COVID-19 Infection Externalities:
Pursuing Herd Immunity or Containment?*

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Abstract

We analyze the externalities that arise when social and economic interactions transmit infectious diseases such as COVID-19. We show that internalizing these externalities would lead society to pursue containment as opposed to letting the disease spread, no matter if immunity to COVID-19 is possible or not. In an SIR model calibrated to capture the main features of COVID-19 in the US economy, we show that private agents perceive the cost of an additional infection to be around $18k whereas the social cost including infection externalities is $55k. It is socially optimal to pursue a "smart containment" strategy that targets the infected and quickly contains the disease at relatively low economic cost. If the infected cannot be targeted, a second-best "blind containment" strategy involving general lockdowns comes at much greater economic cost. The decentralized "herd immunity" strategy leads to a long-lasting economic slump as susceptible individuals reduce their economic activity over several years.

Keywords: COVID-19, infection externalities, social distancing
SIS model, SIR model, epidemiological strategy

JEL Codes: H12, H23, E65, I18

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1 Introduction

The ongoing coronavirus pandemic has presented policymakers around the world with a pivotal challenge: to choose between an uncontrolled spread of the virus or the imposition of public health interventions such as lockdowns that severely infringe upon personal liberties. Some have presented this choice as a trade-off between lives versus livelihoods, arguing that lockdowns may reduce the death toll but come at significant economic cost. Others, by contrast, have argued that containing the virus is a precondition for healthy economic activity so that there is in fact no trade-off between lives and livelihoods.

International experience provides rich examples of the different strategies that countries have employed to address this challenge. Figure 1 depicts the population-wide infection rate for a selected set of countries. Countries in the first row have contained the disease, although with stark differences in peak infection rates, which ranged from 0.004% in China to 0.015% in South Korea and 0.1% in Germany, where Chancellor Merkel justified the country’s containment strategy as follows: “If we show the greatest possible endurance and discipline at the beginning of this pandemic, we will be able to return to economic, social and public life more quickly and sustainably.” In countries in the second row, infection rates are already higher than these levels and still rising. In the US, social distancing guidelines were being relaxed even though infections were still increasing in mid-May. Sweden has officially proclaimed that it will not impose mandatory lockdowns and is opting for a “herd immunity” strategy instead. At the time of writing, the ultimate economic toll of the different strategies is still unknown.

This paper employs an analysis of the externalities involved in disease transmission to evaluate the available public health strategies to deal with the pandemic and inform policy. When infected individuals engage in social or economic activity, they put those with whom they interact with at risk of infection; in doing so, we find that they impose significant externalities – they significantly undervalue the social cost of an additional infection. In the absence of public health interventions, the burden of disease mitigation is on susceptible individuals who rationally engage in social distancing and reduce their economic activity, imposing significant costs on the economy that last for years, until herd immunity is acquired, and costing many lives along the way. By contrast, public health measures that internalize the
externalities quarantine the infected which quickly contains the disease. This enables susceptible individuals to return to their economic activity much sooner. We estimate the benefit of such a “smart containment” strategy to be around $14 trillion for the US economy. In summary, except in the very short run, there is in fact no trade-off between lives and livelihoods.

The novel coronavirus was first identified in Wuhan, China, in December 2019. It jumped from bats via an intermediate host to humans. The virus has officially been named “SARS-CoV-2,” and the disease that it causes has been named “Coronavirus Disease 2019” (abbreviated “COVID-19”). It spreads among humans via respiratory droplets and aerosols as well as by touching infected surfaces. In an uncontrolled outbreak, the disease burden grows exponentially, with cases doubling approximately every six days. The incubation period, i.e. the time between when one is exposed to the virus and when one develops symptoms of disease, is from two to 14 days, with an average of five days. Those infected usually present with a fever, a dry cough and general fatigue, frequently involving a mild form of pneumonia, although there are also many asymptomatic cases. About 15 percent of cases develop more severe pneumonia that requires hospitalization, intensive care, or even mechanical ventilation. Verity et al. (2020) estimate the infection fatality rate to be around 0.67% – as long as the healthcare capacity of a country is not overwhelmed.

This paper analyzes the externalities that arise when economic interactions trans-
mit infectious diseases such as COVID-19. We embed rationally optimizing individual agents into epidemiological models and allow the agents to choose a level of economic activity that also influences the rate of disease transmission. Given the uncertainty about whether lasting immunity to SARS-CoV-2 can be acquired (Li et al., 2020; World Health Organization, 2020), we focus on two different epidemiological models: In section 2, we analyze an SIS (Susceptible-Infected-Susceptible) model, which splits the population into two compartments – susceptible $S$ and infected $I$. Susceptible agents $S$ become infected at a given rate by interacting with infected agents; infected agents $I$ in turn recover at a given rate and return to the pool of susceptible agents $S$. In section 3 we include a third epidemiological compartment $R$ of recovered and resistant agents in our analysis, delivering the SIR (Susceptible-Infected-Recovered) model in the spirit of the first epidemiological model laid out by Kermack and McKendrick (1927).¹

We contrast the behavior of individually rational optimizing agents with what would be chosen by a social planner who has the power to coordinate agents’ decisions. Individual agents who are susceptible to a disease rationally reduce the level of their economic activity so as to reduce the risk of infection. However, infected agents who act individually rationally do not engage in sufficient precautions in their economic and social interactions since they do not internalize that their activities impose externalities upon others by exposing them to the risk of infection. We show in formal propositions that this induces the social planner to value the cost of an extra infection more highly than decentralized agents. Moreover, we describe a corollary that lays out four alternative ways to induce private agents to internalize their externalities.

In our analysis of the SIS model, the decentralized outcome is to converge to an equilibrium in which the disease is endemic. By contrast, a social planner who internalizes the infection externalities contains the infected agents by significantly reducing their activity so as to lower the spread of the disease. In our simulations, we find that for a wide range of parameter values, the social planner does this to a sufficient extent to contain and eradicate the disease from the population. Only if the social cost of a disease is extremely low, lower than the common cold, will the planner allow the disease to become endemic.

¹For more on epidemiological models see Anderson and May (1991) or the overview at https://en.wikipedia.org/wiki/Compartmental_models_in_epidemiology
In our analysis of the SIR model, the decentralized outcome is that the disease spreads until herd immunity is acquired. Along the way, susceptible agents who are wary of becoming infected significantly reduce economic activity. By contrast, a social planner who can target individuals based on whether they are infected or not will pursue a “smart containment” strategy whereby she contains the infected and imposes only minor restrictions on the susceptible agents, imposing substantially lower economic costs than what is implied by the “herd immunity” strategy. When calibrating our SIR model to capture the epidemiological parameters of COVID-19 in the US economy, we find that individually rational agents perceive the cost of an additional infection to be around $18k whereas the true social cost including infection externalities is more than three times higher, around $55k, when the fraction of infected agents is 0.3%. The economic cost of following the “herd immunity” strategy is high: an initial sharp decline in aggregate output by 13% over the first 15 weeks followed by a slow recovery that takes several years. Accounting for both economic losses and the statistical value of lives lost, the “smart containment” strategy saves around $10 trillion compared to the “herd immunity” strategy. By focusing public policy measures on the infected who make up a much smaller fraction of the population in the initial stages of an epidemic, smart containment produces a much milder recession.

A natural concern is that targeting the infected is difficult since many countries, including the US, have suffered from shortages in testing kits, and because COVID-19 has a long incubation period and a significant fraction of infected individuals are asymptomatic. To capture this situation, we analyze a version of our model in which the epidemiological status of individuals is hidden so the planner has to choose a uniform level of economic activity for all agents. Even in that case, the social planner aggressively contains the disease for our baseline parameterization. However this must now be achieved through a reduction in the level of activity of all agents, generating a decline in aggregate output that is much more severe than under smart containment – about a 10% reduction in aggregate activity that is long-lasting since the planner needs to keep activity low to prevent the re-emergence of the disease. If the planner assigns a lower value to lives, e.g. if he only counts the economic losses from losing workers rather than the statistical value of life as in our baseline parameterization, then “blind containment” is no longer optimal and the planner opts to allow the disease to spread, albeit at a slower rate than without
intervention to “flatten the curve,” and achieve herd immunity within the first year of the pandemic.

We also compare the private and social gains from vaccination. Individually rational susceptible agents find vaccines useful for two reasons: first, they no longer face the risk of costly infection and second they no longer need to incur the cost of social distancing to avoid becoming infected. Vaccines are most useful in a society in which social distancing is determined by individually rational behavior since it fast-tracks herd immunity. When 0.3% of the population is infected and no one has acquired immunity yet, the private gain from an individual vaccination is $9k. By contrast, the social gain from an individual vaccination is over 7 times larger, at $70k, when policymakers pursue the herd immunity strategy. Overall, vaccinating the population delivers a gain of $9.5 trillion in the aggregate when policymakers would otherwise follow the herd immunity strategy, or an $7.2 trillion gain when policymakers would pursue the blind containment strategy. If policymakers followed the smart containment, the gains from a vaccine are significantly lower. This illustrates that smart containment and vaccination are substitutes.

**Literature** In the economics literature, our work is most closely related to papers that study the externalities of health interventions for infectious diseases in SIS and SIR models, beginning with Gersovitz and Hammer (2003, 2004), Goldman and Lightwood (2002), and Gersovitz (2011). Georgiy et al. (2011) show cross-country externalities in responding to flu pandemics. Our addition to this early strand of literature is (i) to analyze the economic effects of the specific non-pharmaceutical interventions relevant for COVID-19 – social distancing – and (ii) to contribute a quantitative analysis to the evaluation of COVID-19 infection externalities to better inform the policy debate.

A fast-growing recent literature examines optimal policy in SIR-type models calibrated to COVID-19, including Alvarez et al. (2020), Eichenbaum et al. (2020), Jones et al. (2020), Garibaldi et al. (2020), Keppo et al. (2020), Krueger et al. (2020), Glover et al. (2020), Farboodi et al. (2020), Kapicka and Rupert (2020), and Toxvaerd (2020). Chudik et al. (2020) analyze distancing and infection dynamics in an SIR model and provide evidence for selected regions. We complement these papers by providing analytic results on the externalities that arise in both SIS and SIR models and by quantifying how much individually rational agents undervalue
the cost of infection with COVID-19. Our findings also highlight the crucial role of testing, as suggested in Berger et al. (2020) and Piguillem and Shi (2020), among others. Our estimates are informed by data on the rate of COVID-19 transmission, death rates, and hospital capacity provided by Atkeson (2020), Verity et al. (2020), and others. Our work complements other recent economics papers that analyze the macroeconomic implications of COVID-19, such as the role of aggregate demand policies (e.g. Guerrieri et al., 2020), fiscal policy (e.g. Faria-e-Castro, 2020), labor market effects (e.g. Gregory et al., 2020), and amplification (Caballero and Simsek, 2020) in a pandemic.

2 An SIS Economy

2.1 Model Setup

In this section, we develop an SIS model that introduces a role for economic decision-making, and we perform an analysis of welfare and of the externalities that arise. The SIS framework is a benchmark model in which recovered individuals do not gain immunity. Although we hope that recovering from COVID-19 will provide individuals with immunity, the jury on this question is still out (World Health Organization, 2020). More generally, the SIS model provides the simplest possible setting to illustrate the structure of the externalities problem and allows us to analyze the interactions between economics and epidemiology in utmost clarity. In section 3 below, we will expand on this framework to add a third epidemiological compartment of recovered/resistant individuals to describe the externalities arising when recovered individuals gain immunity.

Epidemiology  Let us denote the mass of susceptible individuals by $S$ and the mass of infected individuals by $I$, and normalize the total population to $N = S + I = 1 \forall t$. By assumption, all individuals in a given category are identical. Time is continuous and goes on forever. We follow the convention in the epidemiological literature of dropping the time subscript of $S$ and $I$ but remind the reader that the variables are, of course, time-dependent. Changes are denoted by $\dot{S}$ and $\dot{I}$. The
evolution of $S$ and $I$ follows the standard epidemiological laws

$$\dot{S} = -\beta(\cdot) IS + \gamma I$$

$$\dot{I} = \beta(\cdot) IS - \gamma I$$

(1) (2)

The term $\beta(\cdot) IS$ captures the flow of susceptible individuals that become infected: it reflects that there is a mass $S$ of susceptible individuals in the population that interact with other agents with meeting intensity $\beta(\cdot)$, and the probability that a given interaction partner is infected and passes on the disease is $\frac{I}{N} = I$. In the economic model block below, we will specify how $\beta(\cdot)$ depends on individual behavior. The term $+\gamma I$ captures that infected individuals recover at rate $\gamma$ and return to the pool of susceptible individuals. The expression for $\dot{I}$ is the mirror image of $\dot{S}$ since the population is constant. Thus it is sufficient to keep track of only one of the two variables – an epidemiological version of Walras’ Law.

**Individual Behavior**  The utility of an individual agent depends on her epidemiological status $i = S, I$ as well as on the level of activity $a_i \in [0, 1]$ that she chooses to take contingent on her status. This level may be interpreted as the extent of economic activity in those areas in which such activity requires physical interaction. Activity level $a_i = 0$ reflects complete isolation whereas $a_i = 1$ captures normal activity. We parameterize the probability of infection $\beta(a_S, a_I) = \beta_0 a_S a_I$ in the spirit of the epidemiological relationships described above, where $\beta_0$ reflects the spread at the maximum level of activity for both types of agents.

In the dynamic decision problem of individual agents, we denote by $I = Pr(i = I)$ the probability of an agent being infected. We observe that each atomistic agent takes as given the activity level and the fraction of infected agents in the population and denote these by $\bar{a}_I$ and $\bar{I}$, where the latter evolves according to the law (2). The individual’s epidemiological status thus satisfies

$$\dot{I} = \beta(a_S, \bar{a}_I) \bar{I} (1 - I) - \gamma I$$

(3)

In equilibrium it will be the case that $\bar{a}_I = a_I$ and $\bar{I} = I$.

\textsuperscript{2}An alternative interpretation is that the decision maker is a household that is small relative to the overall economy, with a fraction $I$ of members infected.
For an individual with initial epidemiological status \( I(0) \), the utility maximization problem is to choose a path of activity levels \( \{a_S, a_I\} \) so as to

\[
\max_{\{a_S, a_I\}} U = \int_t E_i \left[ e^{-rt} u_i(a_i) \right]
\]

subject to (3), where the flow utility of the agent in a given period is \( E_i[u_i(a_i)] \). For now, we capture the utility derived from social activity in reduced form. In our full model below we will describe how activity \( a \) interacts with the economic functions of agents in more detail. We assume that the flow utility of susceptible agents \( u_S(a) = u(a) \) is increasing and concave \( u''(a) < 0 < u'(a) \) up to its maximum level at which it becomes flat so \( u'(1) = 0 \). The flow utility of infected agents is \( u_I(a) = u(a) - c(\bar{I}) \) where \( c(\bar{I}) \) captures the utility loss from being sick and satisfies \( c(0) > 0 \) and \( c'(\bar{I}) \geq 0 \). The latter may reflect congestion effects in the healthcare system, which have been of critical importance during the COVID-19 pandemic in some regions.

We reformulate the individual’s optimization problem in terms of the current-value Hamiltonian

\[
\mathcal{H} = I \left[ u(a_I) - c(\bar{I}) \right] + (1 - I) u(a_S) - V_I \left[ \beta(a_S, \bar{a}_I) \bar{I} (1 - I) - \gamma I \right]
\]

together with the transversality condition \( \lim_{T \to \infty} e^{-rT} V_I \cdot I = 0 \), where \( V_I \) is the current-value shadow cost of an agent being infected. Each agent internalizes that her infection status depends on her choice of interactions with other agents but rationally takes as given the overall fraction of the population infected \( \bar{I} \) and their actions \( \bar{a}_I \), which determine both the risk of infection for susceptible individuals and the congestion effects in the healthcare system. This generates rich externalities, as we will explore subsequently.

In addition to the transversality condition, the individual’s optimality conditions are

\[
u'(a_S) = V_I \cdot \beta_0 \bar{a}_I \bar{I} \]
\[
u'(a_I) = 0 \]
\[rV_I = u(a_S) - u(a_I) + c(\bar{I}) - V_I \beta(\cdot) \bar{I} - V_I \gamma + \dot{V}_I \]
The first optimality condition reflects that the agent equates the marginal utility of activity \(a_S\) to the marginal expected cost of becoming infected, which consists of the lifetime utility loss of infection \(V_I\) times the marginal probability of infection \(\beta_S(\cdot) \bar{I} = \beta_0 a_I \bar{I}\). Ceteris paribus, a larger number of infected agents increases the infection probability \(\beta \bar{I}\) and induces the agent to scale back her economic activity, i.e. to behave in a more cautious manner. The second optimality condition implies that it is individually rational for the infected agent to not take into account the epidemiological effects of her behavior, inducing the maximum level of activity \(a_I = 1\). The third optimality condition reflects the flow shadow cost of being infected versus susceptible: the agent obtains different flow utility and experiences the cost \(c(\bar{I})\); moreover, the agent no longer faces the risk of infection, captured by the term \(-V_I \beta(\cdot) \bar{I}\) and faces the potential prospect of recovery \(-V_I \gamma\); finally, the shadow cost of being infected changes through time as \(I\) changes.

In equilibrium, the probability of infection of an individual agent equals the aggregate fraction of infected agents \(I = \bar{I}\).

**Definition 1** (Decentralized SIS Economy). For given initial \(I(0)\), a decentralized equilibrium of the described SIS system is given by a path of the epidemiological variable \(I\) that follows the epidemiological law (2), as well as paths of action variables \((a_S, a_I)\) and the shadow cost \(V_I\) that satisfy the optimization problem of individual agents.

**Steady State** In steady state, we set \(\dot{I} = 0\) in equation (2), obtaining a non-degenerate infection rate of \(I = 1 - \gamma / \beta(a_S, a_I)\), and set \(\dot{V}_I = 0\) in (7). The optimality condition (6) implies \(a_I = 1\). The three variables \(I, a_S, V_I\) are jointly pinned down by equation (5) as well as the two laws-of-motion set to zero.

**2.2 Social Planner**

Let us now contrast the outcome in a decentralized setting with what would be socially optimal if a planner who must obey the epidemiological laws can determine the path of individual actions \(\{a_S, a_I\}\). The planner would maximize overall social welfare, consisting of the integral over the utility (4) of the unit mass of agents \(j \in [0, 1]\),

\[
W = \int U dj
\]  
(8)
where the epidemiological status of the population follows the epidemiological law (2).

For a given value of initial infections $I(0)$, the problem of the planner can be captured by the current-value Hamiltonian

$$
\mathcal{H} = I [u(a_I) - c(I)] + (1 - I) u(a_S) - W_I \left[ \beta(a_I, a_S) I (1 - I) - \gamma I \right]
$$

plus the transversality condition $\lim_{T \to \infty} e^{-rT} W_I \cdot I = 0$, where $W_I$ is the current-value shadow cost of an agent being infected. The resulting optimality conditions are

$$
\begin{align*}
    u'(a_S^*) &= W_I \cdot \beta_0 a_I^* I \\
    u'(a_I^*) &= W_I \cdot \beta_0 a_S^* (1 - I) \\
    r W_I &= u(a_S^*) - u(a_I^*) + c(I) + Ic'(I) + W_I \cdot \beta(\cdot) (1 - 2I) - W_I \gamma + \dot{W_I}
\end{align*}
$$

where we denote by an asterisk the planner’s choices.

**Definition 2 (Planner’s Allocation in SIS Economy).** For given $I(0)$, the planner’s allocation in the described SIS system is given by a path of the epidemiological variable $I$ that follows the epidemiological law (2) as well as paths of action variables $(a_S^*, a_I^*)$ and the shadow cost $W_I$ that satisfy the planner’s optimization problem.

The optimality condition (5) for $a_S^*$ mirrors the equivalent expression (5) in the decentralized equilibrium – individual agents and the planner both account for the risk of infection of susceptible agents in a similar manner. However, the planner’s shadow price of infection $W_I$ differs from that of decentralized agents, as we will describe shortly. The second optimality condition (10) for $a_I^*$ differs from the optimality condition of private agents (6): the planner captures that the activity of infected agents increases the infection risk of the susceptible, which individual agents disregard. The third optimality condition (11) captures the law of motion of the planner’s shadow price of infection $W_I$. In addition to the costs captured by individual agents in the decentralized equilibrium in equation (7), the extra term $Ic'(I) \geq 0$ reflects that the cost of infections rises in the fraction infected at the aggregate level, and the extra term $W_I \beta(\cdot) (1 - I)$ reflects that the planner internalizes that infected agents will transmit the disease to the susceptible population $(1 - I)$ at rate $\beta(\cdot)$,
where both terms are positive. For a given path of actions \((a_S, a_I)\), this implies that \(W_I > V_I\) so the planner values the cost of acquiring the infection more highly than private agents. We summarize our results as follows:

**Proposition 1** (Infection Externalities in SIS Economy). The planner internalizes the infection externalities of the infected and would choose a lower level of activity for infected agents, \(a_I^* < a_I\). For given actions, the planner experiences a higher social cost of infection than private agents, \(W_I > V_I\).

*Proof.* See discussion above. \(\square\)

Whether the planner will induce more or less activity for susceptible agents than in the decentralized equilibrium for given \(I\) depends on two competing forces: since the infected engage in less activity, the risk of infection for susceptible agents is lower, generating a force toward greater activity; however, for given actions, the planner recognizes a greater social loss from one more individual becoming infected, \(W_I > V_I\), generating a force toward lower levels of activity. By implication, for given \(a_I\), she would choose a lower level \(a_S^* < a_S\) than decentralized agents. But since the planner optimally chooses \(a_I^* < a_I\), it may turn out that \(a_S^* \geq a_S\) in general.

**Corollary 1** (Decentralizing the SIS Economy). The planner can implement her allocation in a decentralized setting in the following ways:

1. by imposing quantity controls on the activities of susceptible and infected agents such that
   
   \[ a_S \leq a_S^*, \quad a_I \leq a_I^* \]

2. by imposing taxes on the activities of susceptible and infected agents \(a_S\) and \(a_I\) such that
   
   \[
   \begin{align*}
   \tau_I &= W_I \cdot \beta_0 a_S^* (1 - I) > 0 \\
   \tau_S &= (W_I - V_I) \beta_0 a_I^* I > 0
   \end{align*}
   \]

3. by imposing the tax (12) on the activity of infected \(a_I\), and a utility penalty on becoming infected of
   
   \[ \tau_V = W_I - V_I > 0 \]

\[12\]
4. by imposing the tax (12) on the activity of infected \(a_I\), and a utility penalty or equivalent tax on being infected such that

\[
\tau_C = Ic' (I) + W_I \cdot \beta (\cdot) (1 - I) > 0
\]

(15)

as well as any appropriate combination of the described instruments \(a^*_S, a^*_I, \) and \(\tau_S, \tau_V, \tau_C\).

If \(I \to 0\) in the long run, observe that \(a^*_S \to 1\) and \(\tau_S \to 0\) but the remaining policy instruments remain strictly positive.

Proof. Point 1. follows immediately from Proposition 1. For the remaining points, observe that the current-value Hamiltonian of individuals who face taxes \(\tau_S\) and \(\tau_I\) on activity levels \(a_S\) and \(a_I\) while being susceptible or infected as well as a tax on being infected \(\tau_C\) is

\[
\mathcal{H} = I \left[ u (a_I) - \tau_I a_I - c (I) - \tau_C \right] + (1 - I) \left[ u (a_S) - \tau_S a_S \right] - V_I [\beta (a_S, \bar{a}_I) \bar{I} (1 - I) - \gamma I]
\]

Given a utility penalty \(\tau_V\) on becoming infected, the resulting optimality conditions are

\[
\begin{align*}
u' (a_S) &= \tau_S + (V_I + \tau_V) \beta_0 \bar{a}_I \cdot \bar{I} \quad (16) \\
u' (a_I) &= \tau_I \quad (17) \\
rV_I = u (a_S) - u (a_I) + c (I) + \tau_C - V_I \beta (\cdot) \bar{I} - V_I \gamma + \bar{V}_I
\end{align*}
\]

(18)

By setting \(\tau_I\) to the value given in (12) and one of the three instruments \(\tau_S, \tau_V, \tau_C\) to the values given in (13) to (15), the optimality conditions of decentralized agents who face the taxes will replicate the optimality conditions (9) and (10) of the planner.

To verify the long-run behavior of \(a^*_S\) and \(\tau_S\) when \(I \to 0\), simply take the limit of expressions 9 and 13 and use the transversality condition.

Formulating the different ways of decentralizing the SIS economy is not necessarily meant to provide hands-on policy guidance (especially for points 3. and 4.)

\[\text{\textsuperscript{3}}\text{In an alternative formulation, we could also impose the tax rates }\tau_S\text{ and }\tau_I\text{ directly on the consumption of goods or on the provision of effort leading to the production of goods. We chosen the given formulation to keep the resulting tax formulas as compact as possible.}\]
Instead, we describe the four different options because they offer four complementary ways to pinpoint the infection externalities in our framework. Clearly, as captured by points 1. and 2., it is the actions of the susceptible and infected that ultimately need to change to implement the socially optimal allocation. However, the sole reason why the behavior of the susceptible is distorted is that they misperceive the social cost of being infected. As point 3. illustrates, this implies that correcting the shadow price of becoming infected by imposing an extra penalty would induce the socially optimal level of activity among the susceptible. Moreover, as portrayed in point 4., the undervaluation of the shadow price of infection arises simply because infected individuals – even once we have induced them to engage in the socially optimal level of activity – do not internalize the potential cost that they impose on others, captured by the right-hand side of (15), which consists both of the increase in the cost $C(I)$ for all agents and the term reflecting the infection externality.

**Steady State** The steady state of the system is obtained by setting $\dot{I} = 0$ and $\dot{W}_I = 0$ in equations (2) and (11). For given $(I, W_I)$, optimality conditions (9) and (10) jointly pin down $a^*_S$ and $a^*_f$.

### 2.3 Calibration

The time units in our calibration are weeks. We set the epidemiological parameters to $\gamma = 1/3$ to reflect an average duration of the disease of three weeks and $\beta_0 = 2.5/3$ to capture a basic reproduction rate $R_0 = \beta_0 / \gamma$ of 2.5, reflecting best available estimates on the spread of the disease without precautionary measures.\footnote{See the discussion in Atkeson (2020) and references therein. Current evidence suggests that COVID-19 has an $R_0$ between 2.0 to 3.25.} We perform our analysis for an initial infection rate of $I(0) = 0.3\%$, which is approximately the officially confirmed infection rate in the US in mid-May 2020.\footnote{For current infection numbers see e.g. \url{https://www.worldometers.info/coronavirus/country/us/}} Except where expressly discussed, our simulation results are continuous in the initial infection rate. Changes in $I(0)$ therefore do not affect our main conclusions.

We set the economic parameter $r$ to reflect a typical annual discount rate of 4%. To capture the effects of the level of activity $a$ on the economy and ultimately on welfare, we assume that there is a unit mass $h \in [0, 1]$ of goods $c_h$, of which a
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recovery rate $\gamma$</td>
<td>$1/3$</td>
<td>Average duration: 3 weeks</td>
</tr>
<tr>
<td>Baseline infection rate $\beta_0$</td>
<td>$5/6$</td>
<td>Reproduction rate $R_0 = \beta_0/\gamma = 2.5$</td>
</tr>
<tr>
<td>Initial infection rate $I(0)$</td>
<td>0.3%</td>
<td>World-o-meter</td>
</tr>
<tr>
<td>Time discount rate $r$</td>
<td>4%/52</td>
<td>Yearly discount rate of 4%</td>
</tr>
<tr>
<td>% goods requiring physical contact $\phi$</td>
<td>25%</td>
<td>From Mitchell (2020)</td>
</tr>
<tr>
<td>Weekly baseline GDP/capita</td>
<td>$1200$</td>
<td>GDP/population from BEA</td>
</tr>
<tr>
<td>Baseline utility cost of infection $c_0^{(1)}$</td>
<td>14</td>
<td>Statistical cost of disease (Table A1)</td>
</tr>
<tr>
<td>Economic losses from infection $c_0^{(2)}$</td>
<td>0.61</td>
<td>Economic loss only (Table A2)</td>
</tr>
<tr>
<td>Slope of cost-of-disease function $\kappa$</td>
<td>10</td>
<td>Ventilator capacity; see text below</td>
</tr>
</tbody>
</table>

Table 1: Parameterization

fraction $\phi$ requires physical contact. Examples for goods that do not require physical contact are real estate services, information services, etc. Conversely, examples of goods that do require physical contact include personal services such as haircuts, hospitality, medical treatments, transportation, etc. Although it is difficult to draw a sharp delineation, we set $\phi = .25$, in line with estimates reported in Mitchell (2020) on the fraction of the economy that is paralyzed by a severe physical lockdown of economic activity.

Producing and consuming $c_h$ units of good $h$ generates disutility $d(c_h)$ and provides consumption utility $\tilde{u}(c_h)$. All the goods together provide the agent with overall flow utility of

$$u = \int [\tilde{u}(c_h) - d(c_h)] dh$$

For any good that does not depend on physical contact, it is optimal to choose the first-best level of output and consumption $c^*$, which satisfies $\tilde{u}'(c^*) = d'(c^*)$. By contrast, for the fraction $\phi$ of goods that do require physical interaction, output and consumption is scaled by the activity variable $a$ so that $c_h = ac^*$. The resulting flow utility of activity level $a$ is

$$u(a) = \phi [\tilde{u}(ac^*) - d(ac^*)] + (1 - \phi) [\tilde{u}(c^*) - d(c^*)]$$

In our numerical application below, we assume log consumption utility $\tilde{u}(c) = \log c$ and linear disutility $d(c) = c$, implying that overall flow utility is $u(a) = \phi [\log a - a]$, omitting a constant term. Observe that this specification satisfies our
earlier assumptions \( \lim_{a \to 0} u'(a) = \infty \) and \( u'(1) = 0 \). Note that we have implicitly assumed that the utility of all individuals of a given epidemiological status is affected equally by a reduction in activity \( a \). This is valid if individuals are well-insured, including if they receive social insurance against idiosyncratic shocks. By contrast, if some individuals lose their jobs and incomes whereas others can continue to work, additional welfare costs arise and negative demand multipliers may be triggered (see e.g. Guerrieri et al., 2020).

**Cost of disease** We consider two different scenarios for the cost of disease. Our baseline scenario consists of our best estimate of the full direct cost of COVID-19 for an individual, which includes the disutility of being sick and, in reduced form, the potential risk of death. We also consider a second scenario that includes only the economic losses arising from sickness and death from COVID-19.

**Baseline scenario** In our baseline parameterization we focus on individuals’ expected losses from the risk of death governed by the statistical value of life. (We find that these costs trump any other costs borne by infected individuals by an order of magnitude.) In the analysis of public policies, e.g., safety regulations or environmental policies, economists routinely have to weigh decisions that compare economic benefits and health costs. Estimates of the implied cost of adverse health events are obtained by evaluating how much individuals are willing to spend to avoid a given risk of an adverse event. Based on guidance from the US Department of Transportation (2012) on the value of a statistical life augmented by consumer price inflation, a current estimate in the US is around \$10.3m at the age of the median worker of approximately 40 years. By comparison, before the pandemic, the weekly level of economic activity in the US as measured by GDP was approximately \$1200/capita. In our model, we assume that this corresponds to the first-best level \( c^* = 1 \) and observe that the marginal utility of consumption at that level satisfies \( \tilde{u}'(c^*) = 1 \). For an average individual, a risk of death of \( \delta = 0.66\% \) for a disease that lasts on average for \( 1/\gamma \) weeks could thus be expressed in terms of a weekly flow utility cost of \$10.3m/\$1200\cdot0.0066\cdot\gamma \approx 19 \).

However, a striking feature of COVID-19 is that the case fatality rate depends strongly on age (Verity et al., 2020), ranging from virtually zero for children and teenagers to 7.8\% for patients of age \( \geq 80 \). Combining Verity et al (2020)’s infection
fatality rates with life expectancy data from the SSA, Table A1 shows that the expected statistical loss of life years for an average infected individual in the US is 0.136 years. Using the procedure described by Atkins and Bradford (2020) and a discount rate of 4%, we translate the $10.3m statistical value of life into a $498k value of a statistical life year. Calculating the present discounted value of this figure across different age cohorts, Table A1 shows that this delivers an expected statistical loss of life valued at $50.0k for a COVID-19 infection, which amounts to a weekly flow utility cost in our baseline scenario of $50.0k/$1200·γ ≈ 14.

We thus parameterize the cost of disease as $c(I) = c_0(1+\kappa I)$ where the base cost of disease is given by $c_0 = 14$. One of the concerns about COVID-19 is its potential to overwhelm the capacity of our healthcare system since about 15% of cases require hospitalization and about 6% of cases require mechanical ventilators; see Giannakeas et al. (2020). (Given the early stage of medical research on COVID-19, there is still considerable uncertainty about these parameter values.) The US currently has only about 200,000 ventilators available (Johns Hopkins Center for Health Security, 2020). Assuming the best available distribution to the places where they are needed and no other demand for ventilators by chronically sick patients, this implies that at most .06% of the population can be served at a given time. If the infection rate rises above $I = .06%/6% = .01$, mortality will rise significantly, as experienced in earlier hotspots such as Wuhan or Northern Italy. We set $\kappa = 1/.01/2 \times 20% \approx 10$ to reflect that the fatality rate would rise by 20% if the number of patients requiring ventilators is twice the available capacity. In summary, the parameters for our baseline calibration of the cost of disease are $(c_0, \kappa) = (14, 10)$.

**Economic loss-only scenario** An alternative scenario that we consider is that decisionmakers care only about the purely economic losses from the disease rather than the broader statistical value of life that captures what individuals would be willing to pay to preserve their life. Table A2 lists average weekly income by age category based on data from the CPS March 2019 Supplement. We assume that total economic losses consist of 50% of infected individuals (accounting for asymptomatic cases\(^6\)) not being able to work for the duration of the disease plus the age-dependent fraction of individuals indicated in the table passing away and losing

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\(^6\)See [https://www.cebm.net/covid-19/covid-19-what-proportion-are-asymptomatic/](https://www.cebm.net/covid-19/covid-19-what-proportion-are-asymptomatic/) for an overview of results on what fractions of individuals are asymptomatic.
the present discounted value of future earnings. The weighted average total income loss from these two sources is $2,197 per infection, which we convert to a flow cost of $c_0 = \frac{2197}{1200} = 0.61$ using our earlier conversion procedure. Observe that this is 95% less than our baseline cost from the statistical value of life. Accounting for the worsened labor market opportunities during a pandemic would reduce the lost flow income numbers even further.

Computational Procedure

Computationally, we solve a system of two non-linear differential equations with boundary conditions using a shooting algorithm. In the decentralized equilibrium, the system is given by \((I, V_I)\) described in (2) and (7), subject to \(I(0)\) and the transversality condition. The system features two steady states: an unstable one at \(I = 0\) and a stable one at \(I \in (0, 1 - \frac{\gamma}{\beta})\). Starting from any \(I(0) > 0\), the system is saddle-path stable leading to the non-degenerate equilibrium. Similarly, the planner’s allocation is given by a path of \((I, W_I)\) described in 2 and 11, subject to \(I(0)\) and the transversality condition. However, unlike the decentralized equilibrium, the planner’s optimality conditions are consistent with multiple steady states and dynamic paths that satisfy the transversality condition, so we instead opt to solve the allocation by computing the planner’s value function given the state \(I\).

2.4 SIS Results

Figure 2 depicts the law of motion for the fraction of infected agents in the population for our baseline calibration (left) and the economic-only loss scenario (right). The decentralized SIS economy converges to a unique steady state for any positive initial \(I(0) > 0\), which occurs where the law of motion intersects with the 45-degree line. This occurs around \(I = 1\%\) in the baseline scenario and around \(I = 20\%\) in the economic-loss only scenario. The left-hand side of Figure 3 shows the policy functions for \(a_I\) and \(a_S\) as a function of \(I\) in the decentralized equilibrium: infected agents disregard the infection externalities and engage in full activity \(a_I = 1\). By contrast, susceptible agents scale down their activity level in proportion to the cost and risk of infection they face, which is proportional to \(I\). Under the baseline scena-

\footnote{For illustration, we compute the law of motion from the continuous-time system on a discrete time grid with step size one, equivalent to a week in our calibration.}

\footnote{There is, of course, also a locally unstable steady state at \(I = 0\), at which the population is wholly disease-free.}
rio, susceptible agents quickly scale back activity as \( I \) rises. The steady state features \( a_S = 0.40 \), or equivalent to a permanent reduction in aggregate economic activity of approximately 15\%. When we consider only the economic loss, susceptible agents still scale back activity in steady state, \( a_S = 0.50 \).

The social planner, by contrast, chooses to contain the disease, both in the baseline and in the economic loss-only scenario. Starting from any \( I(0) > 0 \), the planner decreases the infection rate (green-circled lines in Figure 2) by reducing the activity level of both susceptible and infected agents (right panels in Figure 3), ensuring that \( I \to 0 \) asymptotically. For low \( I \), she focuses her risk mitigation on infected individuals. (If \( I \) rose above approximately one half, the planner would shift her mitigation efforts from infected agents to susceptible agents since this would impose the costs of containment on a smaller fraction of the population.)

The upper panels of Figure 4 simulate the paths of the SIS economy for initial \( I(0) = 0.3\% \) in the baseline parameterization while the bottom panels illustrate the case when only the economic losses from COVID-19 are considered. Under both scenarios, the solutions in the decentralized economy and under the planner diverge – the disease remains endemic in the decentralized economy, with the fraction of infected converging to an interior steady state, whereas the planner follows a strategy of containment. The middle panels show that the lives of susceptible agents quickly return to normal under the planner’s solution, whereas decentralized agents find

\[ \text{Aggregate activity is given by } \phi(Ia_I + Sa_S) + 1 - \phi. \]
Figure 3: Activity as a function of the measure of infected agents, in baseline (top panels) and economic-loss only (bottom panels)
it optimal to progressively reduce their activity as \( I \) rises. To accomplish a rapid containment, the planner isolates infected agents by reducing their economic activity to near zero, which mitigates the harm to the susceptible. The efficient solution relies on the planner’s ability to identify and isolate the sick, highlighting the role of effective testing and tracing, which we will explore in more detail below in section 3.4.

What is particularly interesting is that the shadow cost of an additional infection as perceived by the planner is greater than what decentralized agents perceive. Private agents disregard the infection externalities, whereas the planner recognizes that additional infections cost not only the affected agents but also pose a risk to others. In the baseline scenario, the cost of infection is an increasing function of \( I \) for both the decentralized and the planner’s allocation because more infections imply greater risk for the susceptible as well as more externalities and (for the planner) a higher cost of reducing the activity of infected agents.

The first two scenarios we consider illustrate how robust our findings are with
Figure 5: Law of motion for $I(t)$ under a low cost of disease $(c_0, \kappa) = (0.05, 0)$.

respect to how costly COVID-19 is. However, it is possible for optimal policy to allow the disease to spread and remain endemic with a steady-state featuring $I > 0$. To illustrate this scenario we set the cost of the disease considerably lower at $(c_0, \kappa) = (0.05, 0)$, equivalent to an expected flow cost of $60. Figure 5 illustrates the law of motion for the infection rate in the decentralized equilibrium (blue-solid line) and in the planner’s allocation (green-circled line). The decentralized equilibrium features an increasing path of infections to a steady state close to 60%. However the planner’s policy features a discontinuity around $I(0) = 0.16$: when the initial fraction of the population is sufficiently low, the planner chooses to eradicate the disease as in the baseline scenario. However, when the initial disease burden is higher, it is no longer optimal to incur the cost of eradication, and the planner instead chooses a steady state with a positive disease burden that is slightly below the steady state of decentralized agents, internalizing the infection externalities.

3 An SIR Economy

3.1 Model Setup

We expand the SIS model economy from above to account for the possibility that individuals who recover from COVID-19 acquire resistance to future infection, as suggested e.g. by Li et al. (2020).
**Epidemiology**  We denote the fraction of recovered/resistant individuals by $R$ and normalize the population to $S + I + R = 1$. The epidemiological laws of motion in our SIR model are

\begin{align}
\dot{S} &= -\beta (\cdot) IS \\
\dot{I} &= \beta (\cdot) IS - \gamma I \\
\dot{R} &= \gamma I
\end{align}

(19)  
(20)  
(21)

where the last compartment reflects that infected individuals recover at rate $\gamma$. Recovered/resistant is an absorbing state. In our derivations below, we will keep track of the state variables $I$ and $R$ and note that $S = 1 - I - R$.

**Individual Behavior**  The optimal activity level of resistant individuals $R$ is $a_R = 1$ since they can no longer become infected, generating flow utility $u_R = u(1)$. Given that this is constant, there is no change in the endogenous economic decision variables of agents, and the individual optimization problem continues to be given by equation (4).

Defining by $R = Pr(i = \mathcal{R})$ the individual’s probability of being resistant, the current-value Hamiltonian of individuals in the SIR model is

\[ H = I \left[ u(a_I) - c(\bar{I}) \right] + Ru_R + (1 - I - R) u(a_S) \\
- V_I \left[ \beta(a_I, a_S) \bar{I} (1 - I - R) - \gamma \bar{I} \right] + V_R [\gamma I], \]

(22)

The optimality conditions from the Hamiltonian are\(^{10}\)

\begin{align}
 u'(a_S) &= V_I \cdot \beta(\bar{a}_I \bar{I}) \\
 u'(a_I) &= 0 \\
 rV_I &= u(a_S) - u(a_I) + c(\bar{I}) - V_I \beta (\cdot) \bar{I} - (V_I + V_R) \gamma + \dot{V}_I \\
 rV_R &= u_R - u(a_S) + V_I \beta (\cdot) \bar{I} + \dot{V}_R
\end{align}

(23)  
(24)  
(25)  
(26)

plus the two transversality conditions $\lim_{T \to \infty} e^{-rT} V_I \cdot I = 0$ and $\lim_{T \to \infty} e^{-rT} V_R \cdot R = 0$.

\(^{10}\)Note that we define $V_I$ as a *shadow cost* but $V_R$ as a *shadow value* in the Hamiltonian; therefore the optimality conditions for the two are $rV_I = -\dot{H}_I + \dot{V}_I$ and $rV_R = +\dot{H}_R + \dot{V}_R$.  

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Definition 3 (Decentralized SIR Economy). For given $I(0)$ and $R(0)$, a decentralized equilibrium of the described system is given by a path of the epidemiological variables $I$ and $R$ that follow the epidemiological laws as well as paths of $(a_S, a_I)$ and $V_I, V_R$ that satisfy the optimization problem of individual agents.

3.2 Social Planner

Social welfare in the economy continues to be given by expression (8), where the expected flow utility $E_i[u_i(a_i)]$ of individuals is now calculated over the three epidemiological states $i = S, I, R$. The planner’s Hamiltonian is given by the equivalent to the decentralized Hamiltonian (22) with $\bar{I} = I$, $\bar{R} = R$ and $\bar{a}_I = a_I$, where we denote the shadow prices on the laws of motion for $I$ and $R$ by $W_I$ and $W_R$. The optimal activity level of resistant agents is trivially $a^*_R = 1$. The planner’s optimality conditions for $a^*_S$ and $a^*_I$ are equivalent to (9) and (10) with $S = 1 - I - R$. The optimality conditions describing the evolution of shadow prices are

\[
\begin{align*}
    rW_I &= u(a^*_S) - u(a^*_I) + c(I) + Ic'(I) + W_I \beta(\cdot)(1 - 2I - R) - (W_I + W_R)\gamma + \dot{W}_I \tag{27} \\
    rW_R &= u_R - u(a^*_S) + W_I \beta(\cdot)I + \dot{W}_R \tag{28}
\end{align*}
\]

Definition 4 (Planner’s Allocation in SIR Economy). For given $I(0)$ and $R(0)$, the planner’s allocation in the described SIR system is given by paths of the epidemiological variables $I$ and $R$ that follow the epidemiological laws as well as paths of $(a^*_S, a^*_I)$ and $W_I, W_R$ that satisfy the planner’s optimization problem.

Comparing the allocations of decentralized agents and the planner, we arrive at similar results on the differences in behavior as in the SIS model:

Proposition 2 (Infection Externalities in SIR Model). The planner internalizes the infection externalities of the infected and would choose a lower level of activity for infected agents, $a^*_I < a_I$, but the same (full) level of activity for recovered agent, $a^*_R = a_R = 1$. For given actions, the planner perceives a higher social cost of infection than private agents, $W_I > V_I$, but the same social value of being recovered as private agents.
Proof. See discussion above.

As in our discussion following Proposition 1, the planner’s effects on the activity level of susceptible agents depends on two competing forces: since the infected engage in less activity, the risk of infection for susceptible agents is lower, generating a force toward greater activity; however, for given actions, the planner recognizes a greater social loss from one more individual becoming infected, \( W_I > V_I \), generating a force toward lower levels of activity.

The social planner’s allocation can be decentralized in a similar fashion to what we discussed in Corollary 1 for the SIS economy:

**Corollary 2** (Decentralizing the SIR Economy). The planner can implement her allocation in a decentralized setting in the four ways discussed in Corollary 1. No intervention in the behavior of the recovered is necessary.

### 3.3 SIR Results

We keep the parameterization from the baseline scenario of Section 2.3 but now account for the fact that recovered individuals are resistant to re-infection. Computationally, we solve a non-linear four-dimensional boundary value problem in \((I, R, V_I, V_R)\) with conditions \(I(0) > 0\), \(R(0) = 0\) and the two transversality conditions. The boundary conditions for the planner’s solution are equivalent in the corresponding system in \((I, R, W_I, W_R)\), where again the algorithm must check for a global optimum across potentially multiple paths that satisfy the system given by equations (20), (21), (27), (29), and the boundary conditions.

**Baseline Scenario** Figure 6 illustrates the path of the disease in the decentralized and planner’s allocation starting from an initial infection rate of \(I(0) = 0.3\%\), and setting \(R(0) = 0\). In the decentralized economy, susceptible agents reduce their economic activity but infections continue to rise for the first 12 weeks. As the higher fraction of infected increases the risk for susceptible agents, they continue to reduce their economic activity until infection activity peaks. Subsequently, the rising number of recovered agents in the population together with still very cautious behavior by the susceptible leads to a decline in the fraction of infected, allowing susceptible agents to increase economic activity again. One striking observation is
that even after two years, the epidemic is still ongoing: the fraction of infected in the population is still 0.5%, whereas close to half of the population has recovered and acquired resistance (middle panel).

Taken together, one could say that the extremely cautious behavior of the susceptible has “flattened the curve,” but ultimately the mechanism that overcomes the epidemic is to acquire herd immunity, i.e. to acquire sufficient resistance in the population so that the epidemic dies out. Given the externalities, infected agents simply do not find it individually rational to engage in the severe measures that would be necessary to contain the disease.

The planner, by contrast, aims to contain and eradicate the disease as quickly as possible by reducing activity by the infected to close to zero, even though this imposes a stark utility cost on infected agents given the Inada condition \( \lim_{a \to 0} u_I'(a) = \infty \). However, observe that the isolation of the infected allows the planner preserve greater activity for the susceptible agents – who make up the majority of the population. In short, one could say that the planner’s strategy to overcome the epidemic is containment and eradication, i.e. to drive down the number of infected sufficiently low so that it no longer poses a risk to the susceptible, even though they never acquire herd immunity. This illustrates the stark difference in how the disease is overcome by decentralized agents versus the planner.

The welfare gains from following the optimal containment strategy are large. With an initial infection rate of 0.3% of the population, the gain in social welfare is equivalent to $29k per person, or roughly $10 trillion aggregating across the population in the U.S. The optimal containment strategy effectively saves both lives and lost output since susceptible agents are able to return to near full activity. Even if there is a delay in implementing the optimal strategy, the welfare gains remain substantial. Suppose the disease spreads under no intervention for 24 weeks, which implies an infection rate of \( I = 1.9\% \) and a susceptible population of \( S = 83\% \). The gain in social welfare from moving to a smart containment is equivalent to $25k per person or roughly $8 trillion in the aggregate.

These results crucially hinge on the assumption that the epidemiological status of individuals is observable. In practice, widespread shortages in testing capacity as well as the considerable number of asymptomatic cases that are still potentially able to spread the disease currently make it difficult to implement what we have characterized as the planner’s optimal strategy. For comparison, we consider the
Figure 6: Dynamic paths starting from $I(0) = 0.01$ under the baseline scenario.

In the case in which the epidemiological status of individuals is unobservable in Section 3.4.

To provide additional intuition on the differences between the decentralized outcome and the planner’s solution, the left-hand panel of Figure 7 illustrates how private agents and the planner perceive the marginal cost of an additional infection $V_I$ versus $W_I$. At the initial level of infected $I(0) = 0.3\%$, private agents perceive the cost of infection to be only $18k$ (using the first-order approximation discussed in Section 2.3 to convert utils into dollar values). The planner, by contrast, perceives a much larger social cost of an additional infection corresponding to approximately $55k$ – three times higher than that of decentralized agents – for three reasons: First, she internalizes that infected agents spread the disease to others who in turn pass them on to more agents. Secondly she induces infected agents to starkly reduce their level of economic activity. Third, the planner internalizes that the cost of disease $C(I)$ increases in $I$ as a rising case load risks overwhelming the capacity of the healthcare system.

The right-hand panel of Figure 7 illustrates the policy functions for economic activity $a_S(I,R)$ and $a_I(I,R)$ for varying $I$ while holding $R = 0$. Since an increase in $I$ exposes susceptible agents to higher infection risk, they strongly scale back their economic activity in the decentralized equilibrium. For an infection rate of $I = 0.3\%$, susceptible agents cut back physical activity from a normal level of $1.00$ to $a_S = 0.80$; for $I = 5\%$, they cut activity to $a_S = 0.25$. By contrast, the planner reduces the economic activity of the infected to near zero while maintaining activity for the susceptible near normal levels.
Figure 7: Cost of disease and economic activity (for $R = 0$) as a function of $I$.

Economic Cost Only  Figure 8 illustrates an alternative scenario in which we only consider the purely economic cost of the disease with $c(I) = c_0 = 0.61$. The planner’s solution is nearly identical, with rapid containment and elimination of the disease. By contrast, the disease spreads more rapidly in the decentralized economy since susceptible agents engage in less precautionary behavior when the cost of disease is lower. They cut back on activity in proportion to the fraction $I$, which drives their risk of infection. In the long-run, over 75% of the population experience an infection (middle-panel) compared to a bit over 60% in the baseline. There continues to be a discrepancy between the private and social shadow cost of an infection $V_I$ and $W_I$ – the two differ by a factor of almost six as private agents do not internalize the infection externalities that are now greater, given less precautionary behavior of the susceptible population.

3.4 Hidden Epidemiological Status

Following a containment and elimination strategy that focuses on the infected, as we found optimal in our analysis above, requires that the epidemiological status of individuals is readily identifiable. This has been a challenge, not only because COVID-19 has a long incubation period, up to 14 days, and a significant fraction of infected individuals are asymptomatic (Verity et al., 2020), but also because many countries, including the US, have suffered from shortages in testing kits and tracing...
Figure 8: Dynamic paths starting from $I(0) = 0.3\%$ under $c_0 = 0.61$ and $\kappa = 0$. capabilities. Whereas our baseline model assumed that individuals and the planner can easily target their chosen actions to whether a given individual is susceptible or infected, the reality is that many are unaware of their epidemiological status. To analyze the implications of this lack of information, we now consider the extreme case that the epidemiological status $i$ of an individual is hidden so that the planner needs to choose a uniform level of activity $\hat{a}$ that does not depend on epidemiological status.

This modifies the Hamiltonian (22) of both individual agents and the planner so that there is just a single decision variable $\hat{a}$ that replaces $a_S, a_I$ and $a_R$,

$$H = u(\hat{a}) - Ic(\bar{I}) - V_I [\beta(\hat{a}, \bar{\hat{a}}) \bar{I} (1 - I - R) - \gamma \bar{I}] + V_R [\gamma I]$$

The optimality condition for individual agents with respect to $\hat{a}$ is

$$u'(\hat{a}) = V_I \cdot \beta_0 \bar{I} (1 - I - R)$$

By contrast, the planner’s optimality condition becomes

$$u'(\hat{a}) = 2W_I \cdot \beta_0 I (1 - I - R)$$

Comparing the the two conditions, we find:

**Proposition 3 (Infection Externalities with Hidden Status).** In the model with hidden epidemiological status, the planner internalizes twice the infection risk perceived by decentralized agents for a given cost of infection. Furthermore, for given actions,
Figure 9: Dynamic paths starting from $I(0) = 0.3\%$ under the baseline scenario, including optimal policy under hidden status.

The planner perceives a higher social cost of infection than private agents, $W_I > V_I$.

**Proof.** See proofs and discussion above.

The reason why the planner internalizes twice the expected cost of infection is that she recognizes that it is not only the actions of the susceptible that matter but also the actions of the infected agents.

**Numerical Results in Baseline Scenario** Figure 9 illustrates the dynamic path of the disease starting from an infection rate of $0.3\%$.\footnote{We thank Rob Shimer for a helpful conversation on the long-run dynamics of the planner's problem in this case.} The left and middle panels reproduce the paths of infections and levels of activity from Figure 6 and add (red dash-dotted) lines for the planner who cannot distinguish the epidemiological status of individuals. The planner still contains the virus, but the path of infections is slightly higher compared to the optimal planning scenario. Containment must now be achieved through a reduction in the level of activity of all agents. The middle panel shows that the level of activity of all agents is initially reduced to just above 50% of the normal level, increasing only slightly over the span of 25 weeks, but never rising above 63% over the first two years.

The reason is that the planner chooses an activity level that first contains the disease and then, when the fraction infected is close to zero, ensures that the disease...
Figure 10: The dynamic path of the disease reproduction number, $R_t = \beta(\cdot)S\gamma^{-1}$, starting from $I(0) = 0.3\%$ under the baseline scenario, including optimal policy under hidden status.

The reproduction number remains close to one $R_t \approx 1$, where

$$R_t = \frac{\beta(\hat{a}, \hat{a})S}{\gamma} = \frac{\beta_0 \hat{a}^2 S}{\gamma}$$

Transforming the inequality above, the planner must set activity around $\hat{a} \approx \sqrt{\gamma/\beta_0 S}$ in order to keep the disease contained. Initially, when $S \approx 1$, this implies $\hat{a} \approx \sqrt{\gamma/\beta_0} \approx 0.63$. Figure 10 illustrates the disease reproduction number $R_t$ along the dynamic paths of the three allocations.

The right panel of Figure 9 illustrates the impact on aggregate output (which includes the fraction $1 - \phi$ of output that does not require physical/social interaction). When the planner must resort to blunt measures that are independent of epidemiological status, she induces a recession that is larger than in the decentralized economy. Aggregate output is initially reduced by 11% and returns to 9% below normal after two years. The long-term prospects are grim since the planner does not allow the level of physical activity to go above 63% for many years.

The hidden epidemiological status also raises the social cost of an additional infection, as shown in the left-panel of Figure 11. At an initial infection rate of 0.3%, the social cost of an extra infection is near $90k$, 50% higher compared to when the planner can separately reduce the activity of the infected but 4.5 times as high as what is perceived by decentralized agents who know their epidemiological status. For a lower infection rate 0.001%, the social cost of an extra infection under hidden
status increases to $200k compared to a social cost of $52k when epidemiological status is observable.

The planner recognizes that even for a small initial outbreak, she must impose economic costs across all agents, which leads to large social losses. The right panel of Figure 11 shows that the planner, under hidden epidemiological status (red dash-dotted line), reduces economic activity by 48% at an infection rate of 0.3% and by up to 75% if the fraction infected is 5%.

**Economic Cost-Only Scenario**  When the cost of the disease is lower, the planner finds it optimal to let it spread at a faster pace, albeit slower than in the decentralized equilibrium. Figure 12 illustrates that if the policymaker and individuals only consider the economic loss associated with infections, then optimal policy under hidden status leads to herd immunity within the first two years. The left panel shows that the infection rate peaks around 11% after 11 weeks, versus a peak infection of 18% in the decentralized equilibrium so the planner “flattens the curve.” The middle panel shows that the planner reduces activity for all individuals by up to 26%, then more gradually relaxes restrictions as herd immunity is built within two years. Since the infected still maintain full activity in the decentralized equilibrium, the aggregate output loss is larger under the blind containment policy, as shown in the right panel. Under this scenario, there are still large welfare gains from moving
Figure 12: Dynamic paths starting from $I(0) = 0.3\%$ under the economic-loss only scenario, including optimal policy under hidden status.

to a smart containment strategy – at an initial infection rate of 0.3%, the overall social gain is $3.5k$ per individual or $1.1$ trillion for the US economy.

### 3.5 Private versus Social Gains from Vaccination

The economic damage imposed by the virus will depend heavily on how soon a vaccine is developed. Individually rational susceptible agents have incentives to become vaccinated in order to avoid the risk of infection. In our SIR model, the benefit perceived by an individual of moving from susceptible to resistant is captured by $V_R$ in equation (26). The flow gains over time are captured by two terms. The first term, $u_R - u(a_S)$, captures that recovered agents do not have to distance themselves in order to avoid becoming infected. The second term, $V_I\beta(·)I$, captures the expected gain from avoiding the risk of infection.

The left panel of Figure 13 illustrates the private gain from an individual becoming vaccinated (solid-blue line) when 1% of the population is infected, for $R \in [0, 0.99]$. If no one has immunity initially, the private gain from an individual becoming vaccinated is equivalent to $9k$. The gain falls as more of the population becomes resistant/recovered as this reduces the risk of infection. Around $R = 0.6$ the population acquires herd immunity, and the private gain to an additional vaccination declines rapidly as the infection risk of susceptible individuals becomes negligible.

However, since private agents do not internalize the infection externality, the
social gain of an additional vaccination in the decentralized economy is many times larger, shown as the dashed-purple line in the left panel of 13, which reflects $W_R$, the planner’s willingness to pay to transition an agent from susceptible to resistant/recovered, taking as given the private actions of agents. At zero immunity the social gain from an additional vaccination is $70k, nearly 8 times larger than what private agents are willing to pay. As more of the population becomes resistant the social gains from additional vaccinations fall. Around the level of herd immunity, $W_R$ declines sharply: from $20k at $R = 0.6$ to merely $5k at $R = 0.75$.

The societal gain from the entire population becoming vaccinated is represented as the integral of the marginal social gain (dashsed-purple line) from $R = 0$ to 0.99, equivalent to $29k per individual under the decentralized equilibrium or roughly $9.5 trillion when multiplied by the population of the U.S. Even though it is projected to take a year or more to develop a vaccine, the gains are large. If the disease is allowed to spread without intervention, as captured by the decentralized equilibrium in section (3.1), the model predicts a resistant population of around 30% and an infection rate of 1.2% after one year. The social gain from rolling out a vaccine then is $17k per individual or $5.5 trillion in the aggregate.

The right panel of Figure 13 illustrates that the social gains from vaccination depend crucially on what strategy society adopts to contain the disease. The green line marked with circles plots the social benefit of an extra vaccination, $W_R$, under the assumption that the planner employs the optimal containment strategy descri-
bed in section 3.2. If the disease is successfully contained, the social gain of a vaccine is small. This illustrates that smart containment and vaccination are substitutes. However, when the epidemiological status of individuals is hidden and blunt containment is the only containment strategy available, the social gain from a marginal vaccination $\hat{W}_R$ is significantly larger, around $51k$ at zero immunity. The gain from vaccinating the population is then $22k$ per individual or $7.2$ trillion in the aggregate.

4 Conclusions

We integrate economic activity into epidemiological SIS and SIR models in order to analyze and quantify the externalities that arise. Our main finding is that infected agents who behave individually rationally generate large externalities because they do not internalize the effects of their economic and social activities on the infection risk of others and therefore engage in inadequate social distancing. We calibrate our model to capture the main features of COVID-19 and the US economy, and we find that private agents perceive the cost of an additional infection to be around $18k$ whereas the true social cost is more than three times higher, around $55k$.

Whether or not these externalities are internalized has crucial implications for how society responds to the disease: individually rational agents follow a “herd immunity” strategy whereby the disease spreads through the population until herd immunity is acquired. Along the way, susceptible agents significantly reduce their economic activity, generating a protracted economic slump. By contrast, optimal public health measures follow a “smart containment” strategy to isolate the infected by reducing their social activity close to zero while only slightly reducing the activity of the susceptible. This leads to a quick reduction in the number of infected agents and an overall mild impact on aggregate output. Alternatively, if the planner cannot make policy contingent on the epidemiological status of individuals, for instance because of the difficulty tracking asymptomatic cases of COVID-19 and the lack of sufficient testing, the second-best optimal policy is a “blind containment” strategy that sharply reduces activity across all agents, at significantly larger economic cost.

We leave several possible extensions for future work: First, it would be useful to refine our epidemiological models to account for additional aspects of the SARS-
CoV-2 virus. For example, including separate compartments for exposed agents $E$ and asymptomatic agents $A$ would make it possible to explicitly account for the long incubation period of COVID-19 and the difficulty targeting public health measures at asymptomatic cases. Accounting for spatial heterogeneity would make it possible to better capture the dynamics of the disease in a large country such as the US and to analyze the benefits and costs of travel restrictions. Moreover, since the case fatality rate of COVID-19 differs so strongly for patients of different age, accounting for different age groups would make it possible to analyze how the externalities by age group differ.

Secondly, it would be useful to refine the analysis of the macroeconomic feedback effects of the reductions in social and economic activity that we analyze. For example, Guerrieri et al. (2020) show that feedback effects in a multi-sector economy with financial market imperfections may lead to an amplification of the initial shock generated by social distancing. This provides valuable insights to policymakers on the optimal macroeconomic response that complement the insights on epidemiological containment strategies that we focus on.

References


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<th>Age group</th>
<th>Population</th>
<th>Life Expectancy</th>
<th>Value of statistical life*</th>
<th>Infection fatality rate</th>
<th>E[loss] given infection*</th>
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* in thousands of USD

**Sources:**
Case fatality rate: Verity et al. (2020), Table 1
Table A2: Calculation of population-weighted expected income loss in the US given infection

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<thead>
<tr>
<th>Age group</th>
<th>Population</th>
<th>Current weekly income</th>
<th>PDV of E[income]</th>
<th>Infection fatality rate</th>
<th>Total income loss</th>
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<tr>
<td>0–9</td>
<td>40.01</td>
<td>$ -</td>
<td>$ 559,116</td>
<td>0.002%</td>
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<td>10–19</td>
<td>41.97</td>
<td>$ 29.44</td>
<td>$ 828,056</td>
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<td>$ 102</td>
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<td>20–29</td>
<td>45.43</td>
<td>$ 503.12</td>
<td>$ 1,105,018</td>
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<td>30–39</td>
<td>43.69</td>
<td>$ 882.50</td>
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<tr>
<td>40–49</td>
<td>40.46</td>
<td>$ 1,010.45</td>
<td>$ 924,253</td>
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<td>$ 965.87</td>
<td>$ 585,571</td>
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<td>$ 153.26</td>
<td>$ 45,124</td>
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* in thousands of USD

**Sources:**
Income data: CPS 2019 March Supplement, Total wage and salary earnings
Case fatality rate: Verity et al. (2020), Table 1

**Note:** Total income loss is calculated as
3*[Weekly income]*50% + (PDV of E[income]) * [Infection fatality rate]