto efficient if*

$$xyz$$

Proof. The decentralized equilibrium is Pareto efficient if we can find a set of $\left\{\phi^i\right\}$ such that the allocations of the decentralized equilibrium satisfy the optimality conditions of the global planner. ■

8.3.3 Global Coordination without Transfers

In the second variant, we assume that the planner does not have access to compensatory transfers. This may better reflect the reality of our global system of governance, in which countries rarely compensate each other for the international effects of their policy actions.

We setup a global planning problem without compensatory transfers to determine if there is scope for international cooperation in the setting of imperfectly targeted capital controls. We assume that the global planner places weight ϕ^i on the objective of each country i as described in problem (33) and includes the constraints on global market clearing in each state-contingent security (with multiplier ν^ω),

$$\sum_i m^i b^{i,\omega} = 0 \forall \omega$$

If the planner has access to precisely the same set of instruments as decentralized agents but internalizes the endogeneity of the prices q^ω, then her objective function is

$$\max_{b^{i,\omega},c^i,\tau^i,q^\omega} \sum_i \phi^i \left\{ u\left(c^i\right) + \beta E\left[W^\omega\left(b^{i,\omega}\right)\right] - \lambda^i\left[c^i + \Sigma_\omega q^\omega b^{i,\omega} - y^i\right] - \right.$$
$$\left. - \sum_\omega \mu^{i,\omega}\left[1 - \tau^i - \frac{\beta\pi^\omega V^{\omega\prime}\left(b^{i,\omega}\right)}{q^\omega u'\left(c^i\right)}\right]\right\} + \sum_i m^i \sum_\omega \nu^\omega b^{i,\omega}$$

The global planner's optimality conditions are

$$\begin{aligned}
FOC\left(c^i\right) &: \lambda^i = u'\left(c^i\right) - \left(1-\tau^i\right)\frac{u''\left(c^i\right)}{u'\left(c^i\right)}\sum_\omega \mu^{i,\omega} \\
FOC\left(b^{i,\omega}\right) &: q^\omega\lambda^i = \pi^\omega\beta W^{\omega\prime}\left(b^{i,\omega}\right) + \mu^{i,\omega}\left(1-\tau^i\right)\frac{V^{\omega\prime\prime}\left(b^{i,\omega}\right)}{V^{\omega\prime}\left(b^{i,\omega}\right)} + m^i\nu^\omega/\phi^i \quad \forall\omega \\
FOC\left(\tau^i\right) &: \sum_\omega \mu^{i,\omega} = 0 \\
FOC\left(q^\omega\right) &: \sum_i \phi^i\left[\lambda^i b^{i,\omega} + \mu^{i,\omega}\frac{1-\tau^i}{q^\omega}\right] = 0 \quad \forall\omega
\end{aligned}$$

We find the following result:

Proposition 14 *The decentralized equilibrium in our problem with imperfect targeting is Pareto inefficient.*

Proof. The decentralized equilibrium is Pareto efficient if we can find a set of $\left\{\phi^i\right\}$ such that the allocations of the decentralized equilibrium satisfy the optimality conditions of the global planner.

Combining the first and the third condition we find $\lambda^i = u'\left(c^i\right)$, as we did in the decentralized equilibrium. This allows us to express the new (fourth) optimality condition as

$$\sum_i \phi^i\left[q^\omega b^{i,\omega} u'\left(c^i\right) + \mu^{i,\omega}\left(1-\tau^i\right)\right] = 0$$

We sum this equation over all states ω, and using the third optimality condition we obtain

$$\sum_i \phi^i u'\left(c^i\right)\cdot\sum_\omega q^\omega b^{i,\omega} = 0$$

We need to set $\phi^i = m^i/u'(c^i)$ for this condition to be satisfied by global market clearing condition.

We substitute these welfare weights back into the optimality condition $FOC(q^\omega)$ and find that the first term drops out by market clearing. The equilibrium is Pareto-efficient if

$$\sum_i \frac{m^i \mu^{i,\omega}(1-\tau^i)}{u'(c^i)} = 0 \quad \forall \omega$$

■

[to be completed]

9 Conclusions

This paper has studied the effects of capital controls in a general equilibrium model of the world economy and has delineated under what conditions such controls may be desirable from a global welfare perspective. In our positive analysis, we found that capital controls in one country push down the world interest rate and induce other countries to borrow and spend more. We then analyzed three motives for imposing capital controls. If capital controls are imposed to combat national externalities, then controls are Pareto efficient from a global welfare perspective. As long as national policymakers can impose such controls optimally, there is no need for global coordination of such controls as the Nash equilibrium between national planners is socially efficient. Under fairly mild conditions, capital controls that combat national externalities can make everybody in the world economy better off.

On the other hand, if national planners impose capital controls to exert market power and manipulate a country's terms of trade, then they have beggar-thy-neighbor effects and reduce global welfare.

If we deviate from the assumption that national policymakers can optimally address externalities, for example, if imposing capital controls has distortionary side-effects or if they cannot perfectly target different types of capital flows, then global policy coordination is also desirable. The goal of such coordination is to minimize the aggregate distortions created from capital controls.

Finally, if prudential capital controls are imposed that are designed to mitigate the risk of systemic crises after a surge in capital inflows, we have shown that our insights on technological externalities carry through. In particular, capital controls are Pareto efficient from a global perspective. Under certain circumstances, they may even lead to a global Pareto improvement since they reduce financial instability and create the potential for larger gains from trade in the future.

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A Mathematical Appendix

A.1 Decentralized demand for bonds

The consumer's Euler equation, after substituting the government budget constraint, defines an implicit function

$$F = \left(1-\tau_{t+1}^{i}\right) u'\left(y_t^i + b_t^i - b_{t+1}^i / R_{t+1}\right) - \beta R_{t+1} u'\left(y_{t+1}^i + b_{t+1}^i - b_{t+2}^i / R_{t+2}\right) = 0$$

$$\begin{aligned}
\text{which satisfies } \frac{\partial F}{\partial b_{t+1}^i} &= -\left(1-\tau_{t+1}^i\right) u''\left(c_t^i\right)/R_{t+1} - \beta R_{t+1} u''\left(c_{t+1}^i\right) > 0 \\
\frac{\partial F}{\partial R_{t+1}} &= \left(1-\tau_{t+1}^i\right) u''\left(c_t^i\right) \cdot b_{t+1}^i / \left(R_{t+1}\right)^2 - \beta u'\left(c_{t+1}^i\right) \gtrless 0 \\
\frac{\partial F}{\partial \tau_{t+1}^i} &= -u'\left(c_t^i\right) < 0
\end{aligned}$$

The first partial derivative is always positive, allowing us to implicitly define a demand function $b_{t+1}^i\left(R_{t+1};\tau_{t+1}\right)$.

The second partial derivative is negative as long as saving b^i is sufficiently high. Specifically, we write the condition as

$$\left(1-\tau_{t+1}^i\right) u''\left(c_t^i\right) \cdot b_{t+1}^i / R_{t+1} - \beta R_{t+1} u'\left(c_{t+1}^i\right) < 0$$

We employ the Euler equation to substitute for the second term and rearrange to

$$\begin{aligned}
b_{t+1}^i / R_{t+1} &> \frac{u'\left(c_t^i\right)}{u''\left(c_t^i\right)} \\
\text{or} \quad \frac{b_{t+1}^i / R_{t+1}}{c_t^i} &> \frac{u'\left(c_t^i\right)}{c_t^i u''\left(c_t^i\right)} = -\sigma\left(c_t^i\right)
\end{aligned}$$

i.e. the savings/consumption ratio is greater than the negative of the elasticity of intertemporal substitution $\sigma\left(c^i\right)$, as we stated in Assumption 1. If this inequality is satisfied then the demand function $b_{t+1}^i\left(R_{t+1};\tau_{t+1}\right)$ is strictly increasing in R_{t+1}, which allows us to invert it into a strictly increasing inverse demand function $R_{t+1}\left(b_{t+1}^i;\tau_{t+1}\right)$.

The third partial derivative is always negative – this is because we assumed that the revenue from capital controls is rebated so that there are only substitution effects and no income effects from capital controls.

A.2 Learning-by-Exporting Externalities: General Version

This appendix describes the implications of learning-by-exporting externalities in a general framework in which growth is a function of net exports $y_{t+1}^i = y_t^i + \Delta y_{t+1}^i\left(tb_t^i\right)$ as described in specification (16).

National Planner The optimization problem of a national planner in recursive form is

$$W\left(b_t^i, y_t^i\right) = \max u\left(y_t^i + b_t^i - \frac{b_{t+1}^i}{R_{t+1}}\right) + \beta W\left(b_{t+1}^i, y_t^i + \Delta y_{t+1}^i\left(\frac{b_{t+1}^i}{R_{t+1}} - b_t^i\right)\right)$$

The associated Euler equation is

$$u'\left(c_t^i\right) = \beta R_{t+1} W_b\left(b_{t+1}^i, y_{t+1}^i\right) + \beta W_y\left(b_{t+1}^i, y_{t+1}^i\right) \Delta y_{t+1}^{i\prime}\left(tb_t^i\right)$$

and the envelope theorem implies

$$\begin{aligned} W_b\left(b_t^i, y_t^i\right) &= u'\left(c_t^i\right) - \beta W_y\left(b_{t+1}^i, y_{t+1}^i\right) \Delta y_{t+1}^{i\prime}\left(tb_t^i\right) \\ \text{and} \quad W_y\left(b_t^i, y_t^i\right) &= u'\left(c_t^i\right) + \beta W_y\left(b_{t+1}^i, y_{t+1}^i\right) = \sum_{s=0}^{\infty} \beta^s u'\left(c_{t+s}^i\right) := v_t^i \end{aligned}$$

where we use the short-hand notation v_t^i for the utility value of permanently raising output in period t. Since growth is cumulative, observe that we can express v_{t+1} by iterating forward the envelope condition. Putting together these three equations, we express the Euler equation of consumers as

$$u'\left(c_t^i\right) - \beta v_{t+1}^i \Delta y_{t+1}^{i\prime}\left(tb_t^i\right) = \beta R_{t+1}\left[u'\left(c_{t+1}^i\right) - \beta v_{t+2}^i \Delta y_{t+2}^{i\prime}\left(tb_{t+1}^i\right)\right]$$

Increasing saving between today and tomorrow – while keeping the path of future bond holdings unchanged – increases the trade balance tb_t^i today which leads to positive learning-by-exporting effects captured by $\Delta y_{t+1}^{i\prime}\left(tb_t^i\right)$, but reduces the trade balance tb_{t+1}^i tomorrow, which leads to the opposite effects $-\Delta y_{t+2}^{i\prime}\left(tb_{t+1}^i\right)$ in the following period.

If learning-by-exporting effects remain constant over time, for example in a steady state in which $v_{t+1}^i \Delta y_{t+1}^{i\prime}\left(tb_t^i\right) = \beta R_{t+1} v_{t+2}^i \Delta y_{t+2}^{i\prime}\left(tb_{t+1}^i\right) \forall t$, then the benefit and costs of increasing the trade balance for one period cancel out. In that case, the national planner will not intervene in private saving/exporting decisions in any solution path that satisfies the transversality condition on bond holdings.[21] On the other hand, if learning-by-exporting effects decrease over time, for example because the country approaches the world technology frontier, then $x_{t+1}^i\left(b_{t+1}^i\right) > 0$ and the planner would find it optimal to impose a capital control $\tau_{t+1}^i = x_{t+1}^{i\prime}\left(b_{t+1}^i\right)/u'\left(c_t^i\right) > 0$. In the main text, we explored a special case of this setup in which we assumed that learning-by-exporting effects drop to zero after the initial period, i.e. $\Delta y_t^{i\prime}\left(tb_t^i\right) = 0 \ \forall t \geq 2$.

[21] If learning-by-exporting effects are particularly strong, then the country would benefit from accumulating an ever-increasing amount of bonds that violates the transversality condition on bond holdings. See Korinek and Serven (2010) for a more detailed discussion.

Global Planner

Proposition 15 (Efficiency of Unilaterally Correcting LBE-Externalities) *The global equilibrium in which each domestic planner i corrects domestic learning-by-exporting externalities by imposing the unilaterally optimal capital control τ_{t+1}^{i*} is Pareto efficient.*

Proof. The proof proceeds along similar steps as the proof of the general proposition. The allocation is Pareto efficient if it maximizes global welfare for some vector of country welfare weights $\left\{\phi^i > 0\right\}_{i=1}^N$. It is convenient to express welfare as a function of trade balances

$$\max_{\left\{tb_t^i\right\}} \sum_{i,t} m^i \phi^i \beta^t u\left(y_t^i - tb_t^i\right) \quad \text{s.t.} \quad \sum_i m^i tb_t^i = 0 \forall t$$

where output is a function of accumulated learning-by-exporting effects $y_t^i = y_0^i + \sum_{s=1}^t \Delta y_s^i\left(tb_{s-1}^i\right)$. Assigning the multipliers $\beta^t \lambda_t$ to the constraints, the optimality condition associated to the Lagrangian of this problem is

$$\begin{aligned} \phi^i u'\left(c_t^i\right) &= \lambda_t + \phi^i \Delta y_{t+1}^{i\prime}\left(tb_t^i\right) \sum_{s=t+1}^{\infty} \beta^{s-t} u'\left(c_s^i\right) \\ \text{or} \quad \lambda_t &= \phi^i\left[u'\left(c_t^i\right) - \beta v_{t+1} \Delta y_{t+1}^{i\prime}\left(tb_t^i\right)\right] \end{aligned}$$

Let us normalize $\lambda_0 = 1$ and assign $\lambda_{t+1} = \lambda_t / \left(\beta R_{t+1}\right)$ and $\phi^i = 1/\left[u'\left(c_0\right) - \beta v_1 \Delta y_1^{i\prime}\left(tb_0^i\right)\right]$. Then it can easily be seen that the allocations of the Nash equilibrium among national planners satisfy the optimality conditions of a global planner. ■

A.3 Learning-By-Doing Externalities: General Version

National Planner We describe the recursive second-best problem of a national planner who faces learning-by-doing externalities but cannot impose subsidies on labor,

$$\begin{aligned} \max W\left(b_t^i, A_t^i\right) &= u\left(c_t^i\right) - d\left(\ell_t^i\right) + \beta W\left(b_{t+1}^i, A_t^i + \Delta A_{t+1}^i\left(\ell_t^i\right)\right) \\ \text{s.t.} \quad c_t^i &= A_t^i \ell_t^i + b_t^i - b_{t+1}^i / R_{t+1} \\ A_t^i u'\left(c_t^i\right) &= d'\left(\ell_t^i\right) \end{aligned}$$

where we assign the multiplier λ_t^i to the budget constraint and μ_t^i to the implementability constraint. The optimality conditions of this problem and the associated envelope conditions are

$$\begin{aligned}
FOC\left(c_t^i\right) &: \quad \lambda_t^i = u'\left(c_t^i\right) + \mu_t^i A_t^i u''\left(c_t^i\right) \\
FOC\left(\ell_t^i\right) &: \quad A_t^i \lambda_t^i + \beta W_{A,t+1} \Delta A_{t+1}^{i\prime}\left(\ell_t^i\right) = d'\left(\ell_t^i\right) + \mu_t^i d''\left(\ell_t^i\right) \\
FOC\left(b_{t+1}^i\right) &: \quad \lambda_t^i = \beta R_{t+1} W_{b,t+1} \\
& \quad W_b\left(b_t^i, A_t^i\right) = \lambda_t^i \\
& \quad W_A\left(b_t^i, A_t^i\right) = \lambda_t^i \ell_t^i + \mu_t^i u'\left(c_t^i\right) + \beta W_{A,t+1} = \\
& \quad = \sum_{s=t}^{\infty} \beta^{s-t}\left[\lambda_s^i \ell_s^i + \mu_s^i u'\left(c_s^i\right)\right]
\end{aligned}$$

The planner's Euler equation can be expressed as $\lambda_t^i = \beta R_{t+1} \lambda_{t+1}^i$ or

$$\begin{aligned}
u'\left(c_t^i\right) + \mu_t^i A_t^i u''\left(c_t^i\right) &= \beta R_{t+1}\left[u'\left(c_{t+1}^i\right) + \mu_{t+1}^i A_{t+1}^i u''\left(c_{t+1}^i\right)\right] \\
\text{where} \quad \mu_t^i &= \frac{\beta W_{A,t+1} \Delta A_{t+1}^{i\prime}\left(\ell_t^i\right)}{d''\left(\ell_t^i\right) - \left(A_t^i\right)^2 u''\left(c_t^i\right)}
\end{aligned}$$

Intuitively, the shadow price μ_t^i captures the welfare benefits of learning-by-doing externalities that can be reaped from relaxing the implementability constraint, i.e. from inducing consumers to work harder. The Euler equation reflects that each unit of consumption not only provides consumers the marginal utility $u'\left(c_t^i\right) > 0$ but reduces the incentive to work since it lowers their marginal utility $\mu_t^i A_t^i u''\left(c_t^i\right) < 0$.

If the learning-by-doing externalities remained constant over time, e.g. in a steady state in which $\mu_t^i A_t^i u''\left(c_t^i\right) = \beta R_{t+1} \mu_{t+1}^i A_{t+1}^i u''\left(c_{t+1}^i\right)$, then the externality terms drop out of the Euler equation and the planner has no reason to intervene. However, if learning-by-doing externalities (in absolute value) decrease over time so that $\mu_t^i A_t^i \left|u''\left(c_t^i\right)\right| > \beta R_{t+1} \mu_{t+1}^i A_{t+1}^i \left|u''\left(c_{t+1}^i\right)\right|$, then the national planner finds it optimal to subsidize capital outflows while the externality is at work,

$$\tau_{t+1}^i = -\frac{\mu_t^i A_t^i u''\left(c_t^i\right) - \beta R_{t+1} \mu_{t+1}^i A_{t+1}^i u''\left(c_{t+1}^i\right)}{u'\left(c_t^i\right)} \tag{34}$$

In case learning-by-doing externalities are only active in the first period, this expression simplifies to the one given in (20). Conversely, if the externalities increased over time, the opposite conclusions hold.

Global Planner

Proposition 16 (Efficiency of Unilaterally Correcting LBD-Externalities) *The global equilibrium in which each domestic planner i imposes the unilaterally second-best capital control $\tilde{\tau}_{t+1}^i$ given by (34) to correct domestic learning-by-doing externalities is constrained Pareto efficient, given the restrictions on instruments.*

Proof. The allocation is constrained Pareto efficient if it maximizes global welfare for some vector of country welfare weights $\left\{\phi^i > 0\right\}_{i=1}^N$ while respecting the implementability constraint, $A_t^i u'\left(c_t^i\right) = d'\left(\ell_t^i\right) \forall i, t$. The associated Lagrangian as a function of trade balances is

$$L = \sum_{i,t} m^i \phi^i \beta^t \left\{ u\left(c_t^i\right) - d\left(\ell_t^i\right) - \lambda_t^i \left[c_t^i + tb_t^i - A_t^i \ell_t^i\right] + \mu_t^i \left[A_t^i u'\left(c_t^i\right) - d'\left(\ell_t^i\right)\right] \right\} + \\ + \sum_{i,t} \nu_t \beta^t m^i tb_t^i$$

where productivity is a function of accumulated learning-by-doing effects $A_t^i = A_0^i + \sum_{s=1}^t \Delta A_s^i\left(\ell_{s-1}^i\right)$ and we denote the value of an additional unit of productivity by $W_{A,t} =$. The optimality conditions are

$$FOC\left(c_t^i\right) \quad : \quad \lambda_t^i = u'\left(c_t^i\right) + \mu_t^i A_t^i u''\left(c_t^i\right)$$

$$FOC\left(\ell_t^i\right) \quad : \quad d'\left(\ell_t^i\right) = \lambda_t^i A_t^i - \mu_t^i d''\left(\ell_t^i\right) + \Delta A_{t+1}^{i\prime}\left(\ell_t^i\right) \sum_{s=t+1}^{\infty} \beta^{s-t} \left[\lambda_s^i \ell_s^i + \mu_s^i u'\left(c_s^i\right)\right]$$

$$FOC\left(tb_t^i\right) \quad : \quad \phi^i \lambda_t^i = \nu_t$$

We combine the first and second conditions and substitute the implementability constraint to obtain

$$\mu_t^i = \frac{\beta W_{A,t+1} \Delta A_{t+1}^{i\prime}\left(\ell_t^i\right)}{d''\left(\ell_t^i\right) - \left(A_t^i\right)^2 u''\left(c_t^i\right)}$$

which equals (19) in the problem of national planners. If we normalize $\nu_0 = 1$ and assign $\nu_{t+1} = \nu_t / \left(\beta R_{t+1}\right)$ and $\phi^i = 1/\left[u'\left(c_0^i\right) + \mu_0^i A_0^i u''\left(c_0^i\right)\right]$, then the allocations of the Nash equilibrium among national planners satisfy the constrained optimality conditions of a global planner. ■